

Math 154 Exam 1

Fall 2021

Instructions: Do not open until the exam starts. The exam will run for 45 minutes. The problems are roughly sorted in increasing order of difficulty. Answer all questions completely. Unlike homework questions, you are only required to provide a proof if it is explicitly asked for. You are free to make use of any result in the textbook or proved in class. You may use up to 6 1-sided pages of notes, and may not use the textbook nor any electronic aids. Write your solutions in the space provided, the pages at the end of this handout, or on the scratch paper provided (be sure to label it with your name). If you have solutions written anywhere other than the provided space be sure to indicate where they are to be found.

Please be sure to sit in the seat indicated below for the exam.

Do not discuss this exam with other people until Wednesday, November 3rd.

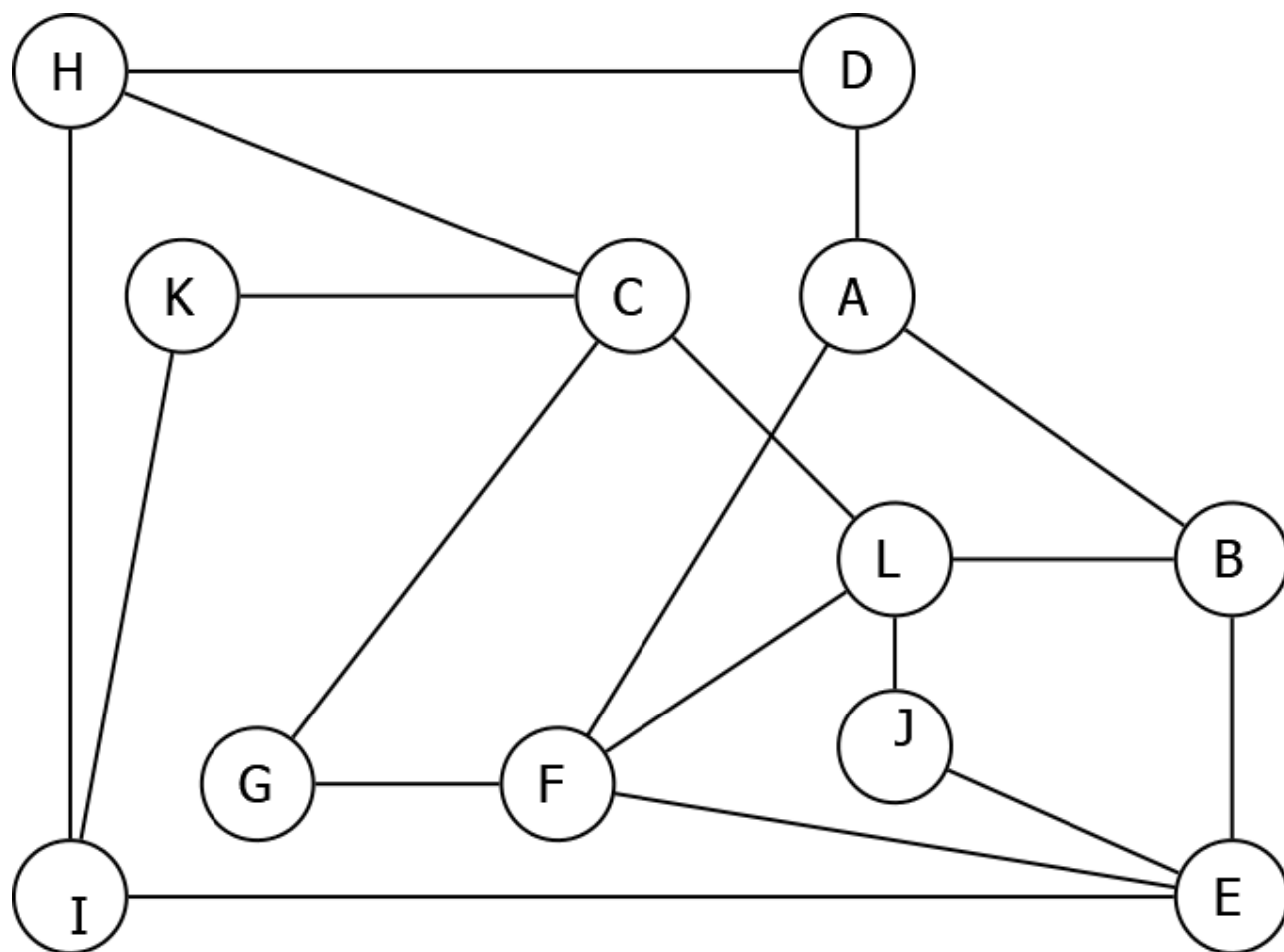
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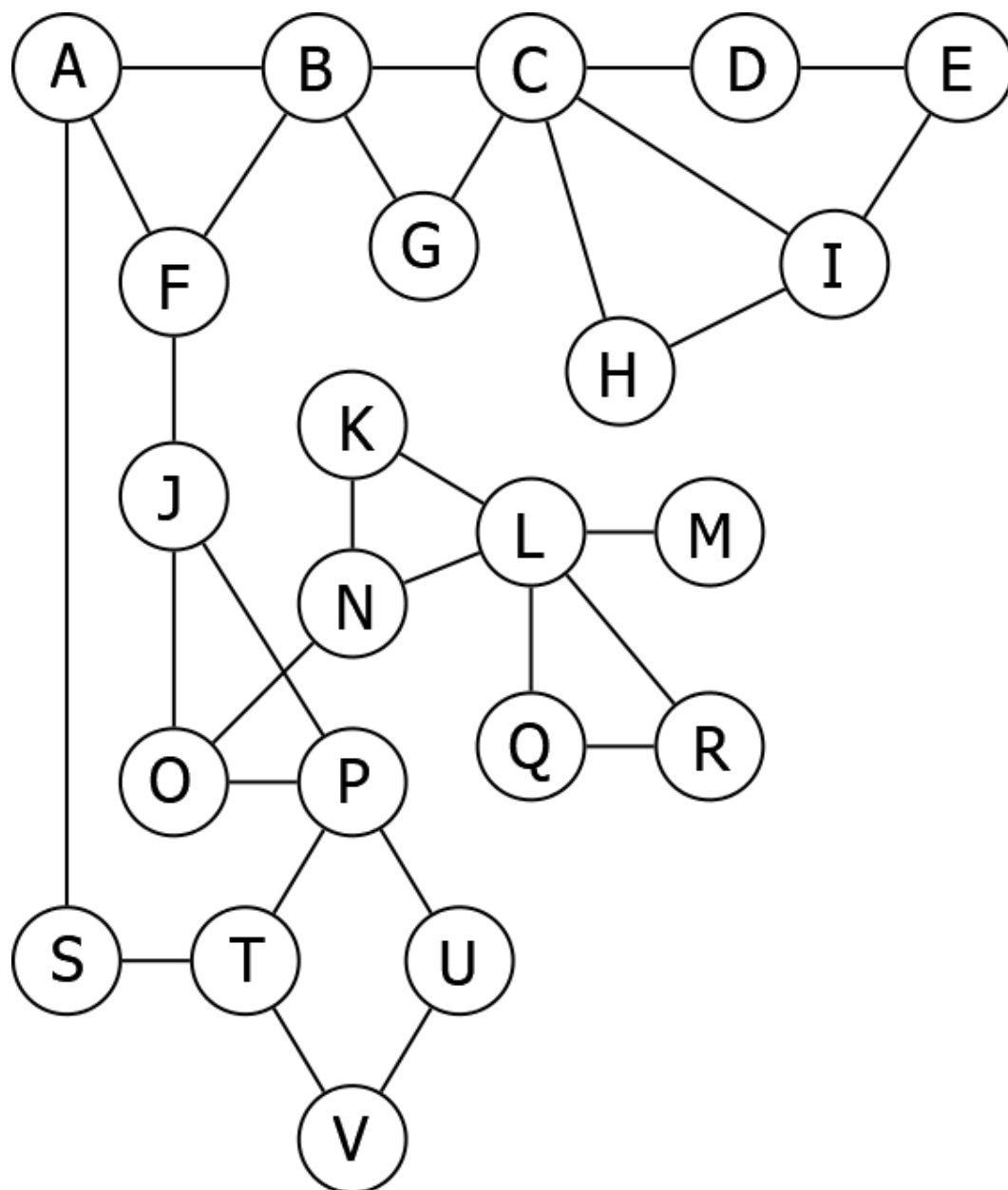
Seat:

Problem	1	2	3	Total
Score				

Question 1 (Bipartite Coloring, 25 points). *Provide a coloring of the vertices of the graph below black and white to show that it is bipartite.*



Question 2 (Block Decomposition, 25 points). *What are the blocks in the graph below?*



Question 3 (Eulerian Vertex Removal, 25 points). *Let G be a connected, Eulerian graph. Suppose that removing any one vertex from G yields a Semi-Eulerian graph. Prove that G is a cycle.*

Question 4 (Vertex Sum MST, 25 points). *Let G be a simple, connected graph so that each vertex of G is assigned a unique weight $w(v)$. An edge e connecting vertices u and v is then given weight $w(e) = w(u) + w(v)$. Let v_0 be the vertex of G with minimum weight. Prove that for any Minimum Spanning Tree T of G contains all of the edges coming out of v_0 .*

