

# Counting Techniques

Spring 2022

A fair bit of this class will involve trying to count various sets of objects. Very often doing so will involve finding the right way to break the problem down in the right way. Here are some tips for approaching it:

- Is there a way to break the objects down into types that can more easily be counted separately? If these types are distinct you can use the addition rule, but if they aren't you'll need to use the more complicated inclusion-exclusion which we will cover later in the class.
- Is there a way to construct an object by making a series of choices, allowing one to apply the (generalized) multiplication rule? Remember that for this to work, the number of options available at each step cannot depend on the choices made in the previous steps. Note that sometimes, making this work involves figuring out the right *order* to make these choices in.
- Can you make the problem easier by making the objects more distinguishable? This will give an over count, but maybe you can determine how much it is overcounting by. More generally, can you relate the count you are trying to find to something easier, using counting in two different ways?
- Can you find other equivalent ways of looking at the thing you are trying to count? Sometimes different formulations will be easier or harder.
- Can you relate the thing you are trying to count to some other known combinatorial quantities like binomial coefficients or Sterling numbers?
- Can you relate the answer to your problem to the answers to simpler versions of the same problem? Maybe you can find a recurrence relation that you can solve.
- Can you use algebra and known combinatorial identities to simplify formulas that you have found?