Math 154 Homework 2

Spring 2020

This homework is due on gradescope by Sunday April 19th at 11:59pm pacific time. Remember to justify your work even if the problem does not explicitly say so. Writing your solutions in LaTeXis recommend though not required.

Please cite any other students with whom you collaborated on any problems.

Question 1 (Bipartite Graphs and Spanning Trees, 30 points). *Suppose that G is a connected bipartite graph and T a spanning tree. Show that the partition of the vertices of G that make it a bipartite graph can be determined uniquely from T.*

Question 2 (Trees of Order 6, 30 points). *Two graphs G and H are* isomorphic *if there is a bijection $f : V_G \to V_H$ between their vertices so that there is an edge between vertices u and v in G if and only if there is an edge between $f(u)$ and $f(v)$ in H. In other words G and H are isomorphic if they are the same up to the way that they are drawn and the names given to the edges/vertices.*

Determine all of the isomorphism types of trees on 6 vertices (in other words, give a list of 6-vertex trees no two of which are isomorphic so that any other 6-vertex tree is isomorphic to one on your list). Make sure to justify that your list is correct.

Hint: It may be useful to consider the degrees of the vertices with degree more than 2 in your graph. By the Handshake Lemma, there cannot be many of them. The full list should contain 6 trees.

Question 3 (Road Capacities and Maximum Spanning Trees, 40 points). *Rayla runs a shipping company. The roads of the city in which she operates are represented by a connected graph G. Unfortunately, some of her trucks are too large to fit on all of the roads. Each road (represented by an edge e in G) has a weight w_e giving the largest size of a truck that can be supported on that road. Rayla wants an easy way to determine for any two vertices v and u in G what the maximum size of a truck would be that could be driven from v to u only on roads without going over the size limit of any road on the path.*

Let T be a maximum spanning tree of G (that is a spanning tree whose total weight is as large *as possible). Show that the largest truck that can make it from v to u is always the same as the minimum edge weight on the unique path from v to u in T.*

Question 4 (Extra credit, 1 point). *Approximately how much time did you spend on this homework?*