CSE 105 Assignment 3
Due April 21 at the beginning of class

You are going to prove that NFAs with ϵ-moves accept the same set of languages as NFAs without ϵ-moves! Here are the theorems:

Theorem 1: Let L be a language accepted by NFA without ϵ-moves M, then there exists an NFA M' with ϵ-moves such that L(M)=L(M').

Theorem 2: Let L be a language accepted by NFA with ϵ-moves M, then there exists an NFA M' *without* ϵ-moves such that L(M)=L(M').

You will have proved the theorems if you can prove the following parts.

Problem 1:
To prove theorem 1, we must show how to construct M' from M. In this case M'=M! Give the construction for Theorem 1 and show that L(M)=L(M').

To prove theorem 2, we start with a construction of M' from M. For NFA M with ϵ-moves where M=(Q,Σ,δ,q_0,F), define M' to be the NFA without ϵ-moves M'=(Q,Σ,δ',q_0,F') where

- $\delta'(q, a) = \epsilon - closure\{p |\ p \epsilon \delta(r, a)$ where r is in $\epsilon - closure(q)\}$
- $F' = F \cup \{q_0|$ if there is a final state in $\epsilon - closure(q_0)\}$

Problem 2:
Show by example how to construct M' from M where M is the NFA with ϵ-moves given in Figure 2.8 on page 24 in Hopcroft and Ullman.

We show that L(M)=L(M') by showing that wϵL(M) iff wϵL(M'). The proof goes like this:

$$\begin{aligned} w\epsilon L(M) \quad & iff \quad \delta(q_0, w) \cap F \neq \emptyset \\ & iff \quad \delta'(q_0, w) \cap F' \neq \emptyset \\ & iff \quad w\epsilon L(M') \end{aligned}$$

The first "iff" and the last "iff" we know by definition of what it means for a string to be in the language accepted by a FSA. **The second "iff" is yet unproven.** Note that to prove this "iff", we need a path on w from the start state to a final state in M iff there is a path on w from the start state to a final state in M' but we don't need the final state in both cases to be the same.

In fact, the final states on the path labelled w **will** be the same unless $w = \epsilon$. For the empty string ϵ, q_0 itself may not be a final state in M, but you may be able to get to a final state from q_0 through some set of ϵ-moves. In that case, $w = \epsilon$ would be accepted by M. In this case, we add q_0 to F' in M'. To prove the second "iff", you can prove the empty string case and the non-empty string case separately. Here are the "lemmas" you need to prove the second "iff":

Lemma 1: For $w = \epsilon$, $w\epsilon L(M)$ iff $w\epsilon L(M')$

Lemma 2: For all $w\epsilon\Sigma^+$, $\delta(q_0, w) = \{p_1, ..., p_k\}$ iff $\delta'(q_0, w) = \{p_1, ..., p_k\}$

Problem 3: Prove Lemma 1.

To prove Lemma 2, we use induction on the length of $|w|$. Note that this induction starts at $|w| = 1$.

Problem 4: Prove the base case of the induction:
For $|w| = 1, \delta(q_0, w) = \{p_1, ..., p_k\}$ iff $\delta'(q_0, w) = \{p_1, ..., p_k\}$

Problem 5: Prove the induction step:
Assume that for $1 \leq |w| \leq n$, $\delta(q_0, w) = \{p_1, ..., p_k\}$ iff $\delta'(q_0, w) = \{p_1, ..., p_k\}$.

Show that for $|w| = n+1$, $\delta(q_0, w) = \{p_1, ..., p_k\}$ iff $\delta'(q_0, w) = \{p_1, ..., p_k\}$.

Problem 6:
Now that you've proven Lemmas 1 and 2, describe how they imply the second "iff".

CONGRATULATIONS! You've proved the entire theorem yourself!