CSE 101 Homework 0

Winter 2021

This homework is due on gradescope Friday January 8th at 11:59pm pacific time. Remember to justify your work even if the problem does not explicitly say so. Writing your solutions in LaTeXis recommend though not required.

Question 1 (Program Runtimes, 20 points). *Consider the following programs:*

```
Alg1(n):
For i = 1 to n
   j = 1
   while j < n
      j = j+1

Alg2(n):
For i = 1 to n
   j = 1
   while j < n
      j = j+i
```

For each of these algorithms, compute the asymptotic runtime in the form $\Theta(-)$*.*

Question 2 (Big-O Computations, 20 points). *Sort the functions below in terms of their asymptotic growth rates. Which of these functions have polynomial growth rates? Remember to justify your answers.*

- $a(n) = n + n^{1/2} + n^{1/3} + \ldots + n^{1/n}$
- $b(n) = 3^{\lceil log_2(n) \rceil}$
- $c(n) = n^2(2 + \cos(n))$
- $d(n) = n^{100}2^{n/2}$
- $e(n) = 2^n$

Question 3 (Walks and Paths, 30 points). *In a graph G we say that there is a* walk *from vertex u to another vertex w if there is a sequence of vertices $u = v_0, v_1, \ldots, v_n = w$ so that (v_i, v_{i+1}) is an edge of G for each $0 \leq i < n$. Prove that if there is a walk from u to w there is a walk where all of the vertices v_i are distinct. Hint: if two are the same show how you can use this to construct a shorter walk.*

Question 4 (Recurrence Relations, 30 points). *Consider the recurrence relation*

$$T(1) = 1, \quad T(n) = 2T(\lfloor n/2 \rfloor) + n.$$

(a) What is the exact value of $T(2^n)$? [10 points]

(b) Give a Θ expression for $T(n)$. Hint: compare its value to that at nearby powers of 2. [10 points]

(c) *Consider the following purported proof that $T(n) = O(n)$ by induction:*

If $n = 1$, then $T(1) = 1 = O(1)$.

If $T(m) = O(m)$ for $m < n$, then

$$T(n) = 2T(\lfloor n/2 \rfloor) + n = O(n) + O(n) = O(n).$$

Thus, $T(n) = O(n)$.

What is wrong with this proof? Hint: consider the implied constants in the big-Os. [10 points]

Question 5 (Extra credit, 1 point)**.** *Approximately how much time did you spend working on this homework?*