Announcement

Homework 1 is online.

You can find it on:

- Course webpage: http://cseweb.ucsd.edu/~dakane/Math154/
- Gradescope
- Canvas

It is due on gradescope at 11:59pm pacific time on Sunday, April 12th.

Last Time

Types of walks:

- Walks
- Paths
- Trails
- Circuits
- Cycles

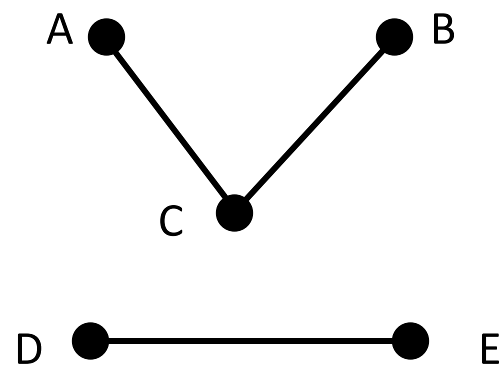
Connectivity

Results:

- Every walk contains a path
- Every graph can be split into connected components

Question: Component of a Vertex

Which vertices are in the connected component of A?



Today

- Classification of Bipartite Graphs
- Introduction to trees
  - Definition

Bipartite Graphs

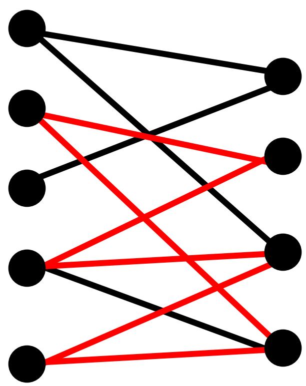
Recall a graph is bipartite if you can split the vertices into two parts so that edges always connect one part to the other.

Equivalently, G is bipartite if you can color vertices black & white so that all edges connect a black vertex to a white vertex.

Theorem (Theorem 1.3): A graph G is bipartite if and only if it has no cycles of odd length.

One Direction

If G is bipartite, each edge crosses sides. Any cycle must do this an even number of times.

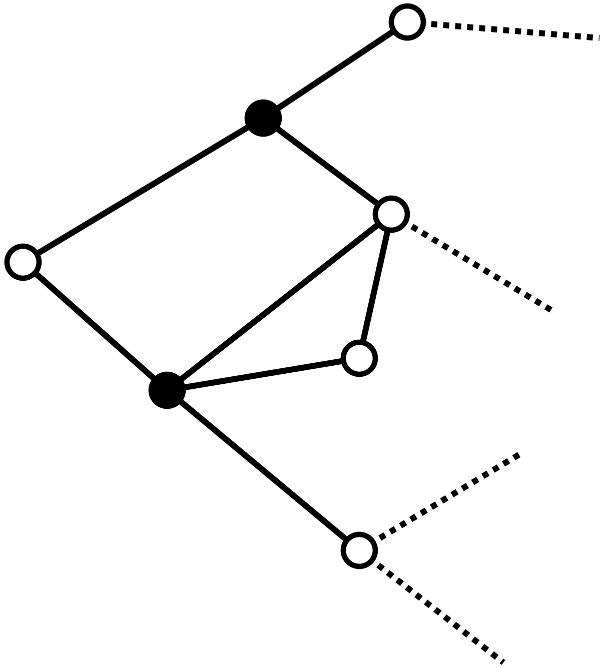


Other Direction

- If G has no odd length cycles, want to show that G is bipartite.
- Deal with each connected component separately. Assume that G is connected.
- Find a black/white coloring of vertices.

Coloring

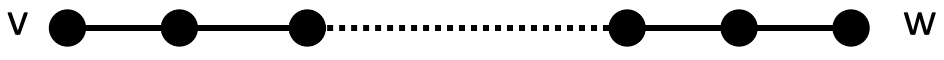
- G is connected.
- Arbitrarily make first vertex white.
- Neighbors must be black
- Keep alternating
- Hope you avoid a contradiction



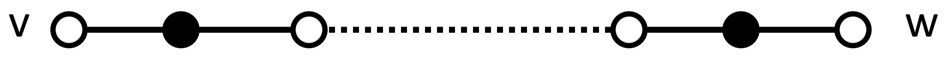
Formally

Pick white vertex v

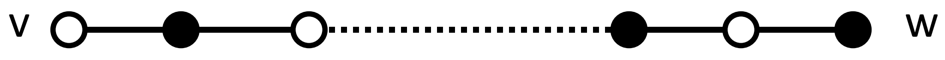
For each w: Find v-w path (G is connected)



If path length is even, color w white.



If path length is odd, color w black.



Consistency

What if there are both even- and odd- length paths?

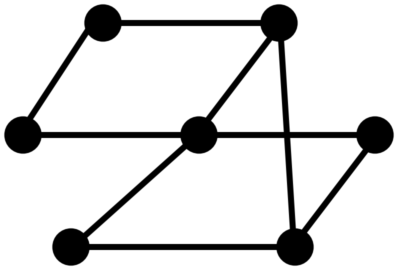
Combine to get odd length loop from v to itself.

Contradiction! G has no odd length cycles!
(and loop can be broken up into cycles)

Question: Bipartite Graphs

Which of the following graphs are bipartite?

A) C_{20}
B) C_{21}
C) K_6
D) P_{17}
E) G as shown



Trees (Chapter 1.3)

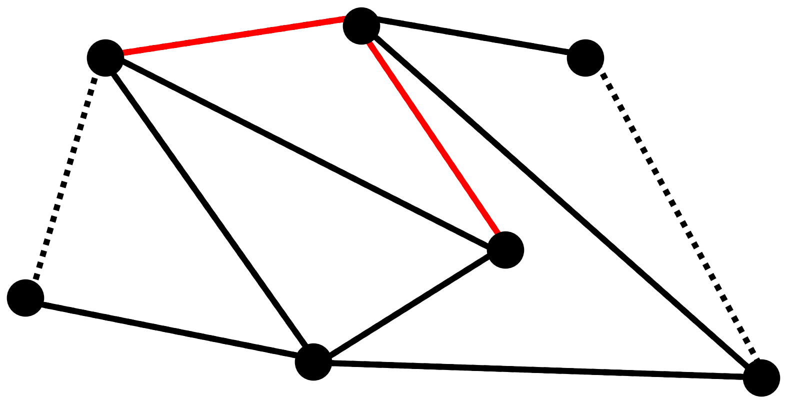
- Definition and motivation
- Basic Properties
- Spanning Trees
- Counting Problems

Fixing Highways I

A storm has taken out a number of major highways. You need to repair highways in order to restore travel as quickly as possible. You want to do this by fixing as few highways as possible.

Fixing Highways II

- Add some of the available edges
- As few as possible
- Want resulting graph to be connected



Creates a cycle.
Doesn't help.

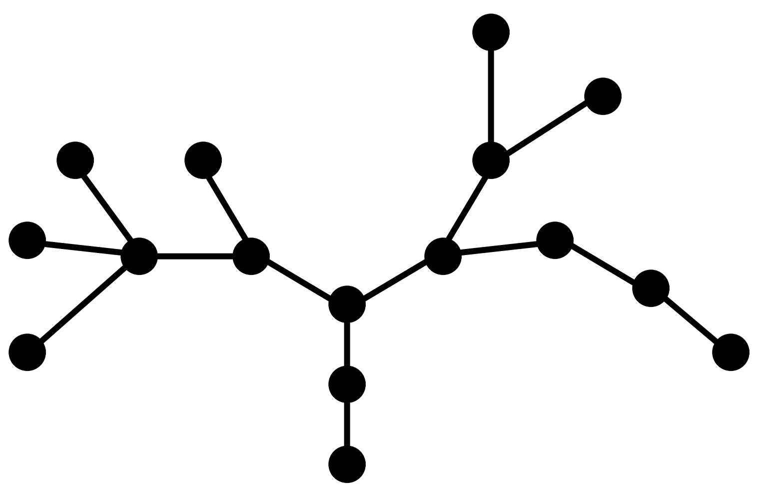
6 edges.
Is this the best?

Trees

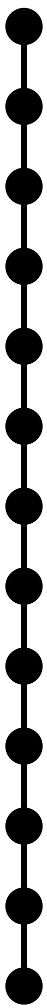
Need a graph that:

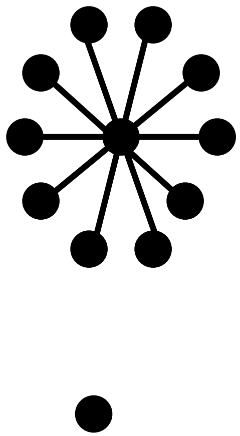
- Connects all vertices
- Doesn't have cycles

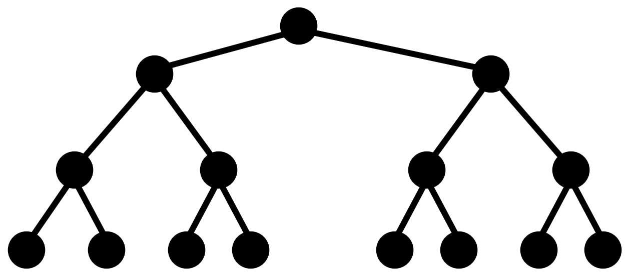
Definition: A *tree* is a connected graph with no cycles. A *forest* is a graph where each connected component is a tree.

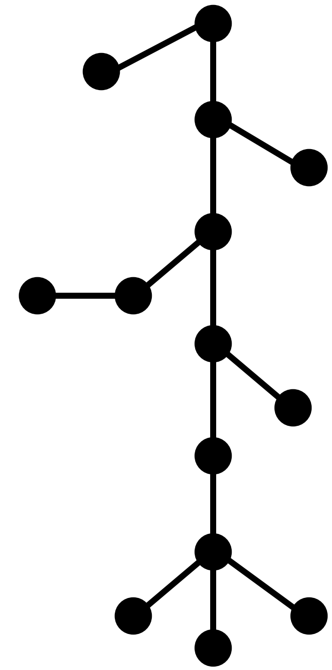


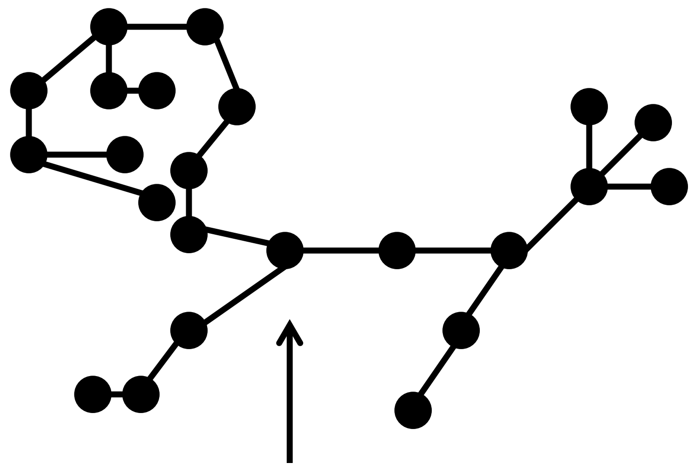
Examples



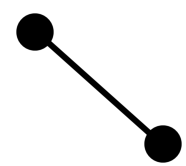








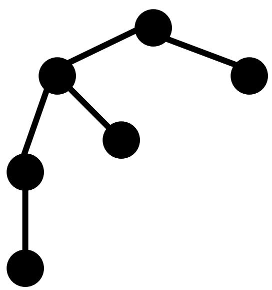
*This tree is well known for its bark.



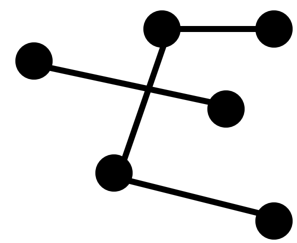
Question: Trees

Which of the following is NOT a tree?

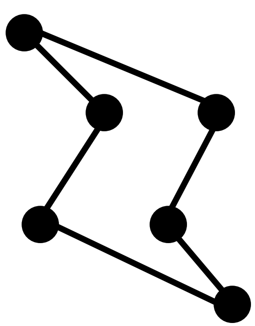
A



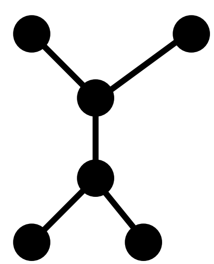
B



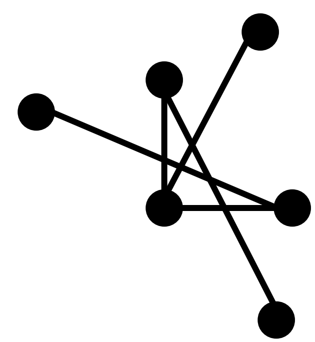
C



D



E



A Lemma

<u>Lemma:</u> Let T be a tree with vertices u and v. There exists a *unique* u-v path in T.

<u>Proof:</u>

- T is connected, so there must be a path.
- If there were two different paths, putting them together would produce a cycle.