Math 154 Homework 4

Spring 2020

This homework is due on gradescope by Sunday May 10th at 11:59pm pacific time. Remember to justify your work even if the problem does not explicitly say so. Writing your solutions in LaTeX is recommend though not required.

Please cite any other students with whom you collaborated on any problems.

Question 1 (Blocks and Planar Graphs, 30 points). *Let G be a finite, connected graph. Show that G is planar if and only if every block of G is planar.*

Question 2 (Cut Sets in Hypercubes, 30 points). *Let G be the hypercube graph where the vertices set is $V = \{0,1\}^n$, the set of length n strings of 0's and 1's, and there is an edge between two vertices if and only if they differ in exactly one location. Prove that in G we have that $\kappa((0,0,\ldots,0),(1,1,\ldots,1)) = n$.*

Question 3 (Bipartite Planar Graphs, 40 points). *(a) Show that a connected, planar graph G is bipartite if and only if each face in a planar embedding of G has an even number of sides. [30 points]*

(b) Ezran draws a number of circles in the plane, separating it into separate regions. Show that it is possible for him to color those regions black and white so that no two adjacent regions are the same color. [10 points]

Question 4 (Extra credit, 1 point). *Approximately how much time did you spend on this homework?*