

Math 154 Homework 1

Fall 2021

This homework is due on gradescope by Friday October 1st at 11:59pm pacific time. Remember to justify your work even if the problem does not explicitly say so. Writing your solutions in L^AT_EX is recommended though not required.

Please cite any other students with whom you collaborated on any problems.

Question 1 (2-Regular Graphs, 25 points). *Show that any finite, 2-regular graph G is a disjoint union of cycles. In particular, show that G has a number of induced subgraphs that are cycles and so that:*

1. *Each vertex is in exactly one of these induced subgraphs.*
2. *No edges connect these subgraphs to each other.*

Question 2 (Properties Inherited by Subgraphs, 25 points). *For which of the following graph properties P does the following hold: If G is a graph satisfying P and H is an induced subgraph of G , then H must also satisfy P . For each property, either give a proof or a counter-example.*

- (a) *G is a complete graph [5 points]*
- (b) *G is a bipartite graph [5 points]*
- (c) *G is a cycle [5 points]*
- (d) *G is a simple graph [5 points]*
- (e) *G is a path [5 points]*

Question 3 (Hypergraph Handshake Lemma, 25 points). *Suppose that you have a hypergraph where each edge is incident on exactly k vertices. Formulate and prove a version of the Handshake Lemma for this type of graph.*

Question 4 (3-Regular Graphs, 25 points). *Show that for every even integer $n \geq 4$ that there is a 3-regular graph with exactly n vertices. What happens if n is odd?*

Question 5 (Extra credit, 1 point). *Approximately how much time did you spend on this homework?*