Inclusion-Exclusion

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1 Inclusion-Exclusion

The inclusion-exclusion principle says that

$$|A_1 \cup A_2 \cup \ldots \cup A_k| = \sum_{\ell=1}^{k} (-1)^{\ell+1} \sum_{1 \le i_1 < i_2 < \ldots < i_\ell \le k} |A_{i_1} \cap A_{i_2} \cap \ldots \cap A_{i_\ell}|.$$

This allows you to find the size of the union of a bunch of (possibly overlapping) sets from the sizes of their intersections.

Things to note:

- You often want to use this to compute the size of the complement of this union. Here the A_i's represent events that you want to avoid.
- This principle is often useful when the A_i's can be thought of as imposing some relatively easy to deal with condition. This can make it easy to count intersections (where you impose multiple conditions simultaneously) but hard to count unions where one of several conditions is imposed.
- The sum contains exponentially many terms, but often all of the ℓ-wise intersections have the same size. Then you can just multiply the size of a single intersection by $\binom{k}{\ell}$.