Announcements

- Homework 1 Due Sunday
- Feedback Survey on Canvas
- Draft slides of upcoming lectures now on course webpage

Last Time

Leaves

- A leaf is a degree-1 vertex in a tree.
- Every tree with at least two vertices has at least 2 leaves.

Spanning Trees

- Every connected graph G has a subgraph T that is a tree connecting all of its vertices
- Breadth First Search produces a spanning tree.

Today

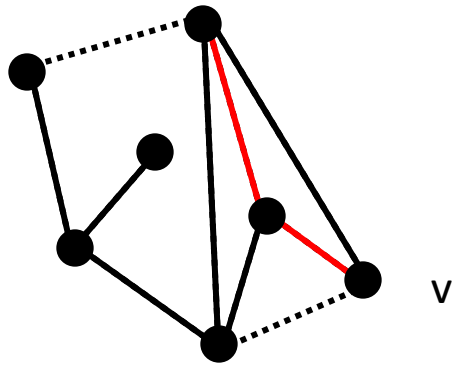
More ways to get spanning trees

- Depth First Search
- Minimum Spanning Trees

Counting Trees

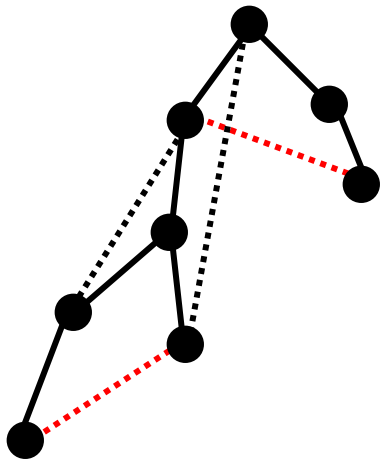
Depth First Search Tree

- Start at a base vertex v
- Follow path from v until cannot extend anymore
- Backtrack until new branch
- Repeat backtrack/extend until nothing else to do



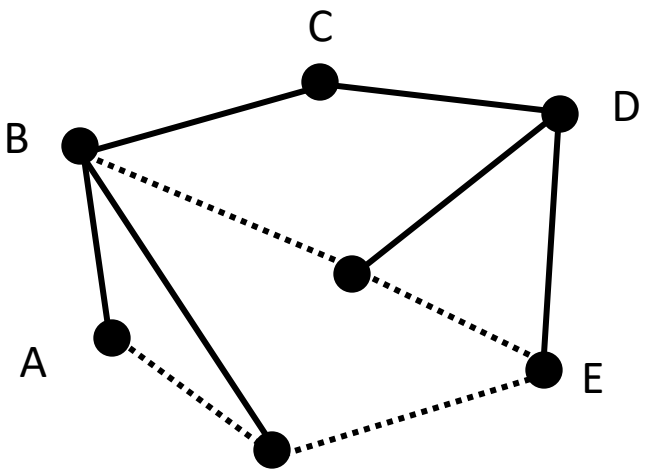
Depth First Search Tree Properties

- G has no extra edges that cross between different branches of the tree.
  - If such an edge existed, it would have been used when exploring the first branch.



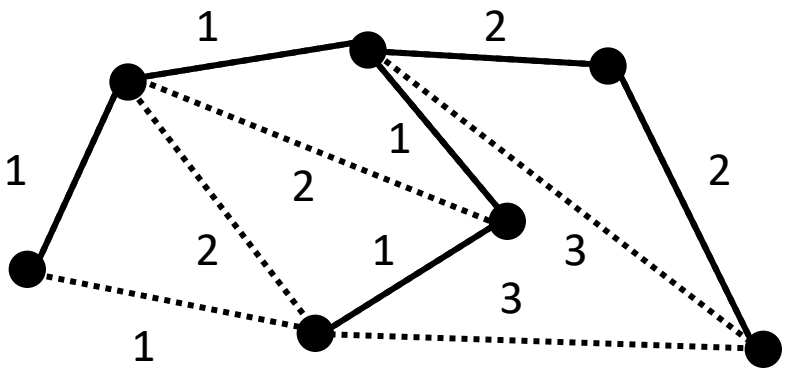
Questions: BFS Trees

The following spanning tree could be a Breadth First Search tree starting from which of the marked vertices?



Minimum Spanning Tree

- Remember Highway Repair problem
- Realistically, some roads harder to fix than others
- Minimum Spanning Tree:
  - Each edge has a weight
  - Want spanning tree with least total weight

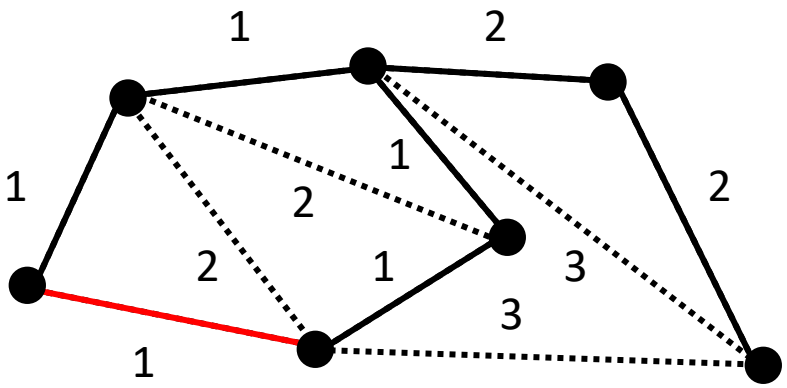


1+1+1+1+2+2=8

How do you find a MST?

Kruskal's Algorithm:

- Repeatedly add lightest edge that does not create a cycle



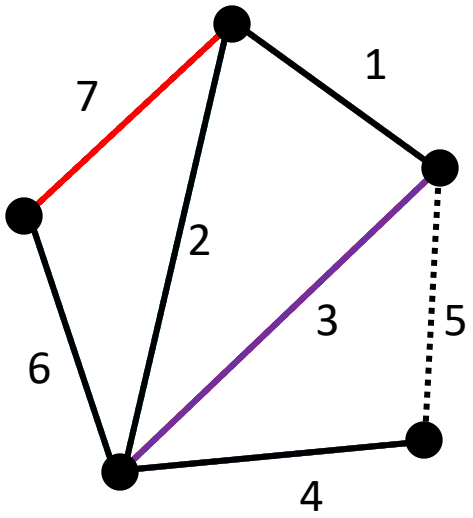
- Clearly creates spanning tree.
- Why minimal?

Proof I

Idea:

- Take *any* spanning tree, turn it into Kruskal's tree
- Change one edge at a time, each time improving weight
- End up with Kruskal, showing that Kruskal's tree is at least as good as what you started with

Proof II



- First edge already in tree
- Second edge not.
  - Adding would create cycle
  - Cycle contains more expensive edge
  - Trade one edge for other
- Third edge already in tree
- Fourth edge not
  - Trade for more expensive edge

Proof III

In general:

- If next Kruskal edge in tree, move on
- If not,
  - Extra edge creates cycle
  - Cycle contains edge not in Kruskal tree
  - Edge more expensive
  - Swap edges give cheaper new tree