

# Math 154 Homework 3

Spring 2020

This homework is due on gradescope by Sunday April 26th at 11:59pm pacific time. Remember to justify your work even if the problem does not explicitly say so. Writing your solutions in L<sup>A</sup>T<sub>E</sub>X is recommended though not required.

Please cite any other students with whom you collaborated on any problems.

**Question 1** (Caley's Theorem and Degree Sequences, 30 points). *Recall that our proof of Caley's Theorem associated to each tree with vertices labelled  $1, 2, \dots, n$  as list of  $n - 2$  numbers from 1 to  $n$ . Show that if a tree  $T$  corresponds to a list of numbers  $L$ , that for every  $1 \leq k \leq n$ , the degree of vertex  $k$  in  $T$  is equal to one plus the number of times that  $k$  appears in the list  $L$ .*

**Question 2** (Eulerian Directed Graphs, 40 points). *We only consider a directed graph to be Eulerian if the circuit that uses each edge exactly once only follows edges in the specified directions (thinking of a directed edge as a 1-way road). The condition for when a directed graph is Eulerian is now somewhat different. Prove that a connected, directed graph  $G$  has an Eulerian circuit if and only if for every vertex  $v$  of  $G$ ,  $d_{in}(v) = d_{out}(v)$ , where  $d_{in}(v)$  is the number of edges pointing into  $v$  and  $d_{out}(v)$  is the number of edges pointing out of  $v$ .*

**Question 3** (4-Regular non-Hamiltonian Graphs, 30 points). *Give an example of a connected, 4-regular, non-Hamiltonian graph. Prove that it is not Hamiltonian.*

**Question 4** (Extra credit, 1 point). *Approximately how much time did you spend on this homework?*