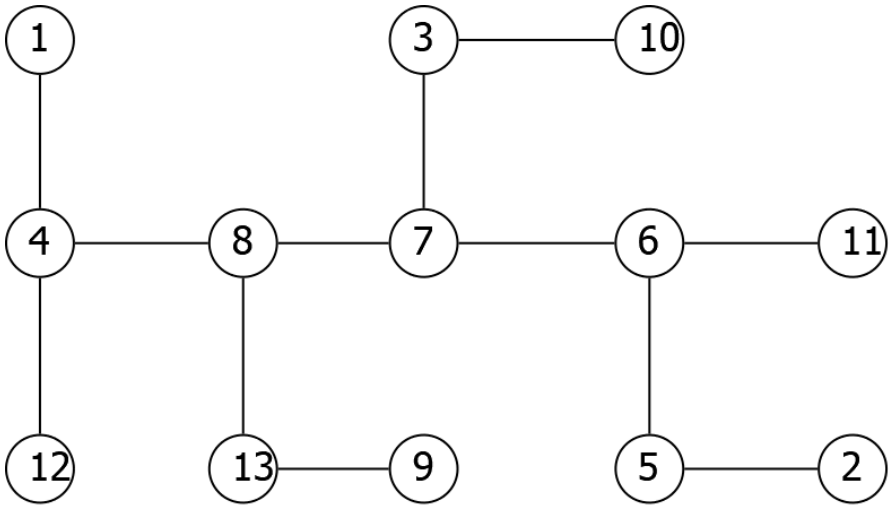
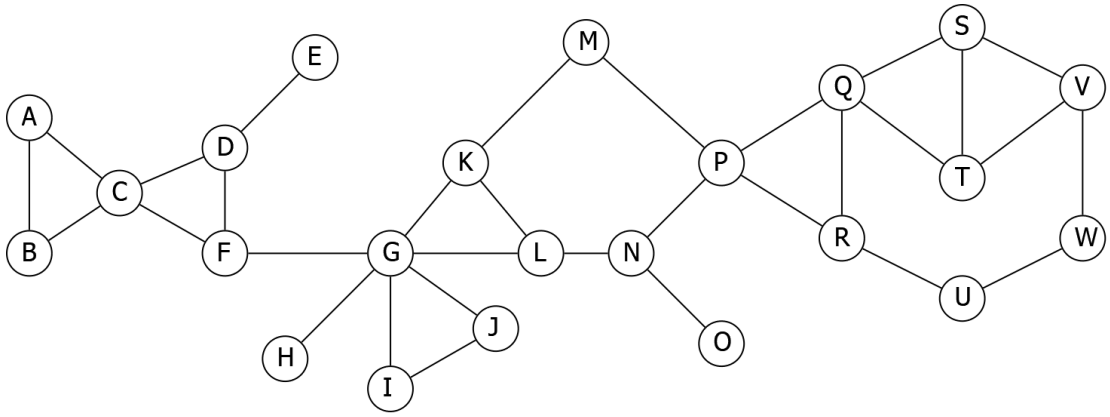
Math 154 Final Exam

Spring 2020

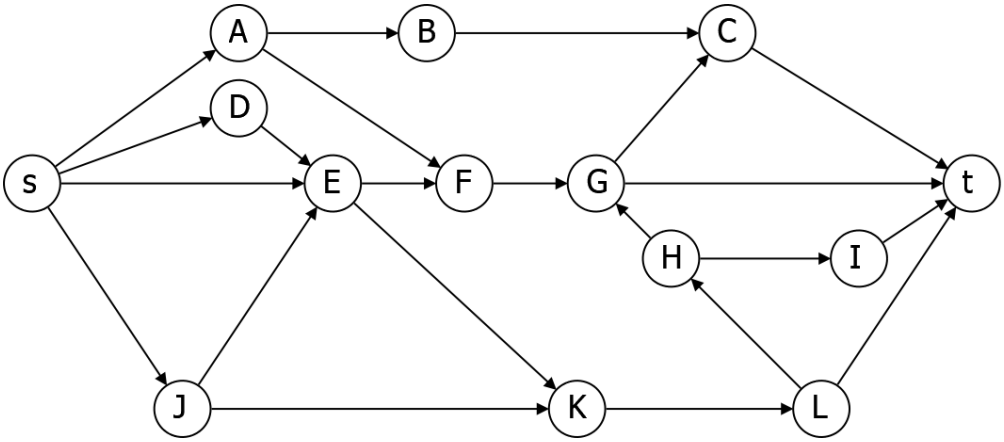
Question 1 (Tree List, 25 points)**.** *Our proof of Caley's Theorem described how to associate a list of numbers to a labelled tree. What is the list corresponding to the tree given below?*



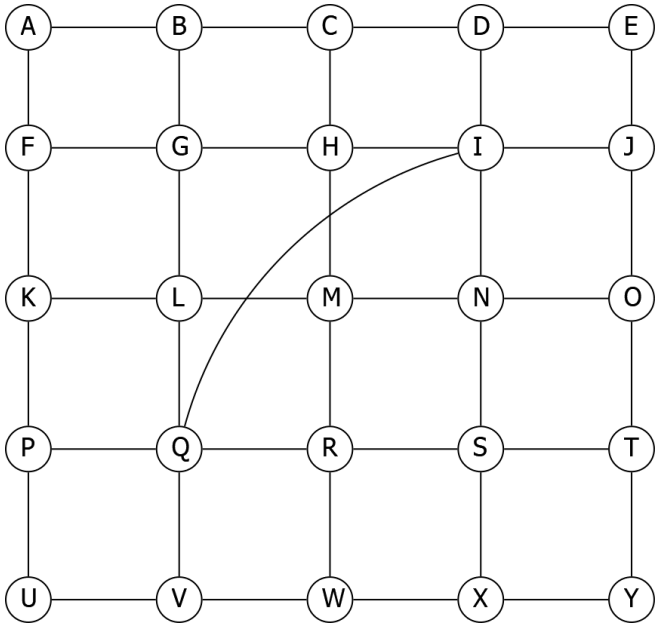
Question 2 (Block Decomposition, 25 points)**.** *What are the blocks in the graph below?*



Question 3 (Minimum Cut, 25 points)**.** *Give a minimum cut for the following network.*



Question 4 (Planarity, 25 points)**.** *Prove that the graph below is not planar.*



Question 5 (Sum of Eulerian Graphs, 25 points)**.** *Suppose that G and H are two connected, Eulerian graphs on the same vertex set that do not share any edges. Show that $G \cup H$ is Eulerian where $G \cup H$ denotes the graph whose edges are all of those in either G or H.*

Question 6 (Chromatic Number of Unions, 25 points)**.** *Let G and H be two graphs sharing the same vertex set. Let $G \cup H$ denote the graph consisting of the union of their edges. Show that $\chi(G \cup H) \leq \chi(G) + \tau(H)$ where $\tau(H)$ denotes the size of the minimum vertex cover of H.*

Question 7 (Perfect Matchings and Degrees of Independent Triples, 25 points)**.** *Let G be a connected graph on $2n$ vertices. Suppose that for any vertices u, v, w of G with no two of them connected by an edge, $d(u) + d(v) + d(w) \geq 3n - 2$. Show that G has a perfect matching.*
Hint: If G doesn't have a perfect matching, you have a set S with $|S| < \Omega(G - S)$. Consider u, v and w to be vertices in the three smallest connected components of $G - S$.

Question 8 (Cycle and Star Ramsey Numbers, 25 points)**.** *Show for any positive integer n that $R(C_{2n}, K_{1,n}) = 2n$.*
Hint: There is a result from early in the course that will be useful here.

Question 9 (Timing)**.** *If you have permission to take the exam at a non-standard time or have OSD accommodations for extended time, please list the time that you started the exam and the time that the exam should be due. Please list both in pacific time. Note that if you do not have pervious permission, you must take the exam at the regular time.*