CSE 203A Homework 3

Spring 2026

This homework is due in class Friday May 15th 11:59pm on gradescope. Make sure to justify all of your answers with a mathematical proof. For algorithms you should both prove correctness (with appropriately small probability of error) as well as appropriate bound on runtime.

Question 1 (Slow Random Walk for 2-SAT, 20 points). *Show that for the random walk algorithm for 2-SAT discussed in class that there are satisfiable instances of 2-SAT on n variables so that for some initial guesses, the expected time to find a satisfying assignment is $\Omega(n^2)$.*

Question 2 (Random Graphs and Expanders, 30 points). *Let n and d be sufficiently large integers with nd even. Let G be a random d-regular multigraph selected by finding a random pairing of the dn vertex-edge pairs. Show that:*

$$\Pr(h(G) < d/3) < 1/n.$$

Hint: you may use the fact that the Chernoff bounds still hold if the X_i's are not independent but instead if $\mathbb{E}[X_i|X_{i-1}, X_{i-2}, \ldots, X_1]$ is a decreasing function of $X_1 + X_2 + \ldots + X_{i-1}$ for each i.

Question 3 (Expander Cover Times, 30 points). *Let G be a d-regular graph on n vertices where all eigenvalues of the adjacency matrix other than the principle one have absolute value at most $d/2$.*

(a) Let S be a set of vertices, and v a specific vertex. Let $h_{v,S}$ be the expected time for a random walk starting at v to reach some vertex in S. Prove that

$$\mathbb{E}_v[h_{v,S}] = O(n/|S|).$$

Hint: Look at eigenvalues of RPR where R is the linear transformation that zeroes out the coefficients of vertices in S. [20 points]

(b) Show that for any v, S that $h_{v,S} = O(n/|S| + \log(n))$. [5 points]

(c) Show that the cover time of G is $O(n \log(n))$. [5 points]

Question 4 (Doubling Graph, 20 points). *Let G be the graph with vertex set $\mathbb{Z}/p$ for some prime p and edges connecting x to $x+1, x-1, 2x, x/2$ mod p. Show that the transition matrix of G has a non-principle eigenvector with eigenvalue $1 - O(1/\log(p))$. Hint: Consider the transition matrix in the Fourier basis.*

Question 5 (Extra credit, 1 point). *Approximately how much time did you spend working on this homework?*