Distinguishability and Balls in Bins

Spring 2022

1 Distinguishability

A lot of subtle issues in counting problems come down to questions of distinguishability. When do we consider two objects (or two constructions of an object) to be the same object? For example, the number of ways to pick a set of k distinct elements from $[n]$ and the number of ways to pick a sequence of k distinct elements from n are quite different. This is because although both product k distinct elements, if you reorder the sequence you get the same set. In fact, if you are careful, this can be used to count the number of subsets. There are $(n)_k$ many sequences, and each subset can be written in $k!$ different ways as a sequence (since there are $k!$) ways to order it. Thus, the number of subsets is $(n)_k/k!$. When dealing with these kinds of issues it helps to be very careful about stating exactly what you are counting and about how many different ways your counting method can generate the same final object.

2 Balls in Bins

The different balls in bins counting problems hinge on notions of distinguishability. If we swap two balls with each other, does that count as the same configuration as before? What if we swap two bins? In general the counting problems that we looked at correspond roughly to:

- Distinguishable balls in distinguishable boxes: Counting strings or sequences
- Indistinguishable balls into distinguishable boxes: Counting compositions or weak compositions
- Distinguishable balls into indistinguishable boxes: Counting set partitions
- Indistinguishable balls into indistinguishable boxes: Counting integer paritions