CSE 101 Homework 0

Winter 2022

This homework is due on gradescope Friday January 7th at 11:59pm on gradescope. Remember to justify your work even if the problem does not explicitly say so. Writing your solutions in \LaTeX is recommended though not required.

Question 1 (Program Runtimes, 20 points). Consider the following two programs:

\textbf{Alg1}(n)
\begin{verbatim}
   for i = 1 to n 
      for j = 1 to 2^n 
         Print(j)
\end{verbatim}

\textbf{Alg2}(n)
\begin{verbatim}
   for i = 1 to n 
      for j = 1 to 2^i 
         Print(j)
\end{verbatim}

For each of these programs give the asymptotic runtime as $\Theta(f(n))$ for some function $f$ and justify your work.

Question 2 (Asymptotic Comparisons, 20 points). Sort the following functions of $n$ in terms of their asymptotic growth rates. In particular, ones should go later in the list if they are larger when sufficiently large values of $n$ are used as inputs. Which of these functions have polynomial growth rates? Remember to justify your answers.

- $a(n) = (\lfloor \log_2(n) \rfloor)!$
- $b(n) = n(n + 100)(n + 10000)$
- $c(n) = e\sqrt{n}$
- $d(n) = 6\log_2(n)$
- $e(n) = 10^{10^{10}} n^2$

Question 3 (Extremal Graph Theory, 30 points). Say that a graph $G$ has a path of length three if there exist distinct vertices $u, v, w, t$ with edges $(u, v), (v, w), (w, t)$. Show that a graph $G$ with 99 vertices and no path of length three has at most 99 edges.

Question 4 (Recurrence Relation, 30 points). Suppose that you have a function $T(n)$ defined by $T(1) = 1$ and
\[ T(n) = T(n - 1) + T(\lfloor n/2 \rfloor) \]
for $n > 1$.

(a) Show that for any positive integer $k$ that $T(n)$ grows faster than $n^k$. Hint: Show that $T(n) \geq \lceil n/2 \rceil T(\lfloor n/4 \rfloor)$. [15 points]
(b) Consider the following “proof” that $T(n) = O(1)$ (note that this contradicts part (a)):

We proceed by strong induction on $n$. Clearly $T(1) = O(1)$, which gives us our base case. If we assume that $T(m) = O(1)$ for all $m < n$, then $T(n) = T(n-1) + T(\lfloor n/2 \rfloor) = 2O(1) = O(1)$. This completes our inductive step and proves that $T(n) = O(1)$ for all $n$.

What is wrong with the above proof? (Hint: Consider what the implied constant in the $O$ term would be.) [15 points]

**Question 5** (Extra credit, 1 point). Approximately how much time did you spend working on this homework?