Balls in Bins Problems

Fall 2026

1 Distinguishability

The basic problem is that of counting how many ways one can put n balls into k bins with each ball assigned to one bin. However, this is complicated by issues of distinguishability. Are all the balls the same? What about the bins? This affects the count as we need to know if swapping two balls counts as a different configuration or not.

2 Functions

If balls and bins are both distinguishable, configurations correspond to functions, which are easily counted.

3 Compositions

If the bins are distinguishable but not the balls, then you can count the number of balls in the $i-th$ bin for each i. This means that our configurations correspond to (weak) compositions. That is ordered sequences of numbers with a fixed sum. These can be counted using the sticks-and-stones idea giving appropriate binomial coefficients.

4 Set Partitions

When the balls are distinguishable but the bins aren't, you can see how the bins partition the balls into different groups. Thus, the configurations correspond to set partitions. The number of set partitions of a set of size n into k nonempty subsets is equal to the Sterling number of the second kind, $S(n, k)$, while the total number of set partitions of a set of size n is given by the Bell number $B(n)$. While neither of these numbers have a clean formula, we did discover some nice recurrence relations that allow you to compute both.

5 Integer Partitions

When neither balls nor bins are distinguishable, you can see an unordered sequence of bin counts. This makes configurations correspond to integer partitions, that is an unordered sequence of numbers (or equivalently a non-increasing sequence of numbers) with a given sum. These also correspond to Ferrer's diagrams and lead to some interesting relations between different kinds of partitions.