

Math 154 Final Exam

Fall 2021

Instructions: Do not open until the exam starts. The exam will run for 180 minutes. The problems are roughly sorted in increasing order of difficulty. Answer all questions completely. Unlike homework questions, you are only required to provide a proof if it is explicitly asked for. You are free to make use of any result in the textbook or proved in class. You may use up to 10 one-sided pages of notes, and may not use the textbook nor any electronic aids. Write your solutions in the space provided, on the backs of pages or blank pages in this handout. If you need more space, you can ask for an extra exam booklet to record answers in. If you do, please make sure to label it with your name and ID. If you have solutions for a problem written anywhere other than the space provided directly below it, be sure to indicate where the rest of the answer is to be found. If you use the backs of any pages, please indicate this on the front of your exam booklet.

Please be sure to sit in the seat indicated below for the exam.

Name:

ID Number:

Seat:

Problem	1	2	3	4	5	6	7	8	Total
Score									

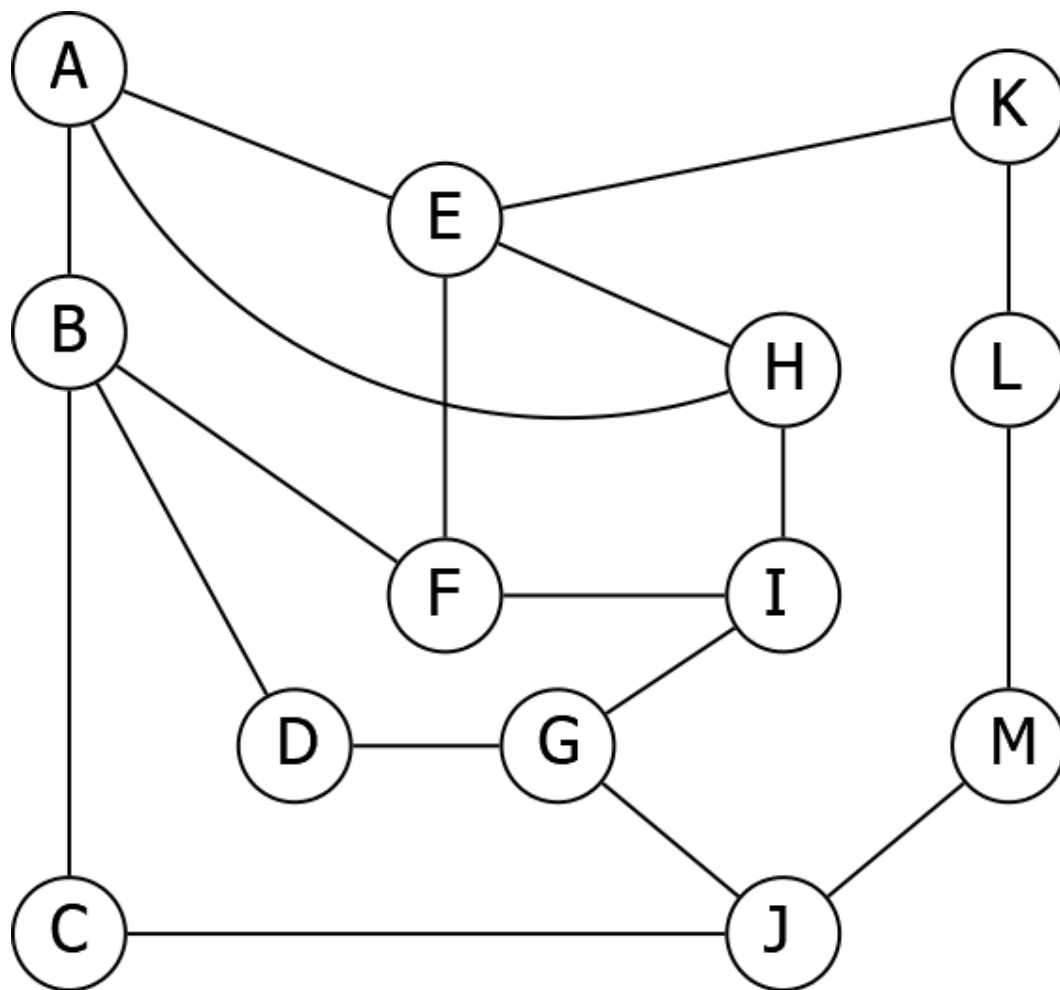
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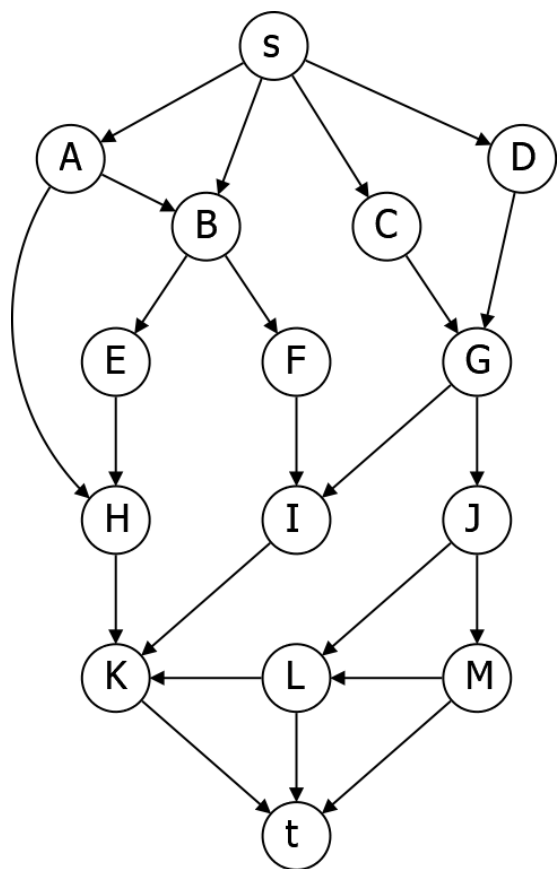
Question 1 (Edge Counting, 25 points). *If a graph has five vertices of degree 7 and eleven vertices of degree 3, and no other vertices, how many edges in total does the graph have?*

Question 2 (Triangle-Free Graphs, 25 points). *What is the greatest number of edges that a K_3 -free graph with 21 vertices can have?*

Question 3 (Ear Decomposition, 25 points). *Show how to write the graph below as a cycle plus ears.*



Question 4 (Minimum Cut Finding, 25 points). *Find a minimum cut in the graph below:*



Question 5 (Eulerian Perfect Matchings, 25 points). *Give an example of an Eulerian graph G with an even number of vertices that does not have a perfect matching. Justify your answer.*

Question 6 (Coloring Almost Planar Graphs, 25 points). *Call a simple graph G almost planar if it can be drawn in the plane with at most one pair of crossing edges.*

(a) *Give an example of an almost planar graph G with $\chi(G) \geq 5$.*

(b) *Show that any almost planar graph G has $\chi(G) \leq 5$.*

Question 7 (Leaves Span Trees, 25 points). *Let T be a finite tree with vertices v and w . Show that there is a leaf u so that the path from v to u contains w .*

Question 8 (Ramsey Number Sums, 25 points). *Prove for positive integers n, m and k that $R(n, m+k-1) \geq R(n, m) + R(n, k) - 1$.*

Hint: Start with a coloring of the complete graph on $R(n, m) - 1$ vertices with no red K_n or blue K_m and a coloring of the complete graph on $R(n, k) - 1$ vertices with no red K_n or blue K_k . Combine them to get a coloring of the complete graph on $R(n, m) + R(n, k) - 2$ vertices with no red K_n nor blue K_{m+k-1} .