CSE 101 Homework 3

Winter 2021

This homework is due on gradescope Friday February 5th at 11:59pm pacific time. Remember to justify your work even if the problem does not explicitly say so. Writing your solutions in LaTeXis recommend though not required.

Question 1 (Public Transit on a Budget, 40 points). *Lars is trying to get around town. He has various options for transportation with the possible routes represented by edges of a directed graph G. Each edge e has a positive integer cost* $\text{cost}(e)$ *dollars and a time it takes to traverse* $\text{time}(e)$*. Lars has a limited number N of dollars and would like to get between two locations (s and t) in as little total time as possible.*

(a) Give an algorithm that given G, s, t, the functions cost *and* time *and the total budget N, determines the shortest time to get from s to t under the budget. For full credit your algorithm should run in time $O(N(|V| + |E|))$ or better. [20 points]*

(b) Suppose that some routes are allowed to have a cost of 0. Provide an algorithm to solve the new version of this problem with runtime $O(N(|V|\log(|V|) + |E|))$ or better. [20 points]

Question 2 (Negative Cycle Finding, 35 points). *We know how to use Bellman-Ford to determine whether or not a weighted, directed graph G has a negative weight cycle. Give an $O(|V||E|)$ time algorithm to find such a cycle if it exists. Hint: If there is such a cycle use Bellman-Ford to find a vertex v with $\text{dist}_{|V|-1}(v) > \text{dist}_{|V|}(v)$ and compute the paths involved. From this you should be able to find a cycle. You may also need to modify your graph some to deal with the possibility of a negative weight cycle not reachable from your chosen starting vertex s.*

Question 3 (Divide and Conquer Recursions, 25 points). *Give the asymptotic runtimes of the following divide and conquer algorithms.*

(a) An algorithm that splits the input into two inputs of a two-thirds the size and then does $\Theta(n)$ extra work. [2 points]

(b) An algorithm that splits the input into five inputs of half the size and then does $\Theta(n^{5/2})$ extra work. [2 points]

(c) An algorithm that splits the input into four inputs of half the size and then does $\Theta(n^2)$ extra work. [2 points]

(d) An algorithm that splits the input into six inputs of a third the size and then does $\Theta(n^{3/2})$ extra work. [2 points]

(e) An algorithm that splits the input into two inputs of a third the size and then does $\Theta(n)$ extra work. [2 points]

(f) An algorithm that splits the input into two inputs of half the size and then does $\Theta(n\log(n))$ extra work. Note: you cannot use the Master Theorem in this case. You may have to do some work to derive the answer. [15 points]

Question 4 (Extra credit, 1 point). *Approximately how much time did you spend working on this homework?*