# Announcements

- Homework 0 due today
- Homework 1 online, due next week
- Jaibei Han's office hours will be Monday, Thursday, and Friday from 4-5pm.
  - The syllabus has been updated.
  - Sorry for the inconvenience.

# Last Time

- Graph Definition
  - Edges connect pairs of vertices
- `Explore/DFS`
  - O(|V|+|E|) runtime
  - `explore(v)` discovers all vertices reachable from v

# Explore and DFS

```
explore(v)
  v.visited ← true
  For each edge (v,w)
     If not w.visited
          explore(w)
DFS(G)
  Mark all v ∈ G as unvisited
  For v ∈ G
    If not v.visited, explore(v)
```

# Today

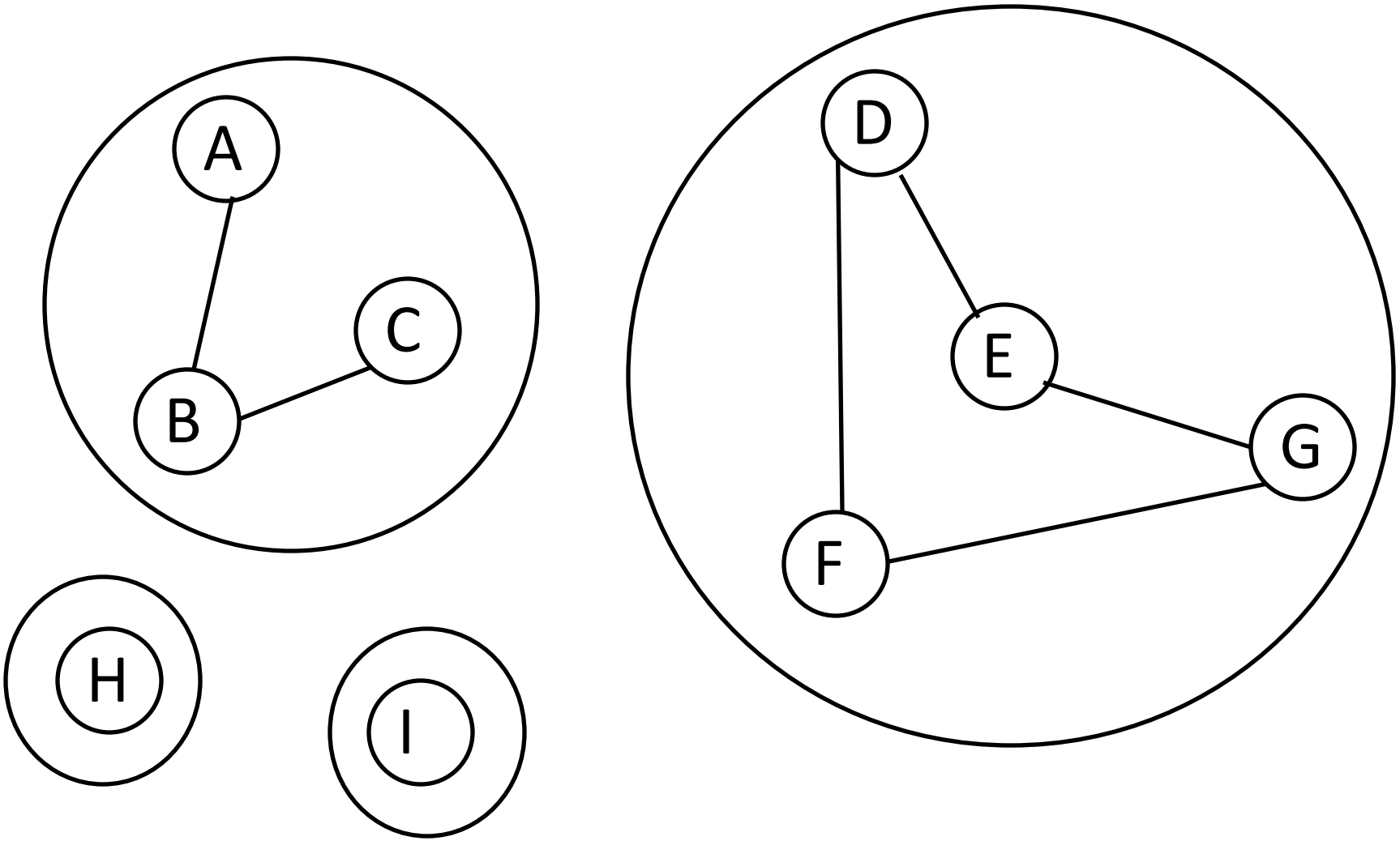
- Connected Components
- Pre- and Post- orderings
- [Directed graphs]

# Connected Components

- Want to understand which vertices are reachable from which others in graph G.
- `explore(v)` finds which vertices are reachable from a given vertex.

**Theorem:** The vertices of a graph G can be partitioned into *connected components* so that v is reachable from w if and only if they are in the same connected component.

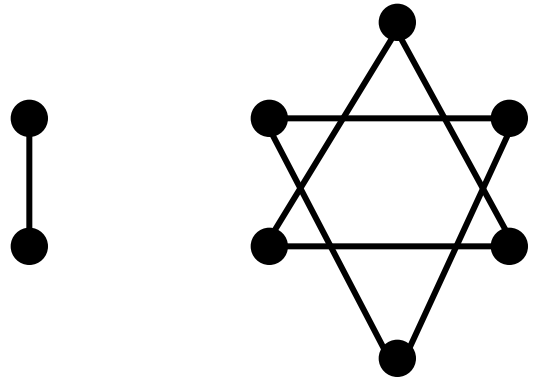
# Example



# Question: Connected Components

How many connected components does the graph below have?

A)0
B)1
C)2
D)3
E)4



# Problem: Computing Connected Components

Given a graph G, compute its connected components.

**Easy:** For each v, run `explore(v)` to find vertices reachable from it. Group together into components.

Runtime: O(|V|(|V|+|E|)).

**Better:** Run `explore(v)` to find the component of v. Repeat on unclassified vertices.

# DFS lets us do this!

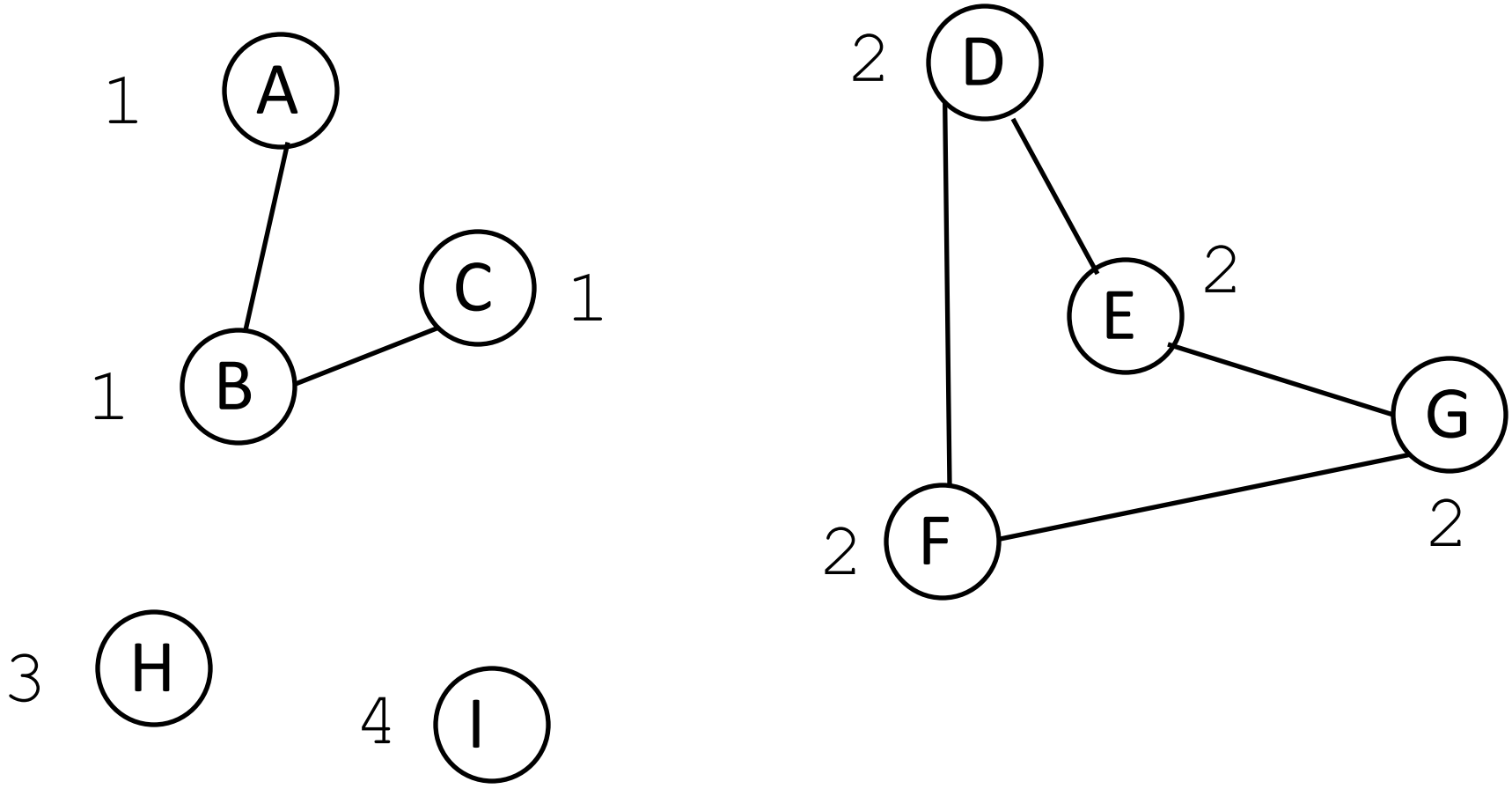
```
ConnectedComponents(G)
  CCNum ← 0
  For v ∈ G
    v.visited ← false
  For v ∈ G
    If not v.visited
      CCNum++
      explore(v)
```

```
explore(v)
  v.visited ← true
  v.CC ← CCNum
  For each edge (v,w)
    If not w.visited
      explore(w)
```

Runtime O(|V|+|E|).

# Example

CCNum: 1 2 3 4



# Discussion about DFS

What does DFS actually *do*?

- No output.
- Marks all vertices as visited.
- Easier ways to do this.

However, DFS also is a useful way to explore the graph. By *augmenting* the algorithm a bit (like we did with the connected components algorithm), we can learn useful things.

# Pre- and Post- Orders

Augment how?

- Keep track of what algorithm does & in what order.
- Have a “clock” and note time whenever:
  - Algorithm visits a new vertex for the first time.
  - Algorithm finishes processing a vertex.
- Record values as `v.pre` and `v.post`.

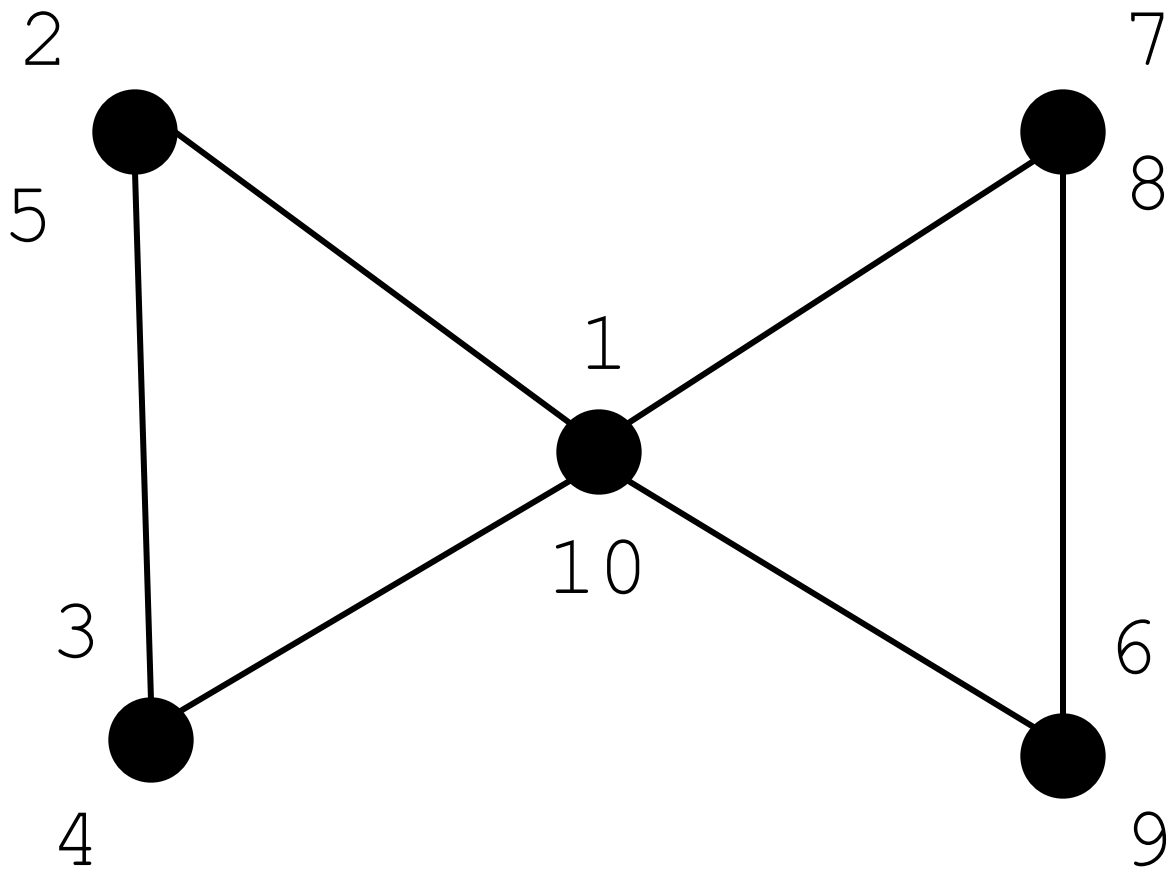
# Computing Pre- & Post- Orders

```
ConnectedComponents(G)
  clock ← 1
  For v ∈ G
    v.visited ← false
  For v ∈ G
    If not v.visited
      explore(v)
```

```
explore(v)
  v.visited ← true
  v.pre ← clock
  clock++
  For each edge (v,w)
    If not w.visited
      explore(w)
  v.post ← clock
  clock++
```

Runtime O(|V|+|E|).

# Example



# What do these orders tell us?

**Prop:** For vertices v, w consider intervals [`v.pre`, `v.post`] and [`w.pre`, `w.post`]. These intervals:

1. Contain each other if v is an ancestor/descendant of w in the DFS tree.
2. Are disjoint if v and w are cousins in the DFS tree.
3. Never interleave (`v.pre` < `w.pre` < `v.post` < `w.post`)

# Proof

- Assume algorithm finds v before w (`v.pre < w.pre`)
- If algorithm discovers w *after* fully processing v:
  - `v.post < w.pre`
  - Intervals disjoint
  - v and w are cousins
- If algorithm discovers w *before* fully processing v:
  - Algorithm finishes processing w before it finishes v
  - `v.pre < w.pre < w.post < v.post`
  - Nested intervals
  - v is ancestor of w

# Question: Possible Intervals

Which pairs of pre-post intervals are possible for DFS?

A) [1,2] & [3,4]

B) [1,3] & [2,4]

C) [1,4] & [2,3]

D) [1,5] & [2,4]

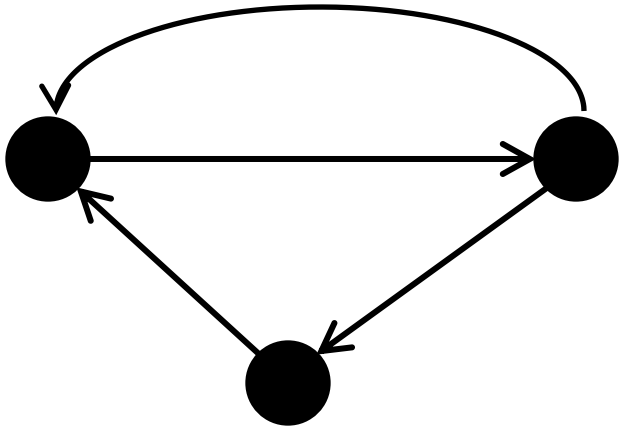
E) [1,6] & [2,5]

# Directed Graphs

Often an edge makes sense both ways, but sometimes streets are one directional.

**Definition:** A directed graph is a graph where each edge has a direction. Goes *from* v *to* w.

Draw edges with arrows to denote direction.



# Question: Directed Graphs

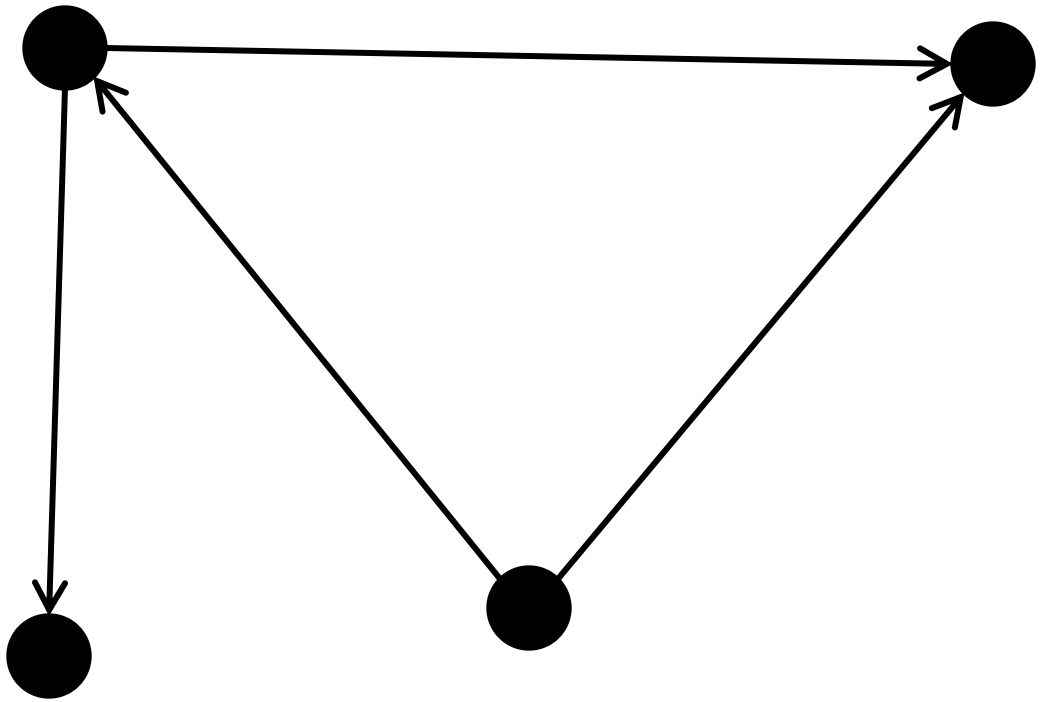
Which of the following make the most sense as directed rather than undirected graphs:

A) The Internet (links connecting webpages)
B) The Internet (wires connecting servers)
C) Facebook (friendships connecting people)
D) Twitter (followings connecting people)
E) Maps (roads connecting intersections)

# DFS on Directed Graphs

- Same code
- Only follow *directed* edges from v to w.
- Runtime still O(|V|+|E|)
- `explore(v)` discovers all vertices reachable from v following only directed edges.

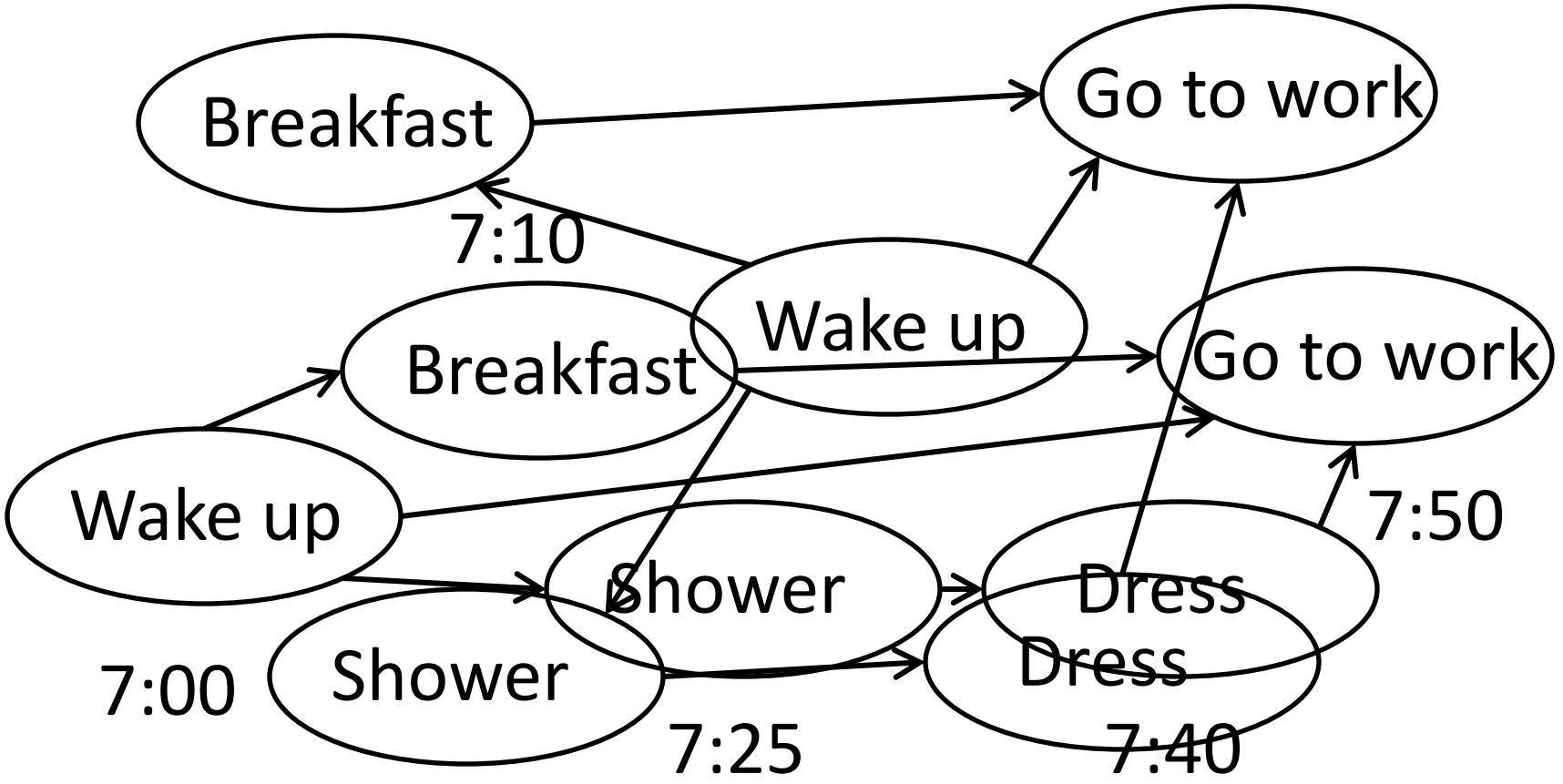
# Example



# Directed Acyclic Graphs

- Directed graphs as dependencies
- Linear orderings
- DAGs definition
- Topological sort

# Dependency Graphs



# Dependency Graphs

A directed graph can be thought of as a graph of dependencies. Where an edge v→w means that v should come before w.

**Definition:** A topological ordering of a directed graph is an ordering of the vertices so that for each edge (v,w), v comes before w in the ordering.

# Question: Existence of Orderings

Does every directed graph have a topological ordering?

A) Yes

B) No

# Counterexample

