

Math 154 Homework 5

Spring 2020

This homework is due on gradescope by Sunday May 31st at 11:59pm pacific time. Remember to justify your work even if the problem does not explicitly say so. Writing your solutions in L^AT_EX is recommended though not required.

Please cite any other students with whom you collaborated on any problems.

Question 1 (Round Robin Tournaments, 40 points). *Callum is running a tournament. In this tournament there are n different teams competing. Callum needs to organize a schedule in which each team plays every other team, but so that no team plays more than one game in any given week. He is trying to arrange for this to happen with the total length of the tournament being as few weeks as possible.*

- (a) *Show that any schedule in which the tournament takes a total of m weeks corresponds to an edge-coloring of K_n with at most m colors. Conclude that for each n the minimum number of weeks needed for this tournament is either $n - 1$ or n . [10 points]*
- (b) *Show that for n odd, there is no schedule for $n - 1$ weeks and that every n week schedule will have exactly one team sitting out each week. [15 points]*
- (c) *Show that for n even, there is a schedule using only $n - 1$ weeks. [15 points]*

Question 2 (Color Maps with Disconnected Regions, 40 points). *The four color theorem states that any map with connected countries each country can be colored one of four colors so that no two countries that share a boarder are the same color. What happens if the countries do not need to be connected?*

- (a) *Show that if the countries can be arbitrary disconnected sets that an unlimited number of colors may be necessary. [10 points]*
- (b) *Show that if each country can have at most 2 disconnected regions that 12 colors is sufficient. Hint: Show that the relevant graph has minimum degree at most 11. [15 points]*
- (c) *Give an example of a map where each country has at most 2 disconnected regions for which 9 or more colors are required. [15 points]*

Question 3 (Number of Squares Attacked by Bishops, 20 points). *On an $n \times n$ chessboard are placed a number of bishops (a piece that can move diagonally). Show that the total number of squares attacked or occupied by these bishops is at least $kn - k^2$ unless there are some $k - 1$ diagonals (of either orientation) so that each bishop is on at least one of these diagonals. Hint: Each bishop is on two diagonals. Although some of these diagonals might be short, for each bishop, the sum of the lengths of its diagonals is at least n . Try to find many bishops that do not share any diagonals.*

Question 4 (Extra credit, 1 point). *Where have I been getting the character names for my homework problems?*