

**Question 1** (Knapsack, 30 points). Consider the knapsack problem where you are allowed to take items more than once, have a total capacity of 10 and have the following items available:

<i>Item</i>	<i>Weight</i>	<i>Value</i>
<i>A</i>	<i>2</i>	<i>4</i>
<i>B</i>	<i>3</i>	<i>5</i>
<i>C</i>	<i>4</i>	<i>9</i>
<i>D</i>	<i>5</i>	<i>10</i>
<i>E</i>	<i>7</i>	<i>16</i>

What is the best combination of items to take?

**Question 2** (Business Plan I, 35 points). *Harold is starting a new business. He begins with \$0 in capital and wants it to be worth as much as possible at the end of  $n$  weeks, at which point he plans to sell. In order to get there, he has  $k$  different strategies he can use to make money. Each strategy takes one week to implement, and can be used multiple times as needed. If Harold has  $x$  dollars at the start of a week and implements the  $i^{\text{th}}$  strategy that week, he will have  $f_i(x)$  dollars at the end of that week for some increasing function  $f_i$  (meaning that  $f_i(x) \geq f_i(y)$  whenever  $x \geq y$ ).*

*Harold has a simple plan for determining which strategies to use in which order. Each week he will note the number of dollars,  $x$ , that he has at the start of the week, and implement the plan with  $f_i(x)$  as large as possible. Prove that this strategy gets Harold as much money as possible by the end of the  $n^{\text{th}}$  week.*

**Question 3** (Business Plan II, 35 points). *The setup of this problem is similar to that in question 2. Please read that question first.*

*Harold later realizes that things are not quite as predictable as he had previously thought. He determines that implementing plan  $i$  in a given week has a probability  $p_{i,j}$  of earning him  $j$  dollars (adding  $j$  to his current amount of money) over the course of that week for each integer  $j$  between 0 and  $m$ . Harold starts with 0 dollars and his goal is to maximize the probability that he will have at least  $m$  dollars at the end of  $n$  weeks.*

*Give an algorithm that given  $n, m$  and all of the  $p_{i,j}$  computes the maximum possible probability with which Harold can achieve this goal. For full credit, your algorithm should run in time  $O(nm^2k)$  where  $k$  is the number of possible strategies available.*