Permutations and Cycle Structure

Spring 2022

Some basic points to keep in mind about permutations and cycle structure:

- Permutations can be thought of in several ways: as orderings of the numbers from 1 to n, as bijections of $[n]$ to itself, or as ways of partitioning $[n]$ into cycles.
- Thinking of a permutation in terms of cycles is useful in understanding what happens when you apply the function repeatedly.
- Using the usual cycle notation, there are many ways to write a permutation. Using canonical cycle notation, there is only one way, and it can be parsed without the parentheses.
- Writing permutations in canonical cycle notation is useful for a number of cycle-related counting problems.
- The unsigned Sterling number of the first kind $c(n, k)$ is the number of permutations of $[n]$ with exactly k cycles.