Announcements

- Exam 2 Solutions Online
- Homework 4 Online, Due Friday
- Change to time and location of Stanlislaw's Tuesday office hours

Today

- Dynamic Programming
- Longest Common Subsequence

Dynamic Programming (Ch 6)

- Background and past examples
- Longest Common Subsequence
- Knapsack
- Chain Matrix Multiplication
- All-Pairs Shortest Paths
- Independent Sets of Trees
- Travelling Salesman

Computing Fibonacci Numbers

Recall:

$$F_n = 1 \text{ if } n = 0 \text{ or } 1$$

$$F_n = F_{n-1} + F_{n-2} \text{ otherwise}$$

Naïve Algorithm

```
Fib(n)
  If n ≤ 1
    Return 1
  Else
    Return Fib(n-1)+Fib(n-2)
```

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```

Far too slow!

Improved Algorithm

```
Fib2(n)
  Initialize A[0..n]
  A[0] = A[1] = 1
  For k = 2 to n
     A[k] = A[k-1] + A[k-2]
  Return A[n]
```

Improved Algorithm

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  Initialize A[0..n]
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Tabulation of answers avoids runaway recursive calls.

Another Example

Something similar happens with our algorithm for shortest paths in DAGs.

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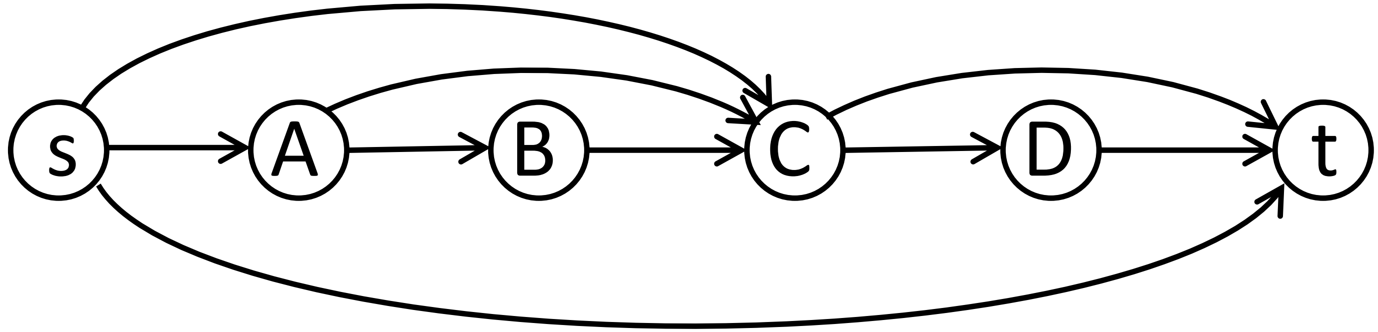
Something similar happens with our algorithm for shortest paths in DAGs.

This was based on the basic recursive formula

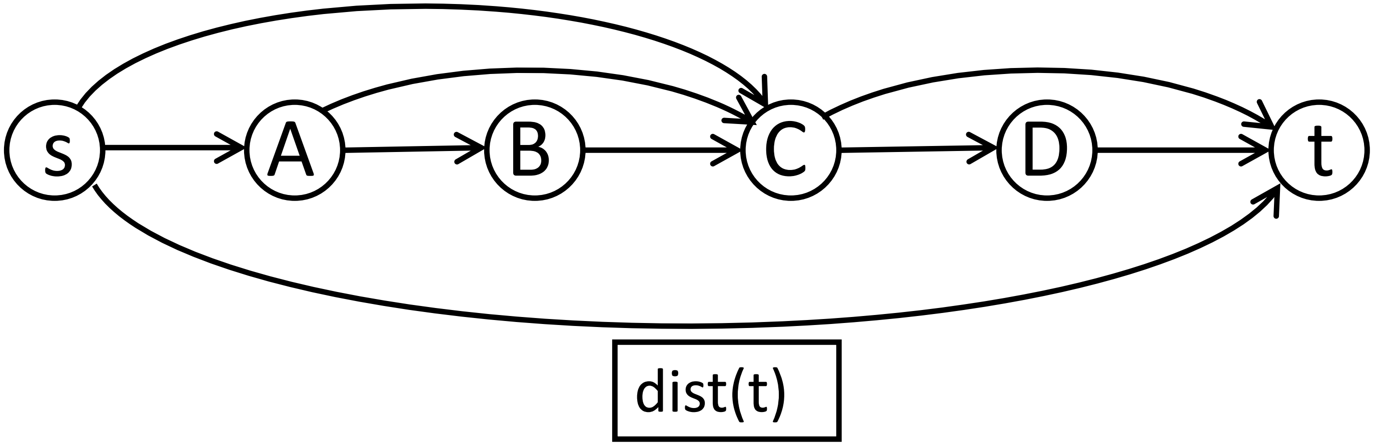
$$\text{dist}(w) = \min_{(v,w)\in E} \text{dist}(v) + \ell(v,w).$$

applied to vertices in topological order.

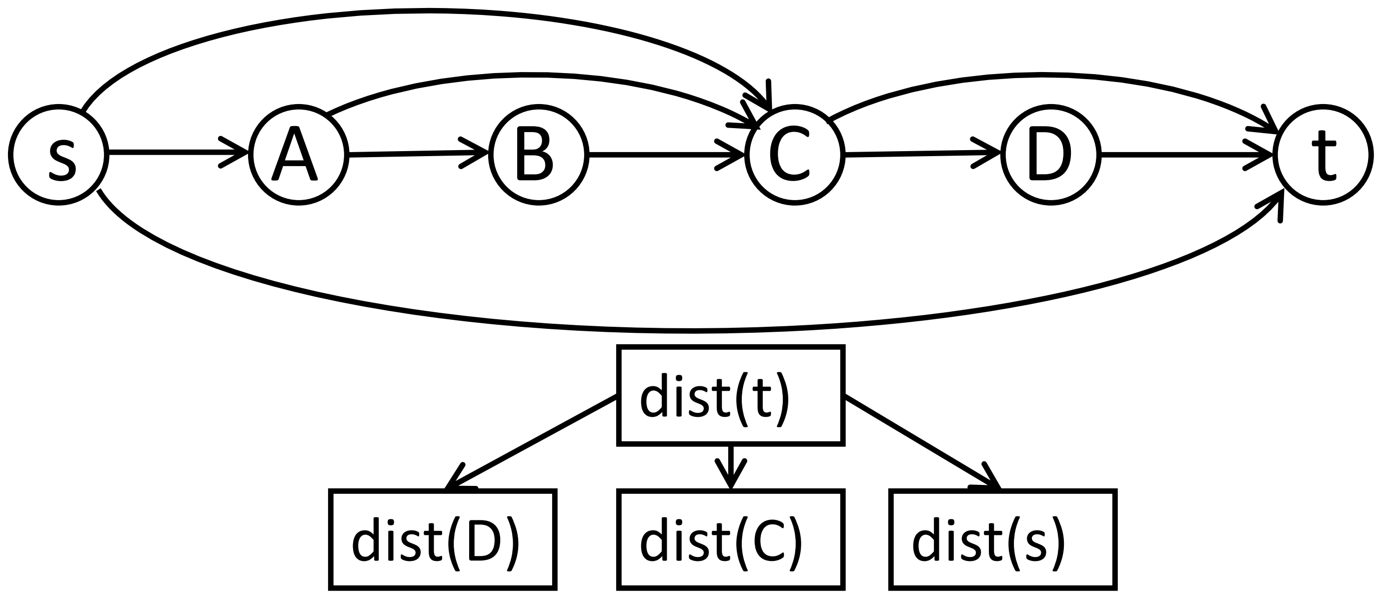
Example



Example



Example



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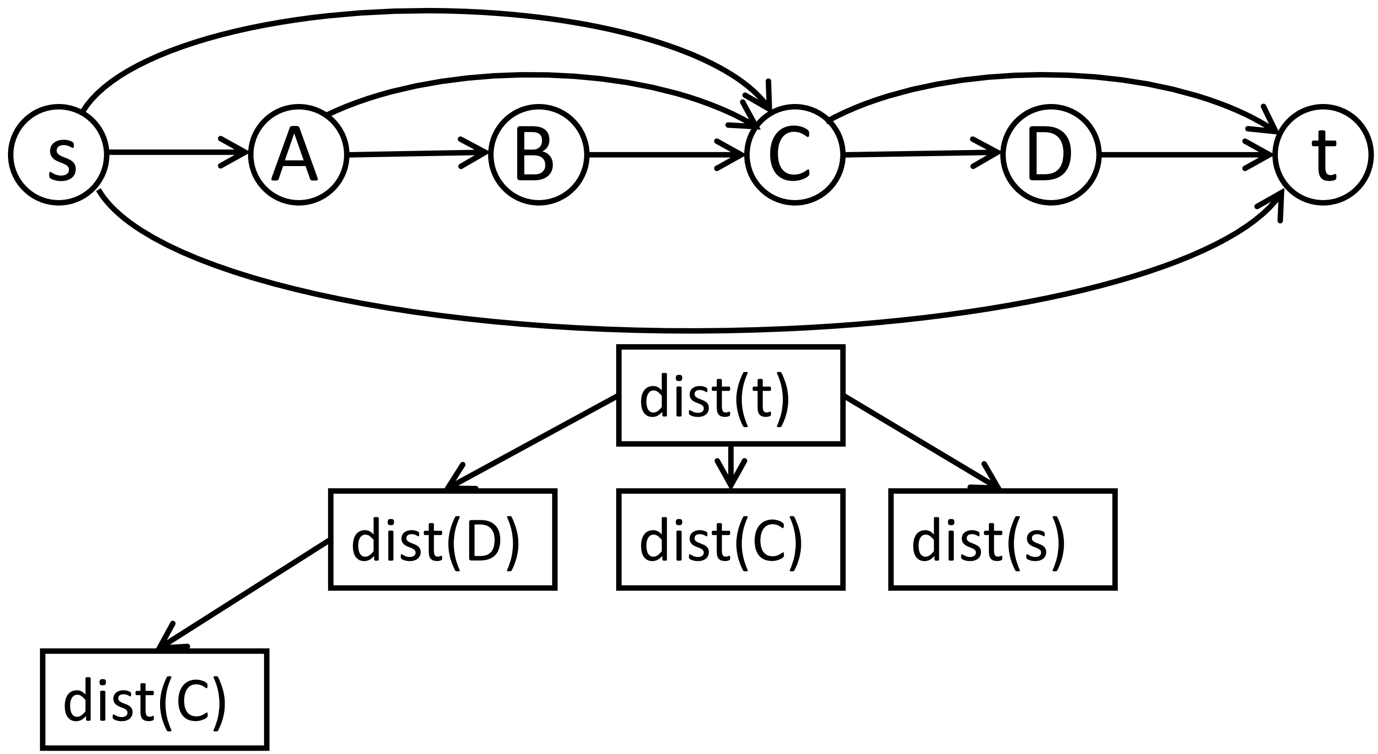

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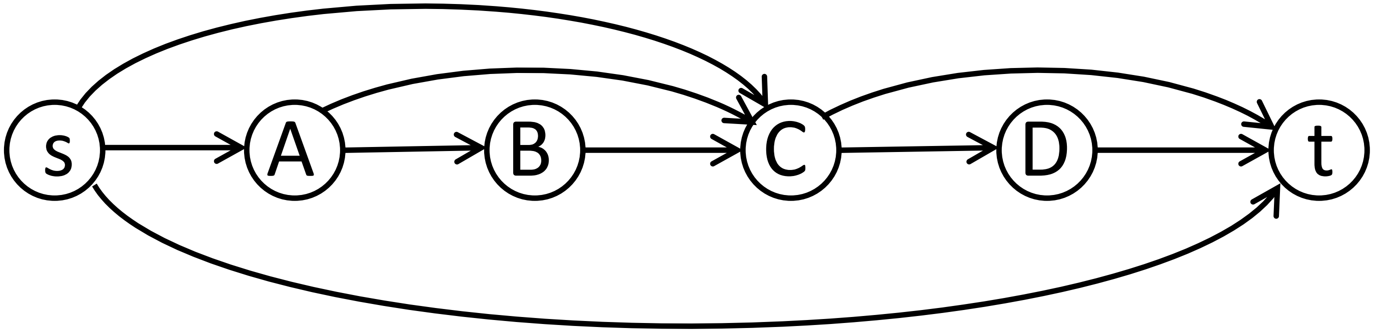

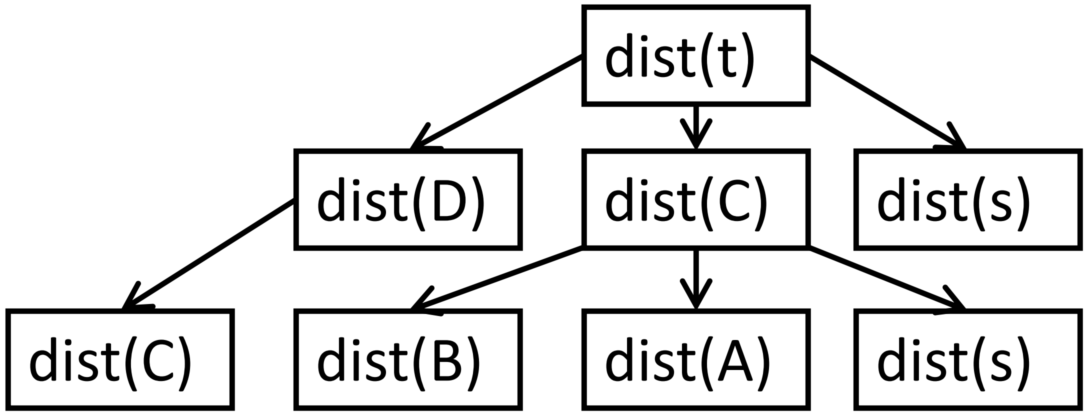

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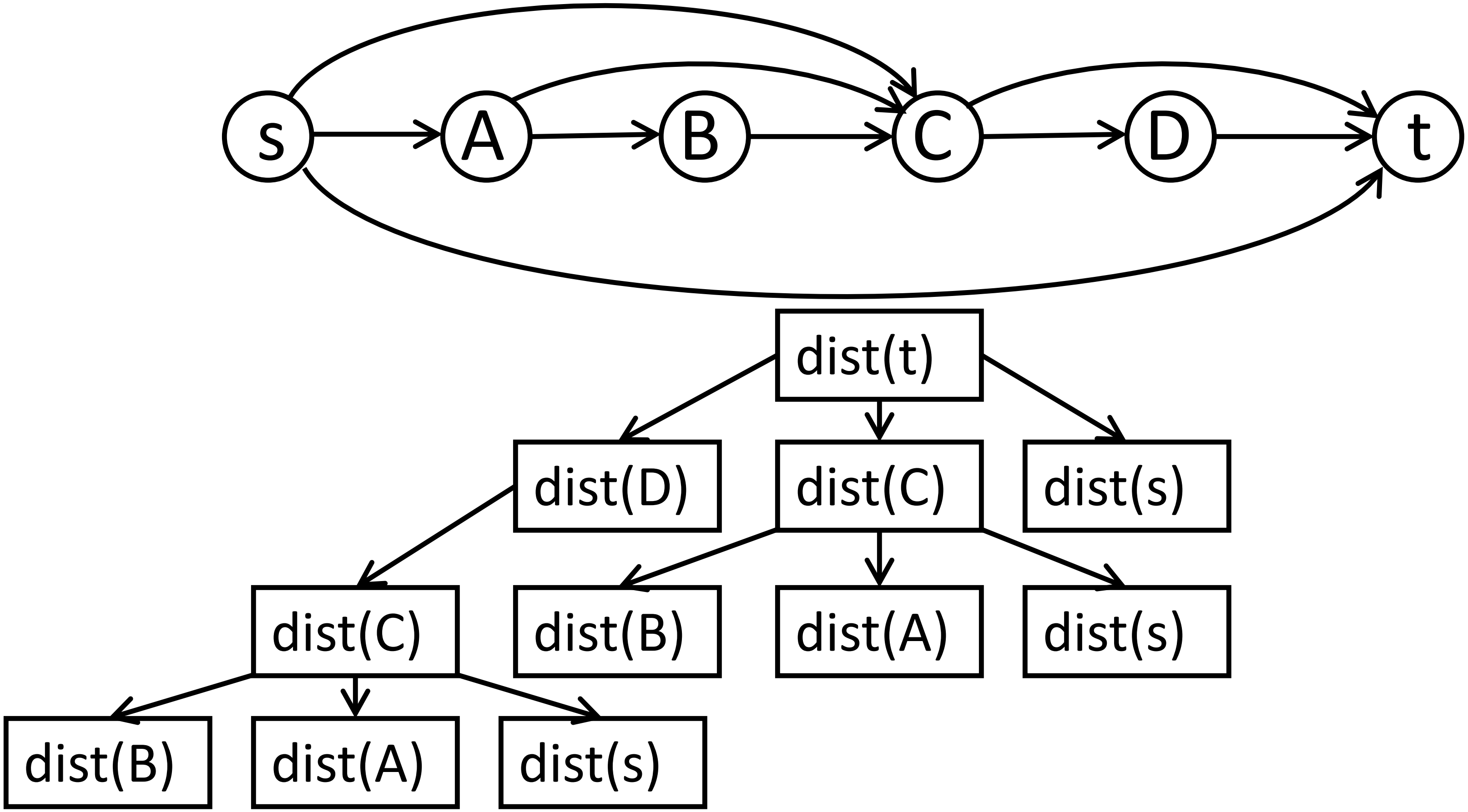
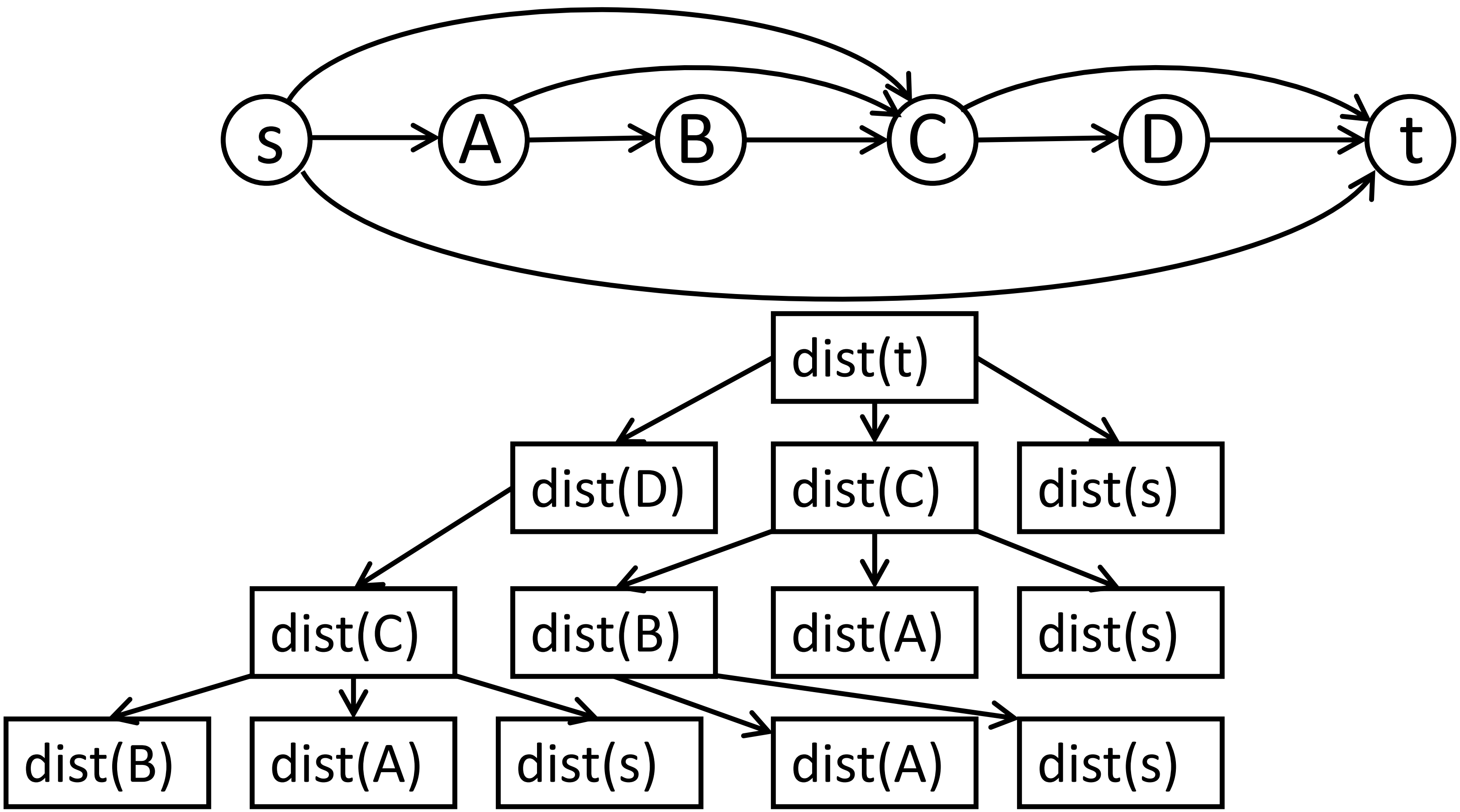

s
A
B
C
D
t
dist(t)
dist(D)
dist(C)
dist(s)
dist(C)
dist(B)
dist(A)
dist(s)
dist(B)
dist(A)
dist(s)

Example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dist(C)
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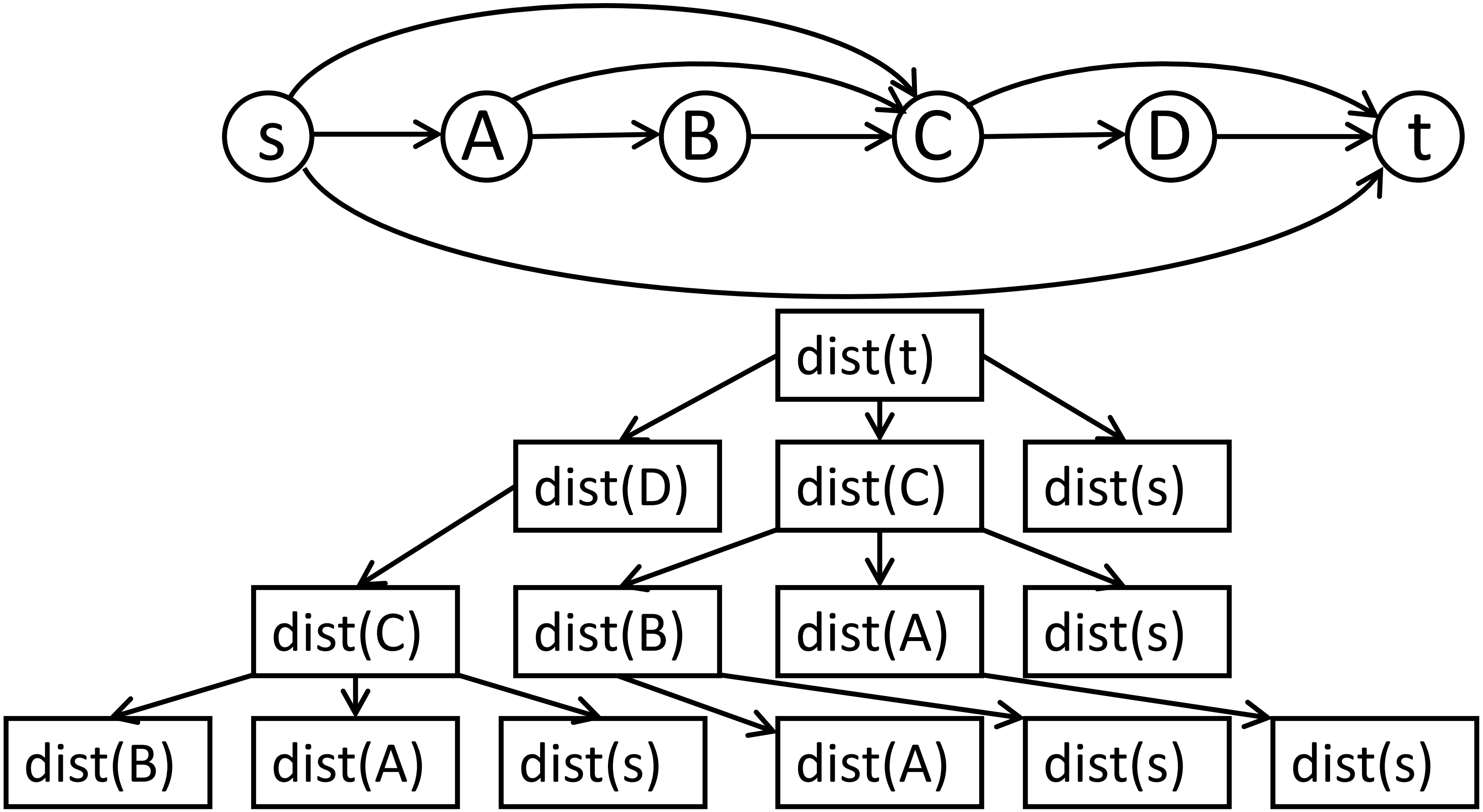
dist(s)
dist(B)
dist(A)
dist(s)
dist(A)
dist(s)

Example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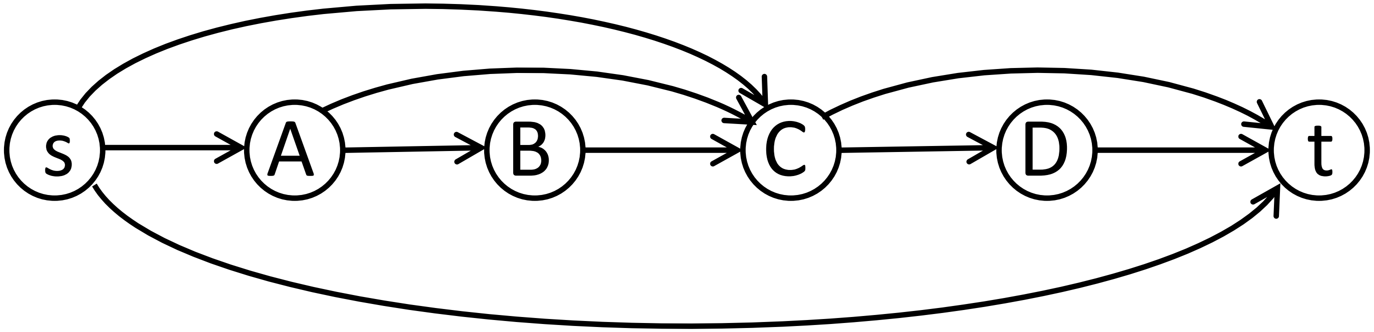
Example



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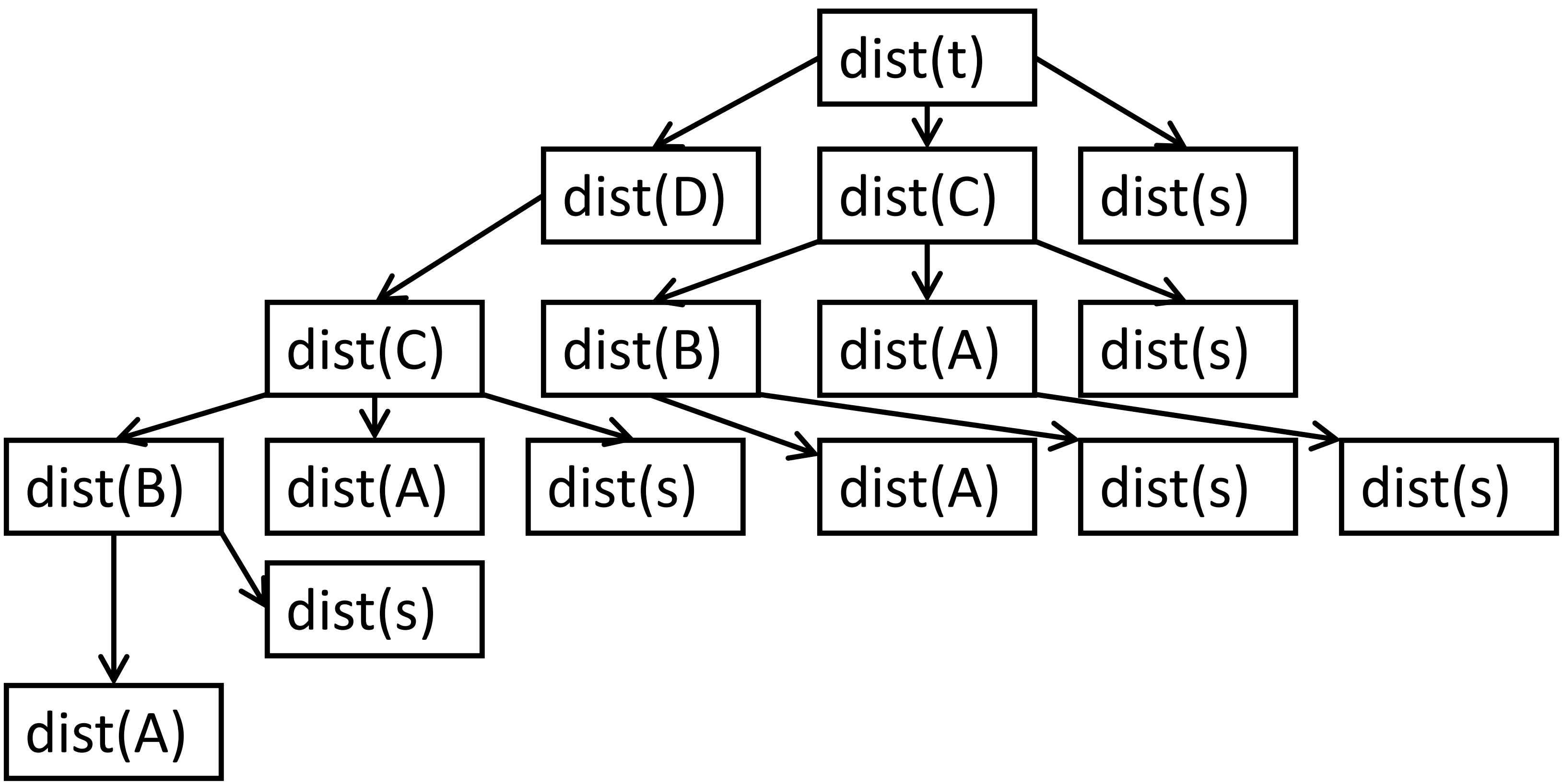

dist(t)
dist(D)
dist(C)
dist(s)
dist(C)
dist(B)
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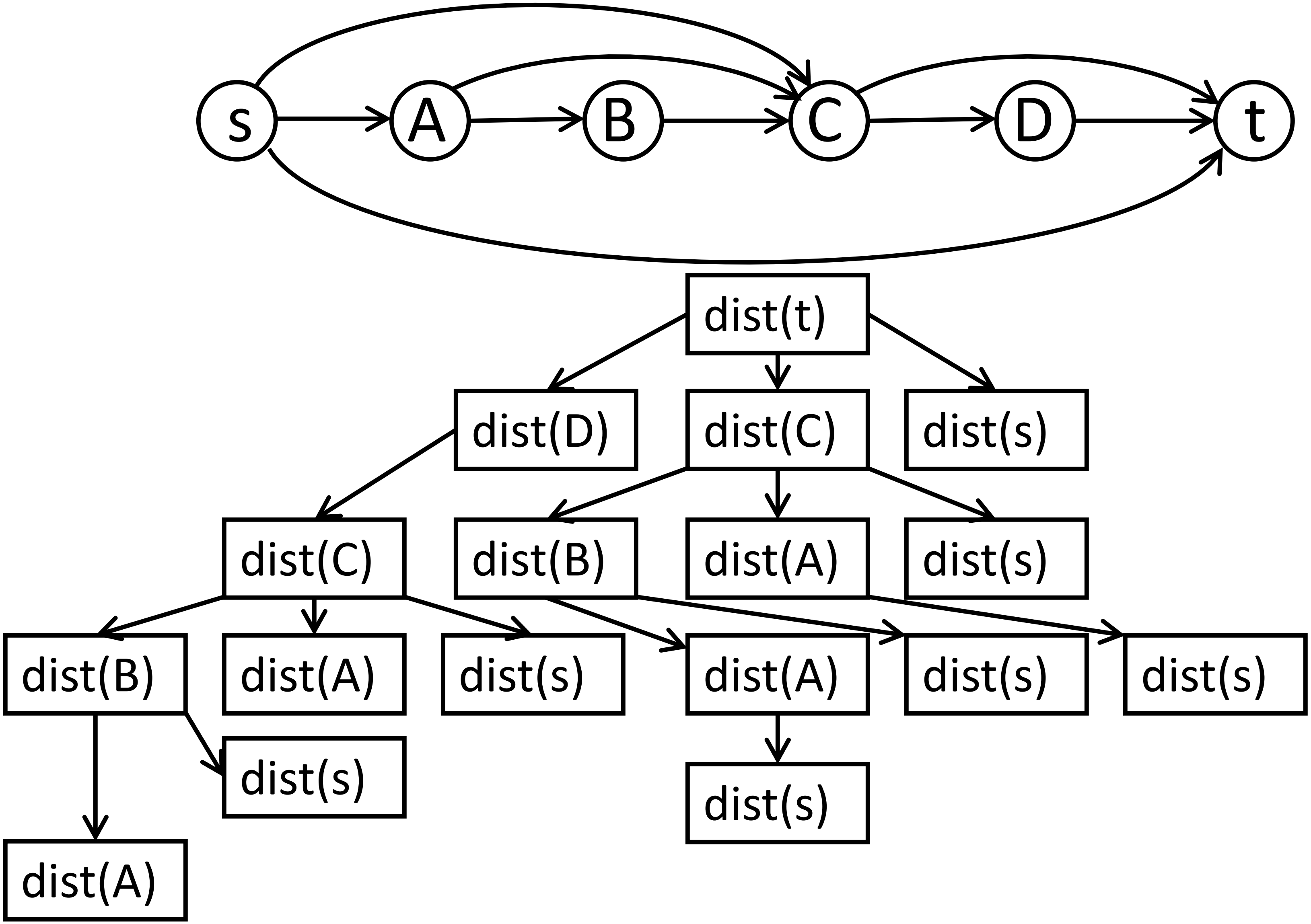
dist(A)

Example


s
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dist(t)
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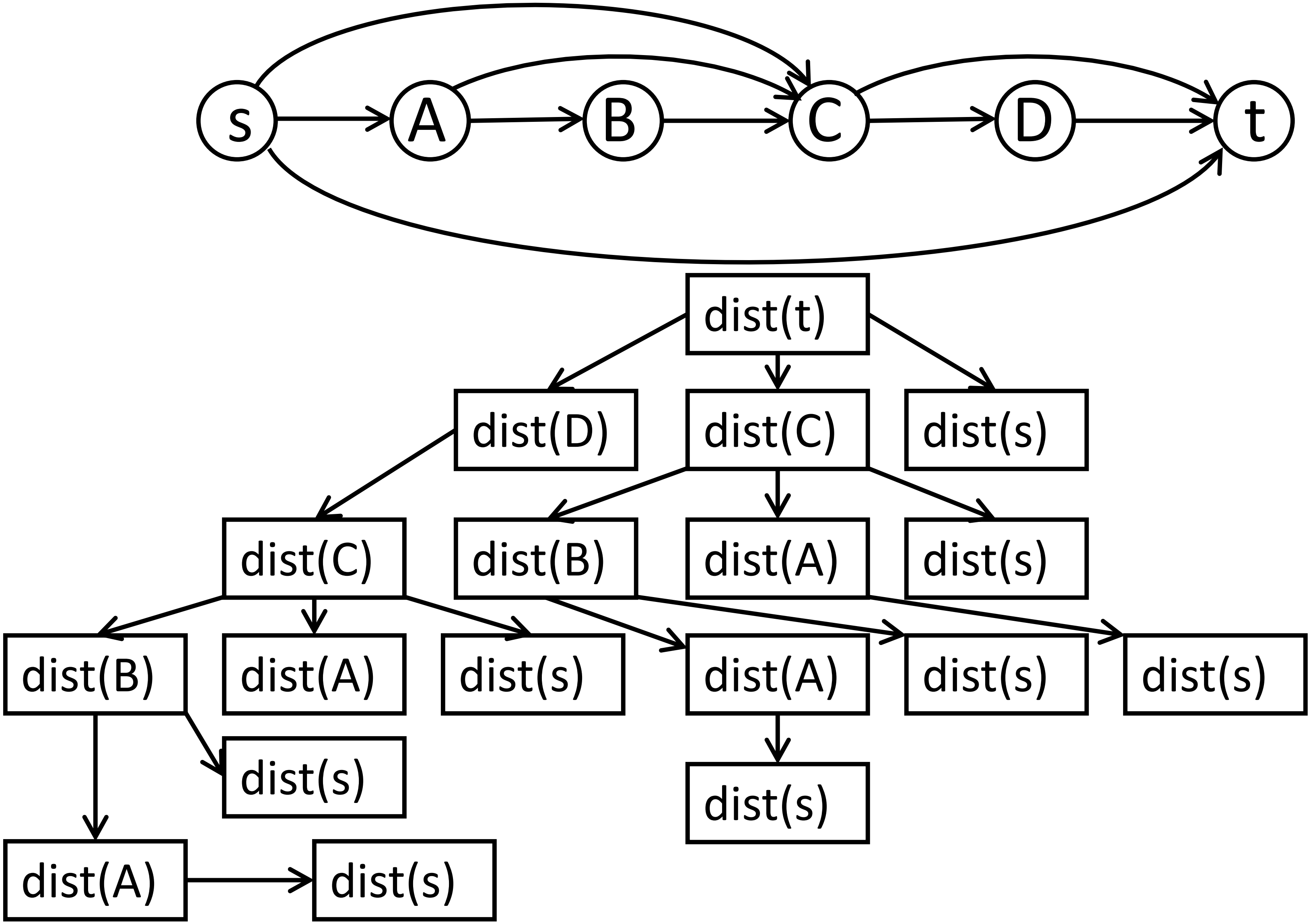
Example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Simplify by Tabulating

Instead of computing these values recursively, compute them one at a time, recording them. Then in the future, you only need to do table lookups.

Dynamic Programming

Our final general algorithmic technique:

1. Break problem into smaller subproblems.
2. Find recursive formula solving one subproblem in terms of simpler ones.
3. Tabulate answers and solve all subproblems.

Question: Dynamic Program

Which of the following algorithms that we have covered so far involves a dynamic program?

A) Bellman-Ford
B) Optimal Caching
C) Computing SCCs
D) Closest Pair of Points
E) Karatsuba Multiplication

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Which of the following algorithms that we have covered so far involves a dynamic program?

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C) Computing SCCs
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E) Karatsuba Multiplication

$$\mathrm{dist}_k(w) = \min_{(v,w)\in E} \mathrm{dist}_{k-1}(v) + \ell(v,w).$$

Subsequences

Given a sequence, say `ABCBA`, a <u>subsequence</u> is the sequence obtained by deleting some letters and leaving the rest in the same order.

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For example, ABCBA would have a subsequence ABCBA = ACB.

Longest Common Subsequence

We say that a sequence is a common subsequence of two others, if it is a subsequence of both.

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For example `ABC` is a common subsequence of `ADBCA` and `AABBC`.

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For example `ABC` is a common subsequence of `ADBCA` and `AABBC`.

Problem: Given two sequences compute the longest common subsequence. That is the subsequence with as many letters as possible.

Question: LCSS

What is the length of the longest common subsequence of `ABCBA` and `ABACA`?

A) 1
B) 2
C) 3
D) 4
E) 5

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What is the length of the longest common subsequence of ABCBA and ABACA?

A) 1
B) 2
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ABCA = ABCBA = ABACA

Case Analysis

How do we compute LCSS($A_1A_2 \ldots A_n$, $B_1B_2 \ldots B_m$)?

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How do we compute LCSS($A_1A_2 \ldots A_n$, $B_1B_2 \ldots B_m$)?

Consider cases for the common subsequence:

1. It does not use A_n.
2. It does not use B_m.
3. It uses both A_n and B_m and these characters are the same.

Case 1

If the common subsequence does not use A_n, it is actually a common subsequence of

$A_1A_2 \ldots A_{n-1}$, and $B_1B_2 \ldots B_m$

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Therefore, in this case, the longest common subsequence would be LCSS($A_1A_2 \ldots A_{n-1}$, $B_1B_2 \ldots B_m$).

Case 2

If the common subsequence does not use B_m, it is actually a common subsequence of

$A_1 A_2 \ldots A_n$, and $B_1 B_2 \ldots B_{m-1}$

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Therefore, in this case, the longest common subsequence would be LCSS($A_1A_2 \ldots A_n$, $B_1B_2 \ldots B_{m-1}$).

Case 3

If a common subsequence uses both A_n and B_m…

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If a common subsequence uses both $\mathtt{A}_\mathtt{n}$ and $\mathtt{B}_\mathtt{m}$…

- These characters must be the same.
- Such a subsequence is obtained by taking a common subsequence of:
  $\mathtt{A}_1\mathtt{A}_2\ldots\mathtt{A}_{\mathtt{n-1}}$, and $\mathtt{B}_1\mathtt{B}_2\ldots\mathtt{B}_{\mathtt{m-1}}$
  and adding a copy of $\mathtt{A}_\mathtt{n}$ = $\mathtt{B}_\mathtt{m}$ to the end.

Case 3

If a common subsequence uses both A_n and B_m…

- These characters must be the same.
- Such a subsequence is obtained by taking a common subsequence of:
  $A_1A_2…A_{n-1}$, and $B_1B_2…B_{m-1}$
  and adding a copy of $A_n = B_m$ to the end.
- The longest length of such a subsequence is
  LCSS($A_1A_2…A_{n-1}$, $B_1B_2…B_{m-1}$)+1.

Recursion

On the other hand, the longest common subsequence must come from one of these cases. In particular, it will always be the one that gives the biggest result.

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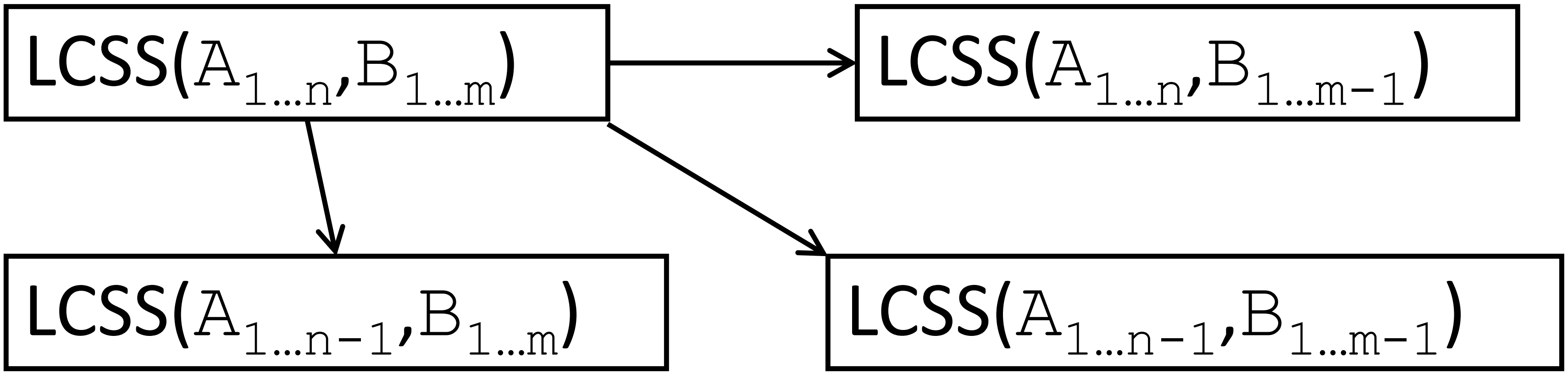
$\text{LCSS}(A_1A_2\ldots A_n, B_1B_2\ldots B_m) =$
$\quad \text{Max}(\text{LCSS}(A_1A_2\ldots A_{n-1}, B_1B_2\ldots B_m),$
$\quad\quad \text{LCSS}(A_1A_2\ldots A_n, B_1B_2\ldots B_{m-1}),$
$\quad\quad [\text{LCSS}(A_1A_2\ldots A_{n-1}, B_1B_2\ldots B_{m-1})+1])$

[where the last option is only allowed if $A_n = B_m$]

Recursion

LCSS($A_{1\ldots n}$,$B_{1\ldots m}$)

Recursion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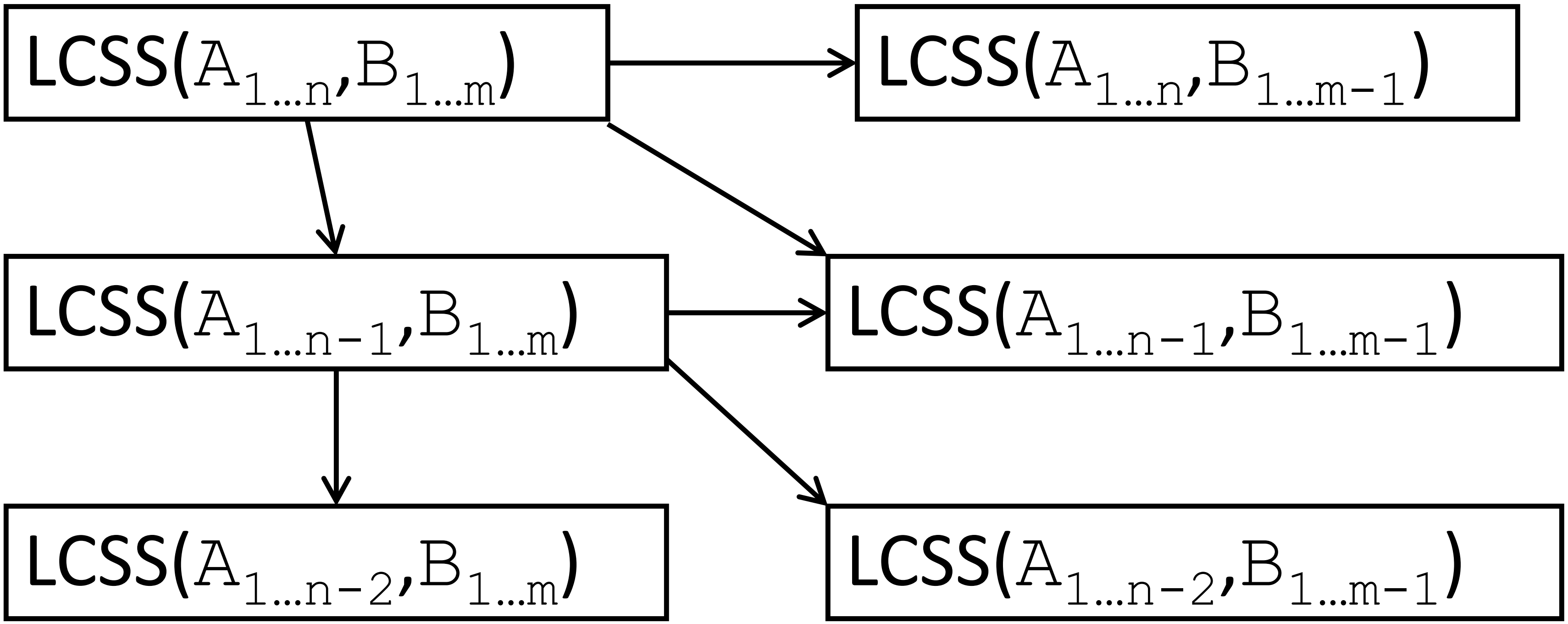
Recursion



LCSS($A_{1…n}$,$B_{1…m}$)
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LCSS($A_{1…n-1}$,$B_{1…m-1}$)
LCSS($A_{1…n-2}$,$B_{1…m}$)
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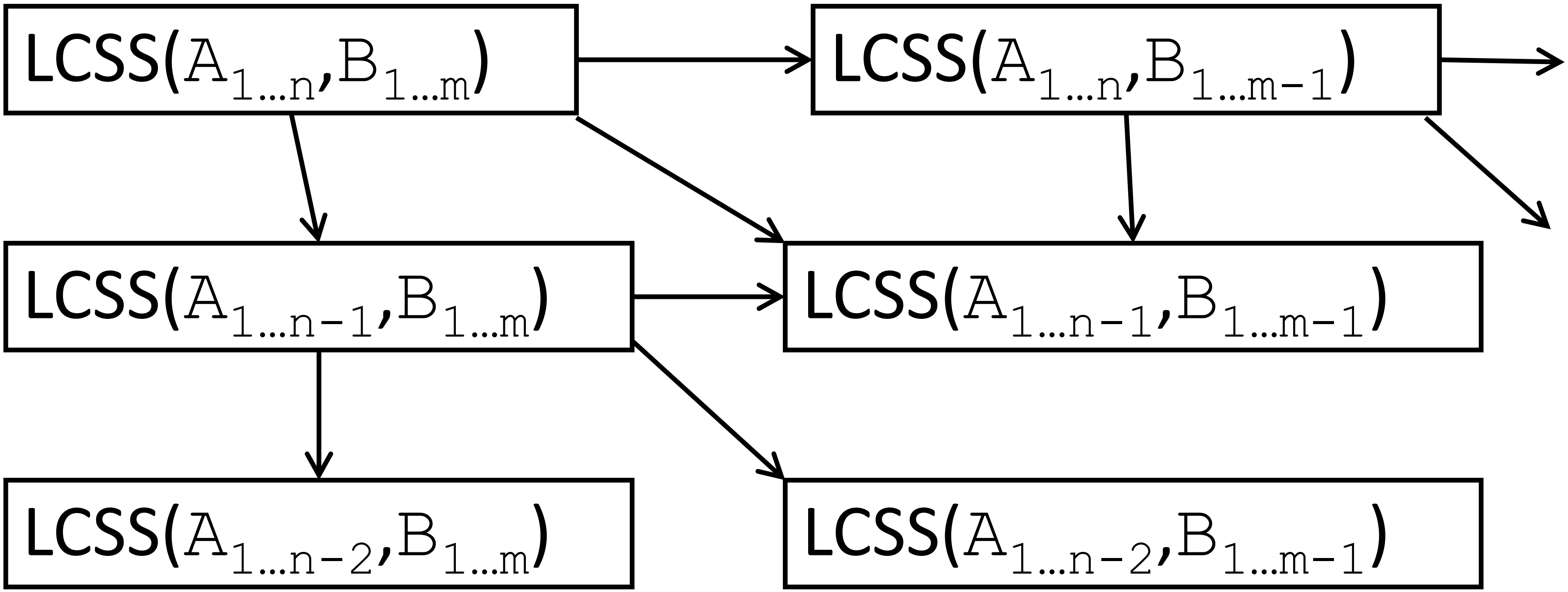
Recursion



LCSS(A1…n,B1…m)
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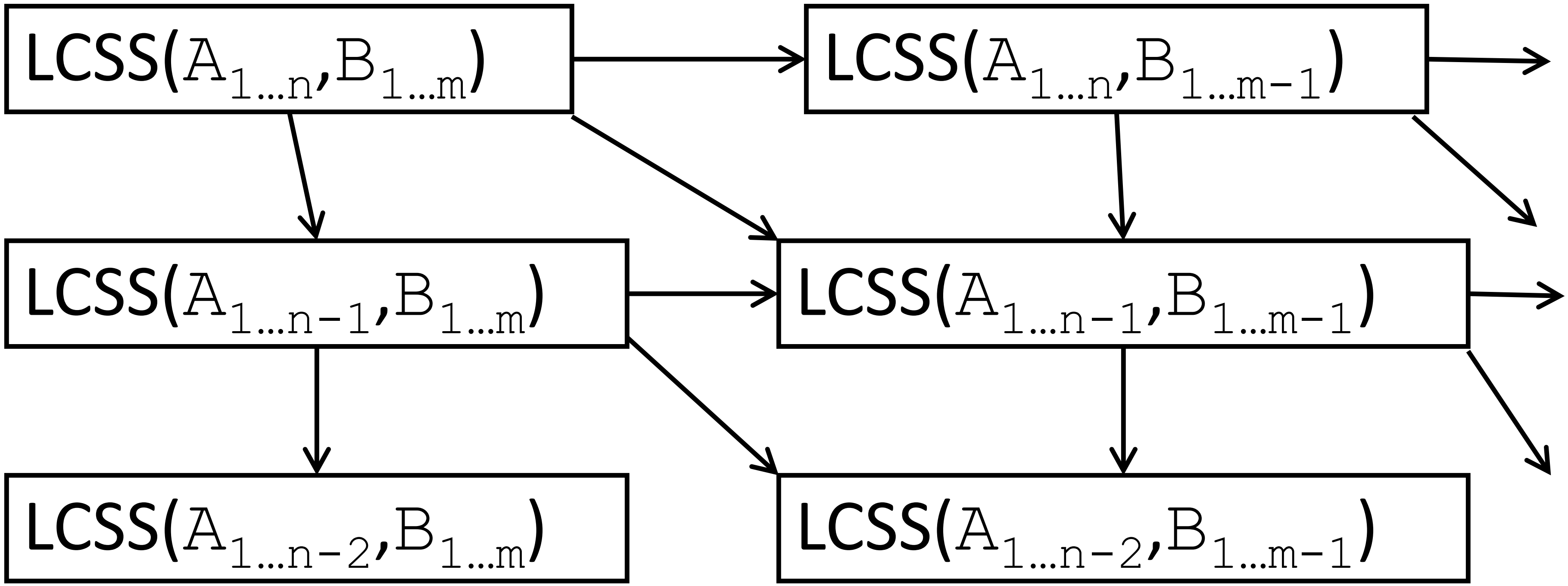
Recursion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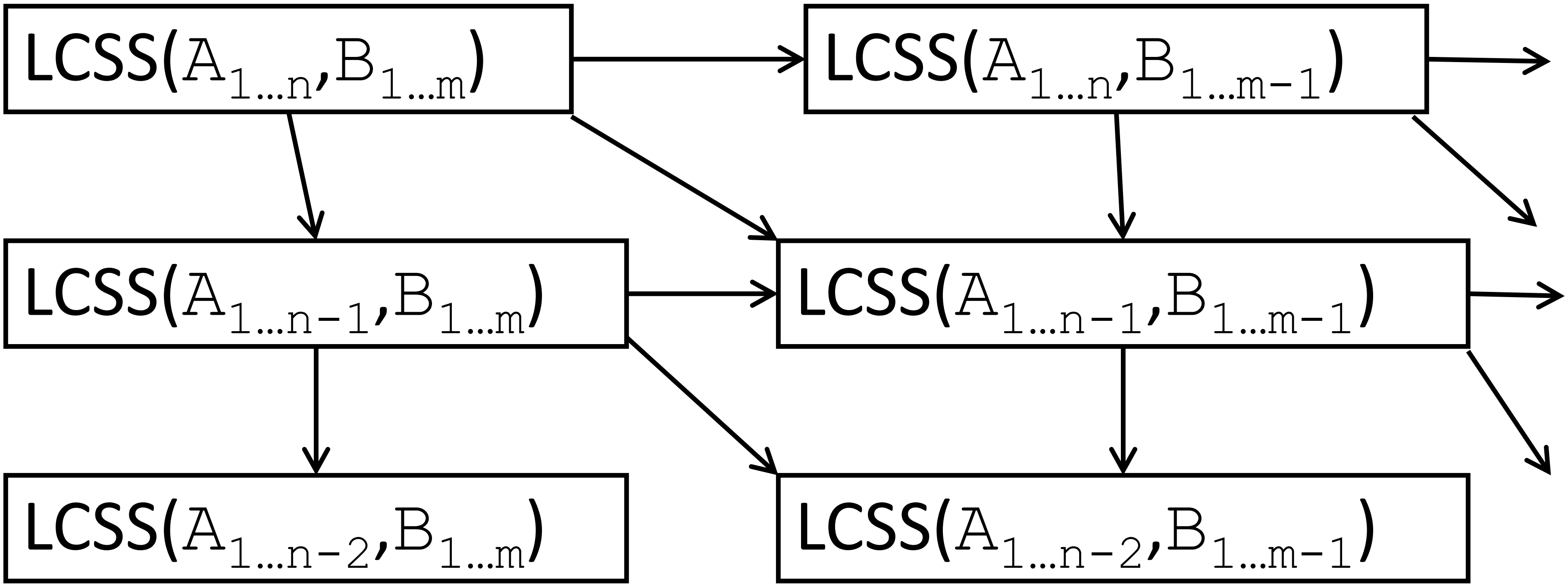
Recursion



Key Point: Subproblem reuse

Only ever see LCSS($A_1A_2...A_k$, $B_1B_2...B_\ell$)

Base Case

Our recursion also needs a base case.

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In this case we have:

LCSS(Ø,$B_1B_2...B_m$) = LCSS($A_1A_2...A_n$, Ø) = 0.

Algorithm

```
LCSS(A1A2…An,B1B2…Bm)
  Initialize Array T[0…n,0…m]
    \\ T[i,j] will store LCSS(A1A2…Ai,B1B2…Bj)
  For i = 0 to n
    For j = 0 to m
      If (i = 0) OR (j = 0)
        T[i,j] ← 0
      Else If Ai = Bj
        T[i,j] ← max(T[i-1,j],T[i,j-1],T[i-1,j-1]+1)
      Else
        T[i,j] ← max(T[i-1,j],T[i,j-1])
  Return T[n,m]
```

Algorithm

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      If (i = 0) OR (j = 0)
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      Else
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  Return T[n,m]
```

O(nm) iterations

Algorithm

$\texttt{LCSS}(A_1A_2\ldots A_n, B_1B_2\ldots B_m)$

  Initialize Array T[0…n,0…m]

    \\ T[i,j] will store $\texttt{LCSS}(A_1A_2\ldots A_i, B_1B_2\ldots B_j)$

  For i = 0 to n

    For j = 0 to m

O(nm) iterations

      If (i = 0) OR (j = 0)

        T[i,j] ← 0

      Else If $A_i = B_j$

        T[i,j] ← max(T[i-1,j],T[i,j-1],T[i-1,j-1]+1)

      Else

        T[i,j] ← max(T[i-1,j],T[i,j-1])

O(1)

  Return T[n,m]

Example

	∅	A	A B	A B A	A B A C	A B A C A
∅						
A						
AB						
ABC						
ABCB						
ABCBA						

Example

	∅	A	AB	ABA	ABAC	ABACA
∅	0					
A						
AB						
ABC						
ABCB						
ABCBA						

Example

	∅	A	AB	ABA	ABAC	ABACA
∅	0	0				
A						
AB						
ABC						
ABCB						
ABCBA						

Example

	∅	A	AB	ABA	ABAC	ABACA
∅	0	0	0			
A						
AB						
ABC						
ABCB						
ABCBA						

Example

	Ø	A	AB	ABA	ABAC	ABACA
Ø	0	0	0	0		
A						
AB						
ABC						
ABCB						
ABCBA						

Example

	∅	A	AB	ABA	ABAC	ABACA
∅	0	0	0	0	0	
A						
AB						
ABC						
ABCB						
ABCBA						

Example

	∅	A	AB	ABA	ABAC	ABACA
∅	0	0	0	0	0	0
A						
AB						
ABC						
ABCB						
ABCBA						

Example

	Ø	A	AB	ABA	ABAC	ABACA
Ø	0	0	0	0	0	0
A	0					
AB						
ABC						
ABCB						
ABCBA						

Example

	Ø	A	AB	ABA	ABAC	ABACA
Ø	0	0	0	0	0	0
A	0	1				
AB						
ABC						
ABCB						
ABCBA						

Example

	∅	A	AB	ABA	ABAC	ABACA
∅	0	0	0	0	0	0
A	0	1	1			
AB						
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Example

	∅	A	AB	ABA	ABAC	ABACA
∅	0	0	0	0	0	0
A	0	1	1	1		
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	∅	A	AB	ABA	ABAC	ABACA
∅	0	0	0	0	0	0
A	0	1	1	1	1	
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A	0	1	1	1	1	1
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∅	0	0	0	0	0	0
A	0	1	1	1	1	1
AB	0					
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Example

	∅	A	AB	ABA	ABAC	ABACA
∅	0	0	0	0	0	0
A	0	1	1	1	1	1
AB	0	1				
ABC						
ABCB						
ABCBA						

Example

	∅	A	AB	ABA	ABAC	ABACA
∅	0	0	0	0	0	0
A	0	1	1	1	1	1
AB	0	1	2			
ABC						
ABCB						
ABCBA						

Example

	∅	A	AB	ABA	ABAC	ABACA
∅	0	0	0	0	0	0
A	0	1	1	1	1	1
AB	0	1	2	2		
ABC						
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ABCBA						

Example

	∅	A	AB	ABA	ABAC	ABACA
∅	0	0	0	0	0	0
A	0	1	1	1	1	1
AB	0	1	2	2	2	
ABC						
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Example

	∅	A	AB	ABA	ABAC	ABACA
∅	0	0	0	0	0	0
A	0	1	1	1	1	1
AB	0	1	2	2	2	2
ABC						
ABCB						
ABCBA						

Example

	∅	A	AB	ABA	ABAC	ABACA
∅	0	0	0	0	0	0
A	0	1	1	1	1	1
AB	0	1	2	2	2	2
ABC	0					
ABCB						
ABCBA						

Example

	∅	A	AB	ABA	ABAC	ABACA
∅	0	0	0	0	0	0
A	0	1	1	1	1	1
AB	0	1	2	2	2	2
ABC	0	1				
ABCB						
ABCBA						

Example

	∅	A	AB	ABA	ABAC	ABACA
∅	0	0	0	0	0	0
A	0	1	1	1	1	1
AB	0	1	2	2	2	2
ABC	0	1	2			
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Example

	∅	A	AB	ABA	ABAC	ABACA
∅	0	0	0	0	0	0
A	0	1	1	1	1	1
AB	0	1	2	2	2	2
ABC	0	1	2	2		
ABCB						
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	∅	A	AB	ABA	ABAC	ABACA
∅	0	0	0	0	0	0
A	0	1	1	1	1	1
AB	0	1	2	2	2	2
ABC	0	1	2	2	3	
ABCB						
ABCBA						

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	∅	A	AB	ABA	ABAC	ABACA
∅	0	0	0	0	0	0
A	0	1	1	1	1	1
AB	0	1	2	2	2	2
ABC	0	1	2	2	3	3
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A	0	1	1	1	1	1
AB	0	1	2	2	2	2
ABC	0	1	2	2	3	3
ABCB	0					
ABCBA						

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∅	0	0	0	0	0	0
A	0	1	1	1	1	1
AB	0	1	2	2	2	2
ABC	0	1	2	2	3	3
ABCB	0	1				
ABCBA						

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	∅	A	AB	ABA	ABAC	ABACA
∅	0	0	0	0	0	0
A	0	1	1	1	1	1
AB	0	1	2	2	2	2
ABC	0	1	2	2	3	3
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ABCBA						

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A	0	1	1	1	1	1
AB	0	1	2	2	2	2
ABC	0	1	2	2	3	3
ABCB	0	1	2	2		
ABCBA						

Example

	∅	A	AB	ABA	ABAC	ABACA
∅	0	0	0	0	0	0
A	0	1	1	1	1	1
AB	0	1	2	2	2	2
ABC	0	1	2	2	3	3
ABCB	0	1	2	2	3	
ABCBA						

Example

	∅	A	AB	ABA	ABAC	ABACA
∅	0	0	0	0	0	0
A	0	1	1	1	1	1
AB	0	1	2	2	2	2
ABC	0	1	2	2	3	3
ABCB	0	1	2	2	3	3
ABCBA						

Example

	Ø	A	AB	ABA	ABAC	ABACA
Ø	0	0	0	0	0	0
A	0	1	1	1	1	1
AB	0	1	2	2	2	2
ABC	0	1	2	2	3	3
ABCB	0	1	2	2	3	3
ABCBA	0					

Example

	∅	A	AB	ABA	ABAC	ABACA
∅	0	0	0	0	0	0
A	0	1	1	1	1	1
AB	0	1	2	2	2	2
ABC	0	1	2	2	3	3
ABCB	0	1	2	2	3	3
ABCBA	0	1				

Example

	Ø	A	AB	ABA	ABAC	ABACA
Ø	0	0	0	0	0	0
A	0	1	1	1	1	1
AB	0	1	2	2	2	2
ABC	0	1	2	2	3	3
ABCB	0	1	2	2	3	3
ABCBA	0	1	2			

Example

	∅	A	AB	ABA	ABAC	ABACA
∅	0	0	0	0	0	0
A	0	1	1	1	1	1
AB	0	1	2	2	2	2
ABC	0	1	2	2	3	3
ABCB	0	1	2	2	3	3
ABCBA	0	1	2	3		

Example

	∅	A	AB	ABA	ABAC	ABACA
∅	0	0	0	0	0	0
A	0	1	1	1	1	1
AB	0	1	2	2	2	2
ABC	0	1	2	2	3	3
ABCB	0	1	2	2	3	3
ABCBA	0	1	2	3	3	

Example

	∅	A	AB	ABA	ABAC	ABACA
∅	0	0	0	0	0	0
A	0	1	1	1	1	1
AB	0	1	2	2	2	2
ABC	0	1	2	2	3	3
ABCB	0	1	2	2	3	3
ABCBA	0	1	2	3	3	4

Example

	∅	A	AB	ABA	ABAC	ABACA
∅	0	0	0	0	0	0
A	0	1	1	1	1	1
AB	0	1	2	2	2	2
ABC	0	1	2	2	3	3
ABCB	0	1	2	2	3	3
ABCBA	0	1	2	3	3	4

Example

	∅	A	AB	ABA	ABAC	ABACA
∅	0	0	0	0	0	0
A	0	1	1	1	1	1
AB	0	1	2	2	2	2
ABC	0	1	2	2	3	3
ABCB	0	1	2	2	3	3
ABCBA	0	1	2	3	3	4

Example

	∅	A	AB	ABA	ABAC	ABACA
∅	0	0	0	0	0	0
A	0	1	1	1	1	1
AB	0	1	2	2	2	2
ABC	0	1	2	2	3	3
ABCB	0	1	2	2	3	3
ABCBA	0	1	2	3	3	4

Example

	∅	A	AB	ABA	ABAC	ABACA
∅	0	0	0	0	0	0
A	0	1	1	1	1	1
AB	0	1	2	2	2	2
ABC	0	1	2	2	3	3
ABCB	0	1	2	2	3	3
ABCBA	0	1	2	3	3	4

Example

	∅	A	AB	ABA	ABAC	ABACA
∅	0	0	0	0	0	0
A	0	1	1	1	1	1
AB	0	1	2	2	2	2
ABC	0	1	2	2	3	3
ABCB	0	1	2	2	3	3
ABCBA	0	1	2	3	3	4

Example

	∅	A	AB	ABA	ABAC	ABACA
∅	0	0	0	0	0	0
A	0	1	1	1	1	1
AB	0	1	2	2	2	2
ABC	0	1	2	2	3	3
ABCB	0	1	2	2	3	3
ABCBA	0	1	2	3	3	4

Example

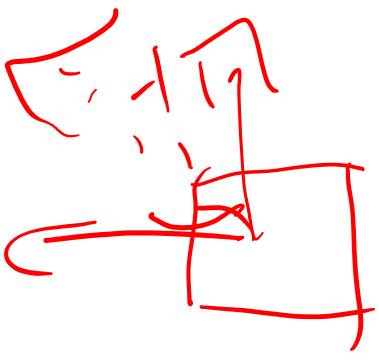
	∅	A	AB	ABA	ABAC	ABACA
∅	0	0	0	0	0	0
A	0	1	1	1	1	1
AB	0	1	2	2	2	2
ABC	0	1	2	2	3	3
ABCB	0	1	2	2	3	3
ABCBA	0	1	2	3	3	4

Example

	∅	A	AB	ABA	ABAC	ABACA
∅	0	0	0	0	0	0
A	0	1	1	1	1	1
AB	0	1	2	2	2	2
ABC	0	1	2	2	3	3
ABCB	0	1	2	2	3	3
ABCBA	0	1	2	3	3	4

Example

	∅	A	AB	ABA	ABAC	ABACA
∅	0	0	0	0	0	0
A	0	1	1	1	1	1
AB	0	1	2	2	2	2
ABC	0	1	2	2	3	3
ABCB	0	1	2	2	3	3
ABCBA	0	1	2	3	3	4



Example

	Ø	A	AB	ABA	ABAC	ABACA
Ø	0	0	0	0	0	0
A	0	1	1	1	1	1
AB	0	1	2	2	2	2
ABC	0	1	2	2	3	3
ABCB	0	1	2	2	3	3
ABCBA	0	1	2	3	3	4

Example

	∅	A	AB	ABA	ABAC	ABACA
∅	0	0	0	0	0	0
A	0	1	1	1	1	1
AB	0	1	2	2	2	2
ABC	0	1	2	2	3	3
ABCB	0	1	2	2	3	3
ABCBA	0	1	2	3	3	4

String:

ABCA

Proof of Correctness

Prove by induction that each value assigned to T[i,j] is the correct value for LCSS($A_1 A_2 \ldots A_i, B_1 B_2 \ldots B_j$).

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Prove by induction that each value assigned to T[i,j] is the correct value for LCSS($A_1 A_2 \ldots A_i, B_1 B_2 \ldots B_j$).

Base Case: When i or j is 0 we assign 0.

Inductive Step: Assuming that previous values are assigned correctly, T[i,j] gets correct value because of recursion for LCSS and inductive hypothesis (and that we have previously filled in T[i-1,j], T[i,j-1] and T[i-1,j-1]).

Notes about DP

- General Correct Proof Outline:
  - Prove by induction that each table entry is filled out correctly
  - Use base-case and recursion

Notes about DP

- General Correct Proof Outline:
  - Prove by induction that each table entry is filled out correctly
  - Use base-case and recursion
- Runtime of DP:
  - Usually [Number of subproblems]x[Time per subproblem]