CSE 101 Homework 5

Winter 2021

This homework is due on gradescope Friday February 26th at 11:59pm pacific time. Remember to justify your work even if the problem does not explicitly say so. Writing your solutions in LaTeXis recommend though not required.

Question 1 (Minimum Spanning Subgraph, 30 points). *When introducing the Minimum Spanning Tree problem, we considered the problem of finding the least costly set of edges that connects a graph G. We showed that if the edge weights are all non-negative that these edges would necessarily form a tree. However, if negative edge weights are allowed, this no longer needs to be the case. Give an algorithm that given a weighted, undirected graph G, provides a subset S of edges so that any vertex in G can be reached from any other using only edges in S and so that subject to this, the total weight of S is as small as possible. For full credit, your algorithm should run in time $O(|V|\log(|V|) + |E|)$ or better.*

Question 2 (Multiple MSTs, 40 points). *(a) Give an example of a graph that has more than one minimum spanning tree. [5 points]*

(b) Show that for any graph G with minimum spanning trees T and T' that for each weight w, T and T' contain the same number of edges of weight w. Hint: Run an exchange-like argument to slowly turn T into T' without changing the number of edges of any weight. [35 points]

Question 3 (LCSS without Double Letters, 30 points). *Say that a string has a double letter if two consecutive letters in the string are the same. Give an algorithm that given two strings of length n, $A = a_1a_2\ldots a_n$ and $B = b_1b_2\ldots b_n$, finds the longest sequence $C = c_1c_2\ldots c_m$ so that C is a subsequence of both A and B and has no double letters. For full credit, your algorithm should run in time $O(n^3)$ or better.*

Question 4 (Extra credit, 1 point). *Approximately how much time did you spend working on this homework?*