Pigeonhole Principle and Basic Counting Techniques

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1 Pigeonhole Principle

The pigeonhole principle is a powerful technique for showing that you can find two things that are "close". Some things to keep in mind when trying to use it:

- What are your pigeons? What are the objects that you need to be close?
- What are your holes? You need to pick holes so that two pigeons landing in the same hole means that they are close in whatever sense you care about.
- How are you mapping pigeons to holes?
- How many pigeons/holes are there? You need to have more pigeons than holes in order for pigeonhole to apply, and many more for generalized pigeonhole.

2 Basic Counting Techniques

Basic counting problems often require careful thought about how to decompose the object being counted into things that can be handled by the standard rules. Sometimes this requires making "choices" in the right order. Sometimes, this requires finding a new way to think about the thing being counted. Sometimes it requires relating what you are counting to something easier to analyze. Sometimes it requires extensive case analysis.

2.1 Basic Rules

1. Addition Rule: If you can break your problem into disjoint sets of possibilities, your answer should be the sum of the sizes of the sets.
2. (Generalized) Multiplication Rule: If you can break down construction of your object into making a series of choices where the number of options for each choice does not depend on the answers to the previous choices, the final count will be the product of the number of options for each choice.
3. Exponentials and Factorials: Often occur when repeatedly applying the multiplication rule.
4. Counting Things in Two Different Ways: Generally a powerful technique. If you can find two ways to count the same thing, the results must be equal.
5. Binomial Coefficients: $\binom{n}{k}$ is the number of subsets of size k of a set of size n. These show up frequently.