

Math 154 Exam 2

Fall 2021

Instructions: Do not open until the exam starts. The exam will run for 45 minutes. The problems are roughly sorted in increasing order of difficulty. Answer all questions completely. Unlike homework questions, you are only required to provide a proof if it is explicitly asked for. You are free to make use of any result in the textbook or proved in class. You may use up to 6 one-sided pages of notes, and may not use the textbook nor any electronic aids. Write your solutions in the space provided, on the backs of pages or blank pages in this handout. If you need more space, you can ask for an extra exam booklet to record answers in. If you do, please make sure to label it with your name and ID. If you have solutions for a problem written anywhere other than the space provided directly below it, be sure to indicate where the rest of the answer is to be found.

Please be sure to sit in the seat indicated below for the exam.

Do not discuss this exam with people other than course staff until Wednesday, November 24th.

Name:

ID Number:

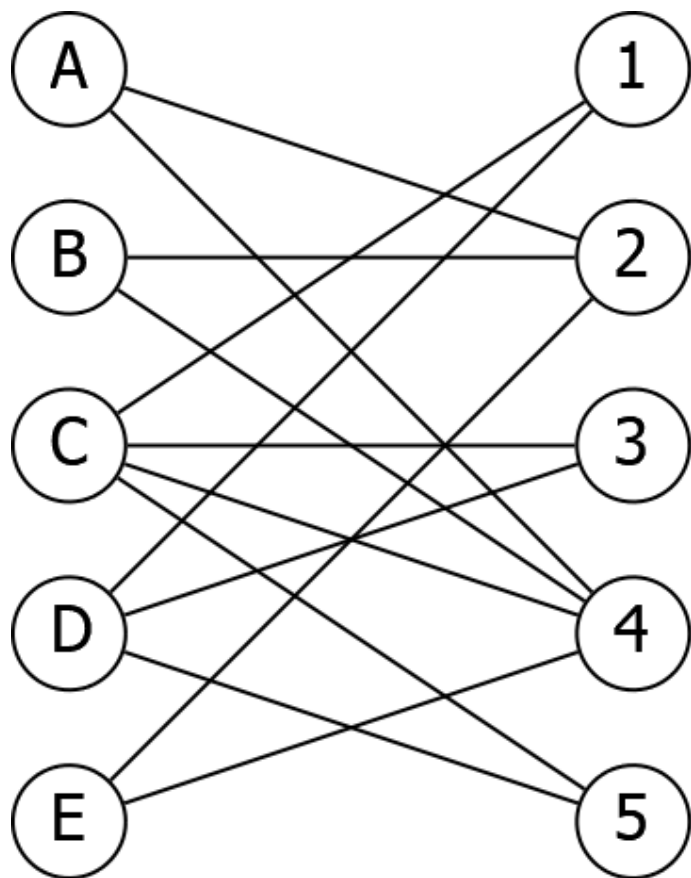
Seat:

Problem	1	2	3	4	Total
Score					

This page is left blank for scratch work.

Question 1 (Edges in Planar Graph, 25 points). *Let G be a connected planar graph all of whose faces are pentagons. If G has 62 vertices, how many edges does it have?*

Question 2 (Perfect Matching, 25 points). *Find a perfect matching in the graph below or show that none exists.*



Question 3 (High Degree Planar Graphs, 25 points). *Let G be a simple, planar graph with $n \geq 5$ vertices, three of which are u, v and w . Show that $\deg(u) + \deg(v) + \deg(w) \leq 2n + 2$.*

Question 4 (Max Edges for Chromatic Number, 25 points). *Let n and k be positive integers. What is the greatest number of edges that a simple graph G with nk vertices and chromatic number at most k can have? Justify your answer.*