In-person lecture room: CSE building 4258

HW2 will be available today or tomorrow (Stay tuned)

$f: M \to \mathbb{R}$ scalar function $\Leftrightarrow$ $f \in \Omega^0(M)$

$\alpha_p \in T_p^*M$ is a covector $\Leftrightarrow$ $\alpha \in \Omega^1(M)$

$\forall p \in M$

Define d (d_0)

$d: \Omega^0(M) \longrightarrow \Omega^1(M)$

s.t. $(df)_p[\vec{v}] = \dfrac{\partial f}{\partial \vec{v}}$ directional derivative $\forall \vec{v} \in T_pM$, $p \in M$

intuitively

$$= \lim_{\varepsilon \to 0} \frac{f(p + \varepsilon \vec{v}) - f(p)}{\varepsilon}$$

more precisely: construct any curve $\gamma: \mathbb{R} \to M$

s.t. $\gamma(0) = p$

$\gamma'(0) = \vec{v}$

$$\tilde{f}(t) = f(\gamma(t)) \qquad \tilde{f}: \mathbb{R} \to \mathbb{R}$$

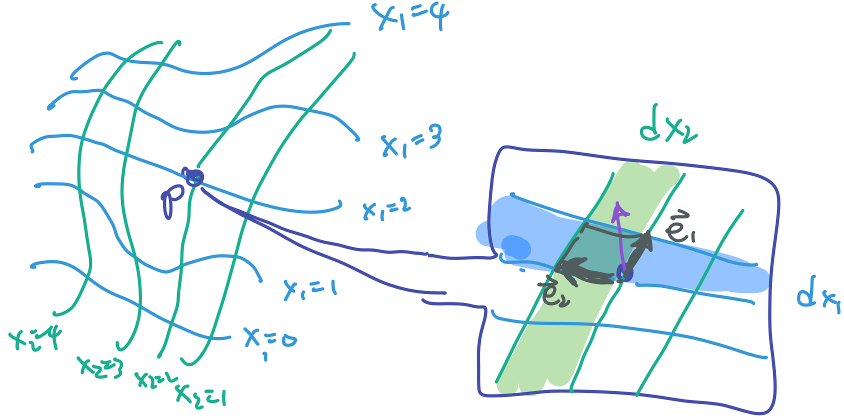
$$(df)_p[\vec{v}] = \frac{d}{dt}\tilde{f}\,\Big|_{t=0}.$$

M: mfd. (nD)

A (local) coordinate on M is a set of n scalar functions (0-forms)

$x_1, x_2, \dots, x_n$ $\qquad x_i \in \Omega^0(M)$

$x_i: M \to \mathbb{R}$.



By taking d, we obtain 1-forms

$dx_1, dx_2, \cdots, dx_n \in \Omega^1(M)$

Given a velocity $\vec{v} \in T_pM$

$dx_i(\vec{v})$ is the rate of change of coord value x_i

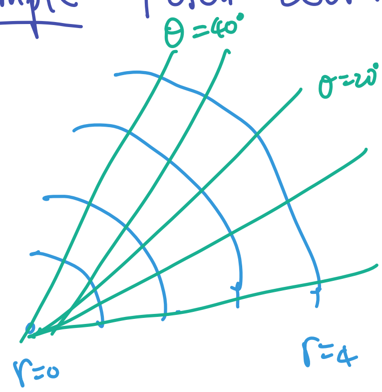
$dx_1, \cdots, dx_n$ form a basis $\vec{e}_1, \cdots, \vec{e}_n$ for T_pM dual to $dx_1, \cdots, dx_n$

<u>Thm (Chain Rule)</u> If $x_1, \cdots, x_n$ is coord. $\rightsquigarrow$ $dx_1, \cdots, dx_n$ basis for T_p^*M.

Then $\forall f \in \Omega^0(M)$, $\quad (df)_p = \underbrace{\frac{\partial f}{\partial x_1}} dx_1 + \underbrace{\frac{\partial f}{\partial x_2}} dx_2 + \cdots + \underbrace{\frac{\partial f}{\partial x_n}} dx_n$

$f = f(x_1, \cdots, x_n)$

<u>Example</u> Polar coordinate



$$r = \sqrt{x^2+y^2} \rightsquigarrow dr = \frac{x}{\sqrt{x^2+y^2}} dx + \frac{y}{\sqrt{x^2+y^2}} dy$$

$$\theta = \tan^{-1}\left(\frac{y}{x}\right) \rightsquigarrow d\theta = -\frac{y}{x^2+y^2} dx + \frac{x}{x^2+y^2} dy$$