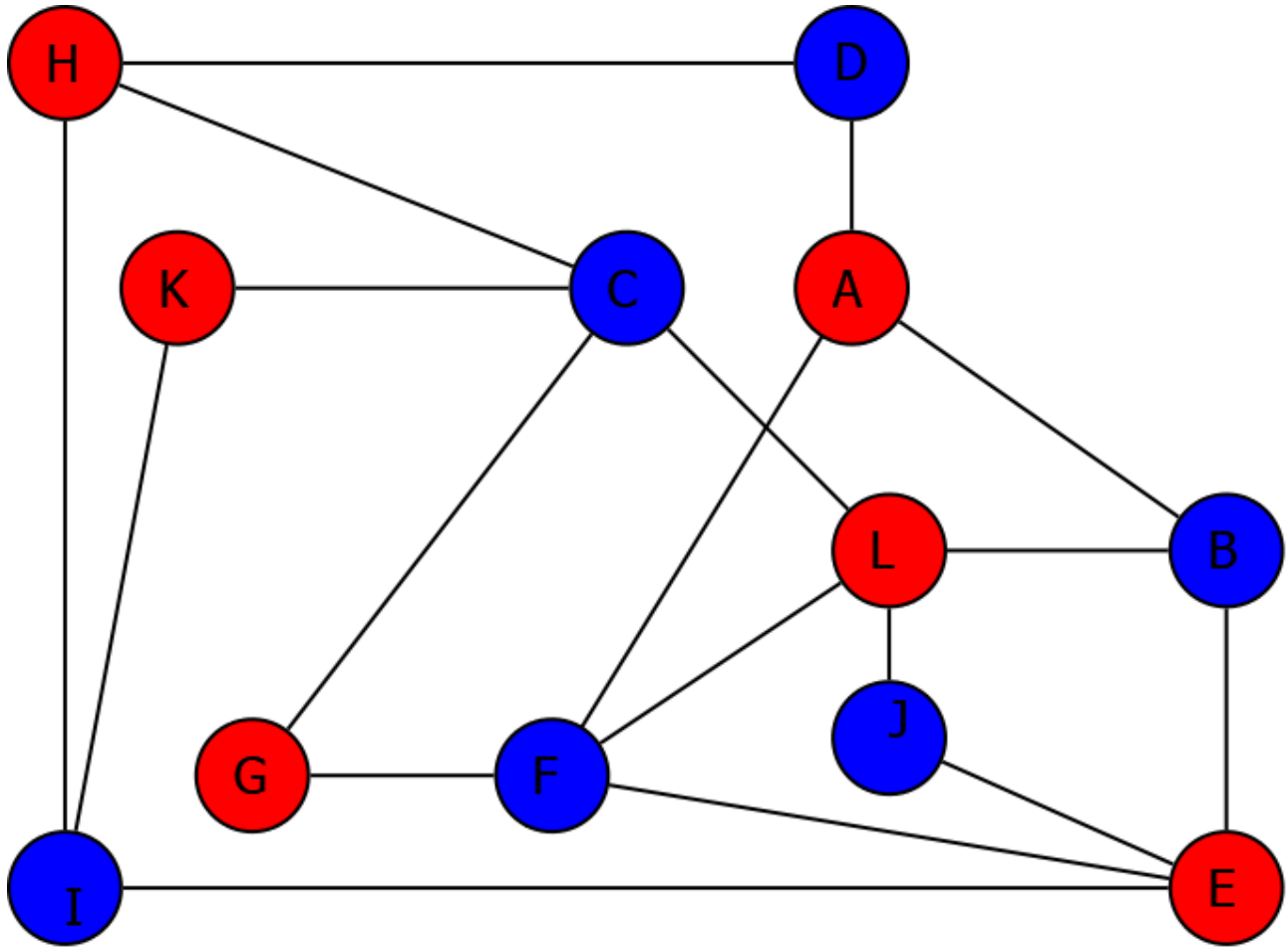
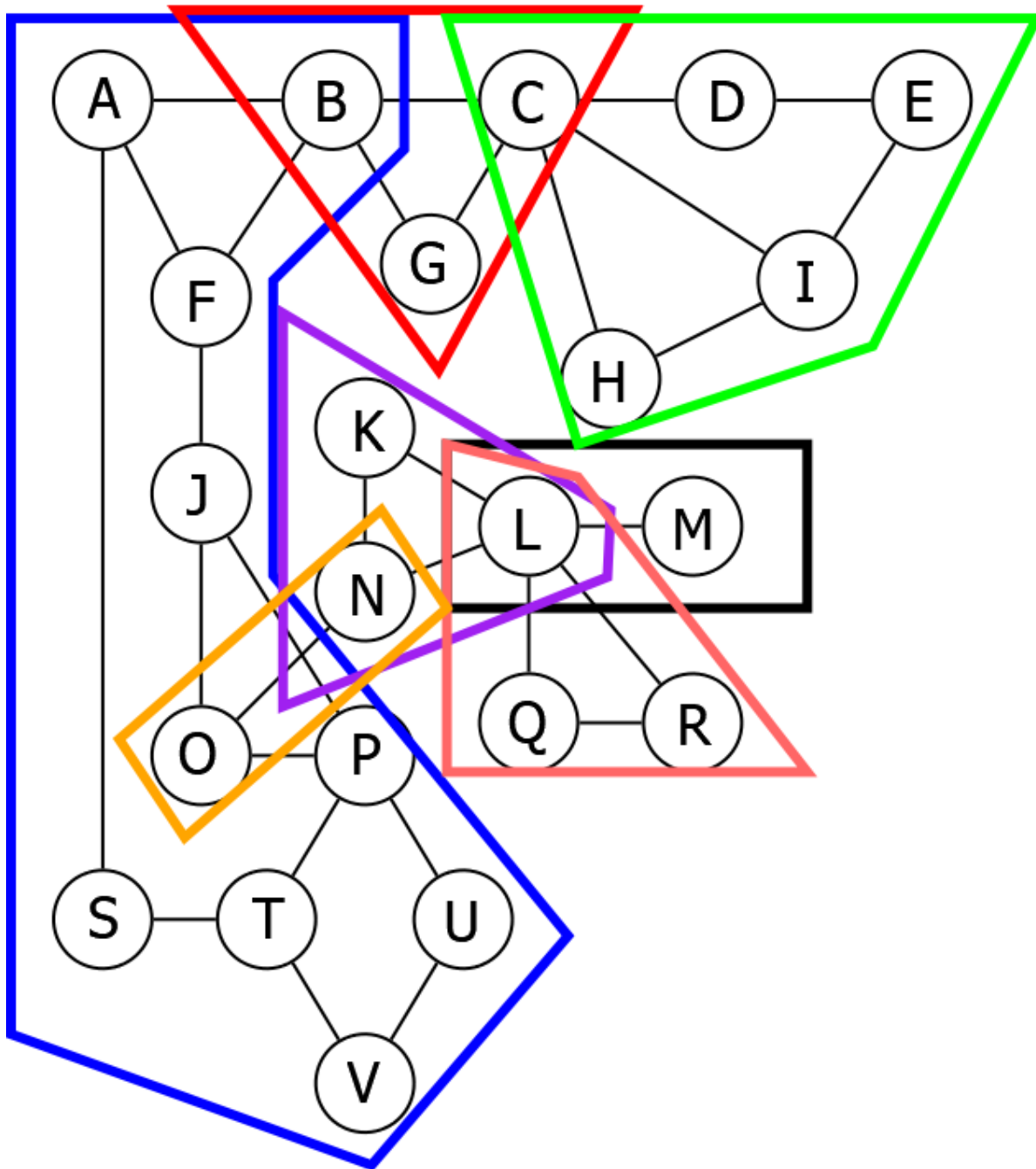


Question 1 (Bipartite Coloring, 25 points). *Provide a coloring of the vertices of the graph below black and white to show that it is bipartite.*



By coloring one vertex and alternatively coloring its neighbors, we get the coloring above.

Question 2 (Block Decomposition, 25 points). *What are the blocks in the graph below?*



Question 3 (Eulerian Vertex Removal, 25 points). *Let G be a connected, Eulerian graph. Suppose that removing any one vertex from G yields a Semi-Eulerian graph. Prove that G is a cycle.*

Since G is Eulerian, all vertices have even degree. Removing a vertex v decreases the degree of all neighbors of v , making them odd. Since $G - \{v\}$ must be semi-Eulerian, v must have degree 2 for all v . This means that G 's Eulerian circuit must pass through each vertex exactly once, and thus G must be a cycle.

Question 4 (Vertex Sum MST, 25 points). *Let G be a simple, connected graph so that each vertex of G is assigned a unique weight $w(v)$. An edge e connecting vertices u and v is then given weight $w(e) = w(u) + w(v)$. Let v_0 be the vertex of G with minimum weight. Prove that for any Minimum Spanning Tree T of G contains all of the edges coming out of v_0 .*

Suppose that this is not the case. Suppose that a minimum spanning tree T does not include the edge $e = (v_0, u)$ for some u . Adding e to T creates a cycle. Let the vertex after u in this cycle be t . Removing the edge (u, t) from T yields a new tree T' with

$$w(T') = w(T) + w(v_0, u) - w(u, t) = w(T) + w(v_0) + w(u) - (w(u) + w(t)) = w(T) + w(v_0) - w(t) < w(T).$$

This contradicts the assumption that T was a minimum spanning tree.

