# CSE 167 (FA22) Computer Graphics: Colors

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# Color Science

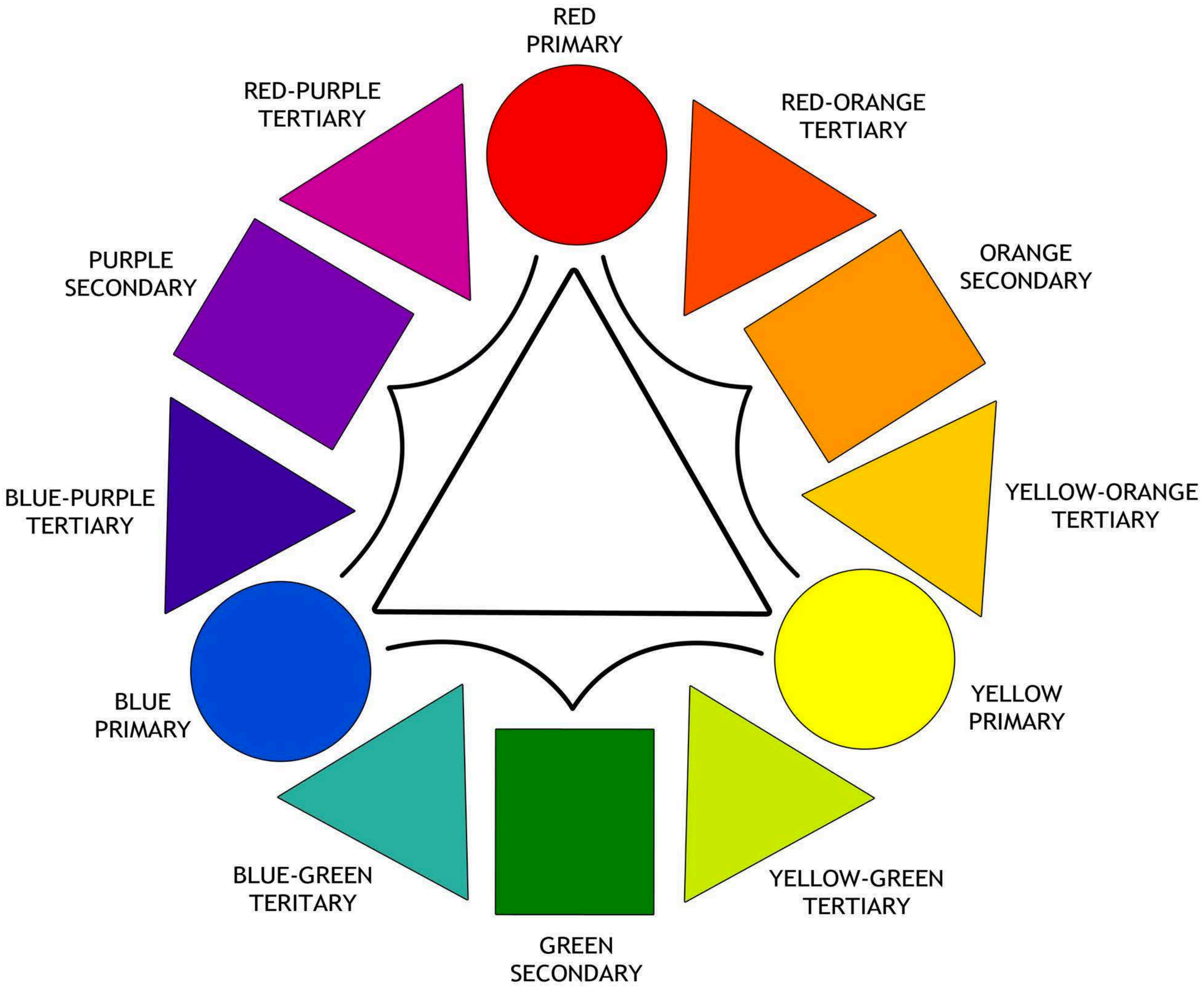
- Color science
- Inner product between functions
- Physical color
- Perceived color
- Color space and gamut

# Color

- Phenomenological description
- What causes color?
- How can we measure color?
- How can we describe color?
- How can we control color?

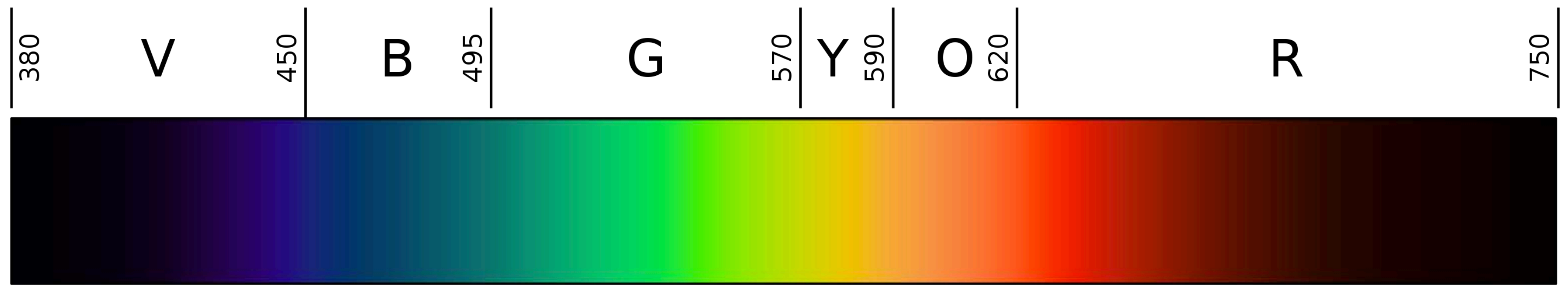
# Discovering color (pigment)

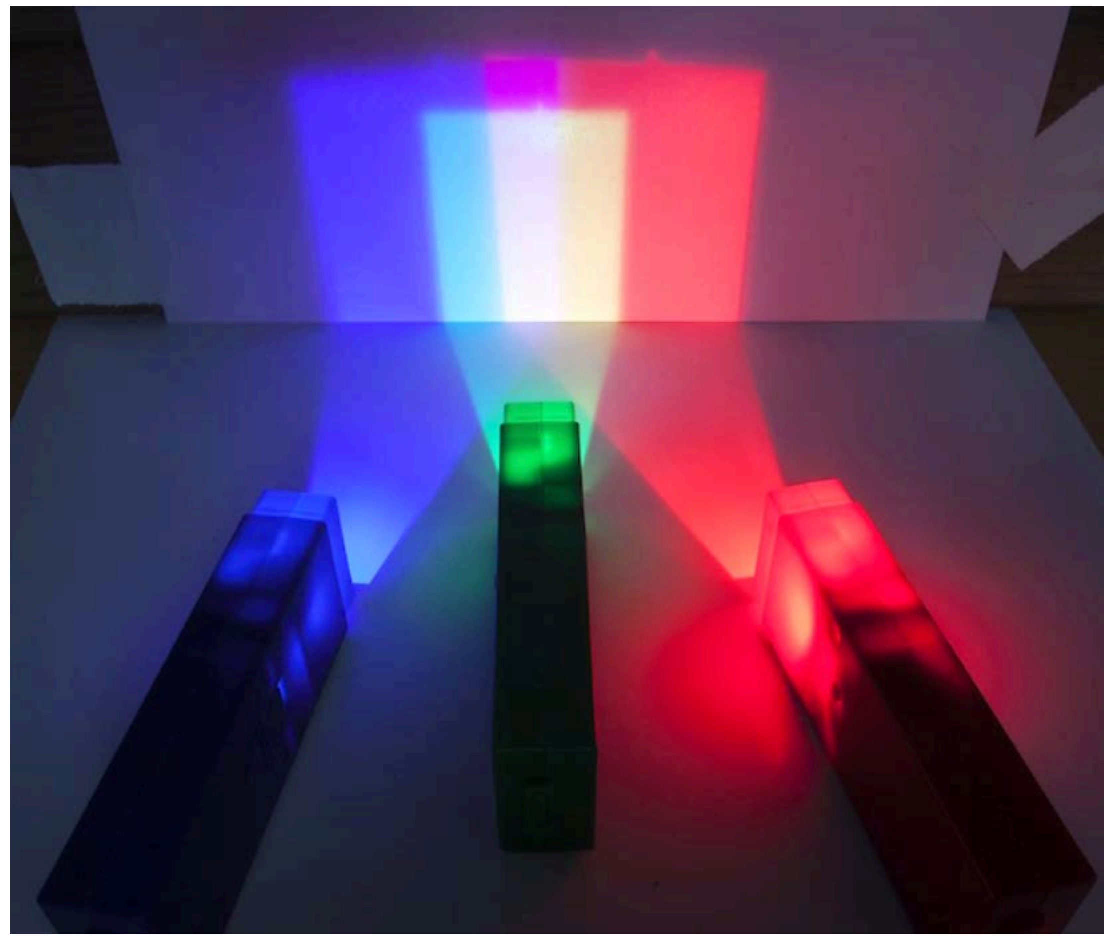
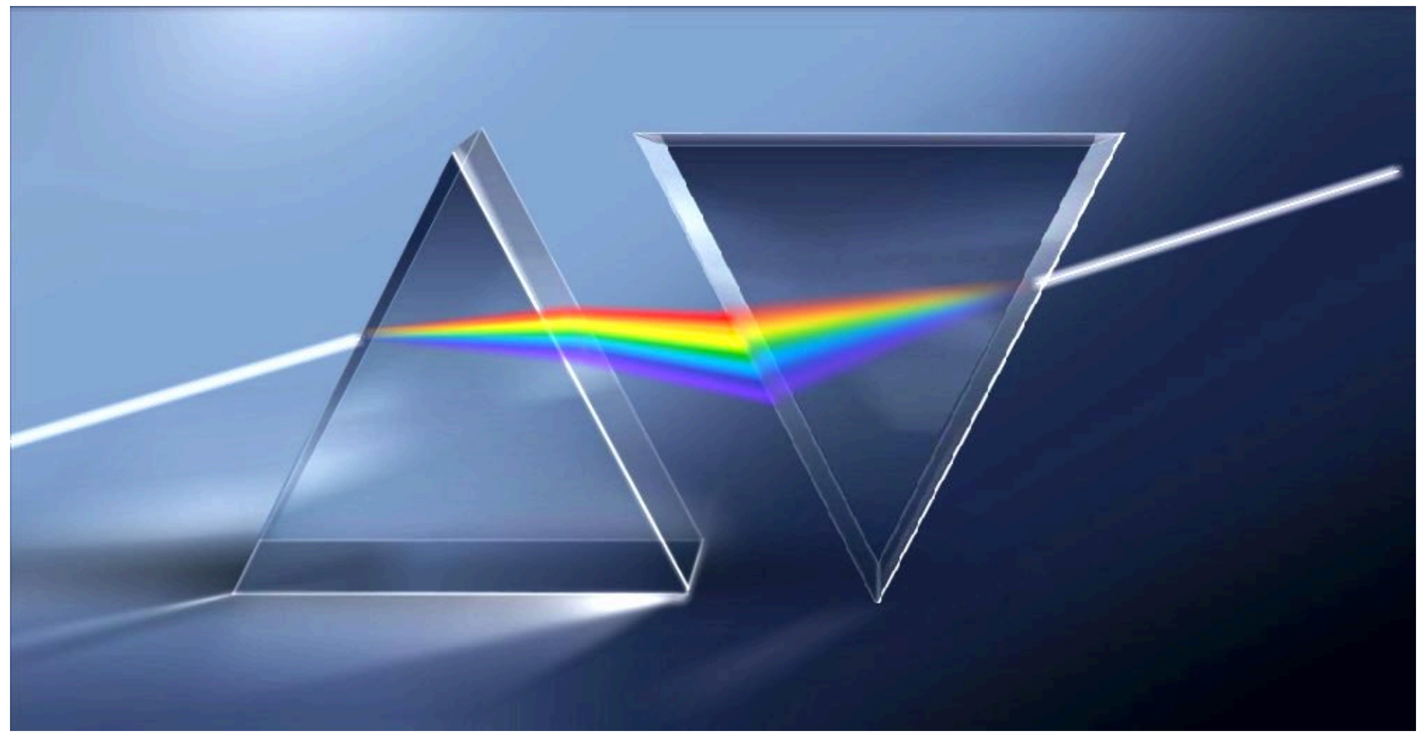
- Artists studied color for millennia
- They knew that mixing 3 primary color creates a full range of hues



# Discovering color (light)

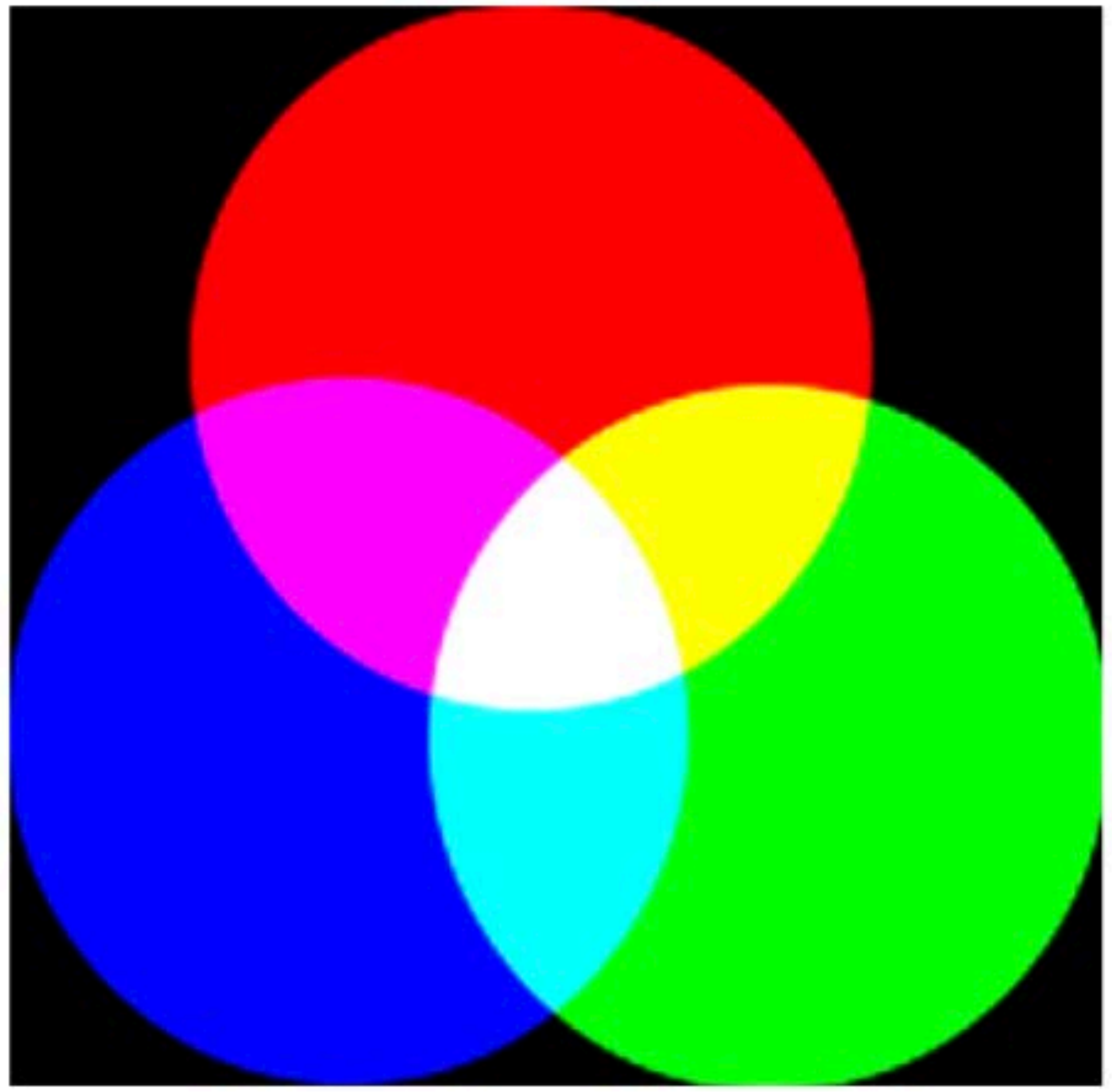
- Isaac Newton experimented with prism
- White light contains all the “rainbow color”
- Colored light combines into white
- C. Huygens (1678), T. Young (1800): light are waves with wavelength around 380–760nm
- Young, Helmholtz, Maxwell: 3 primary color of light

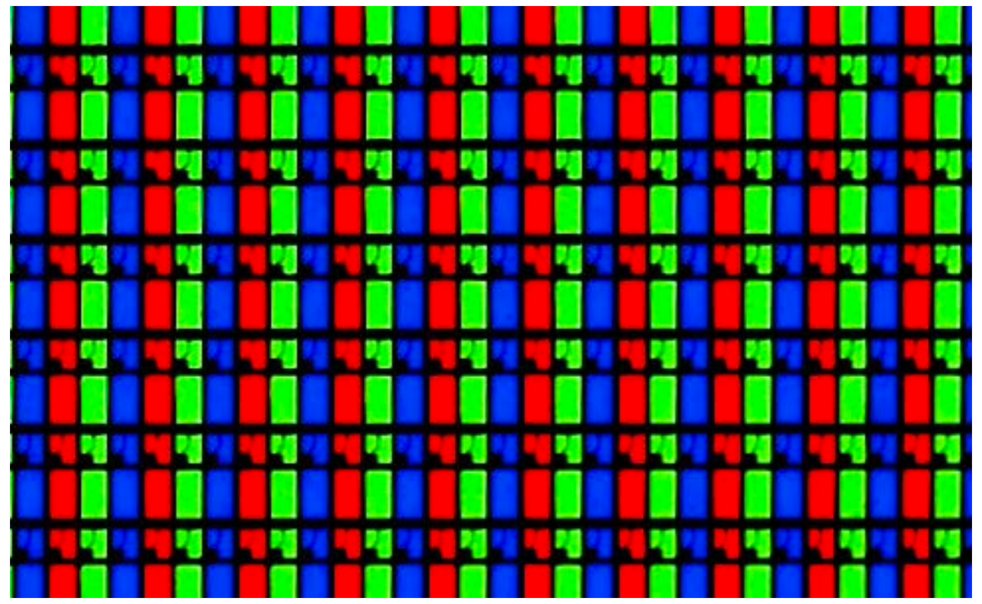



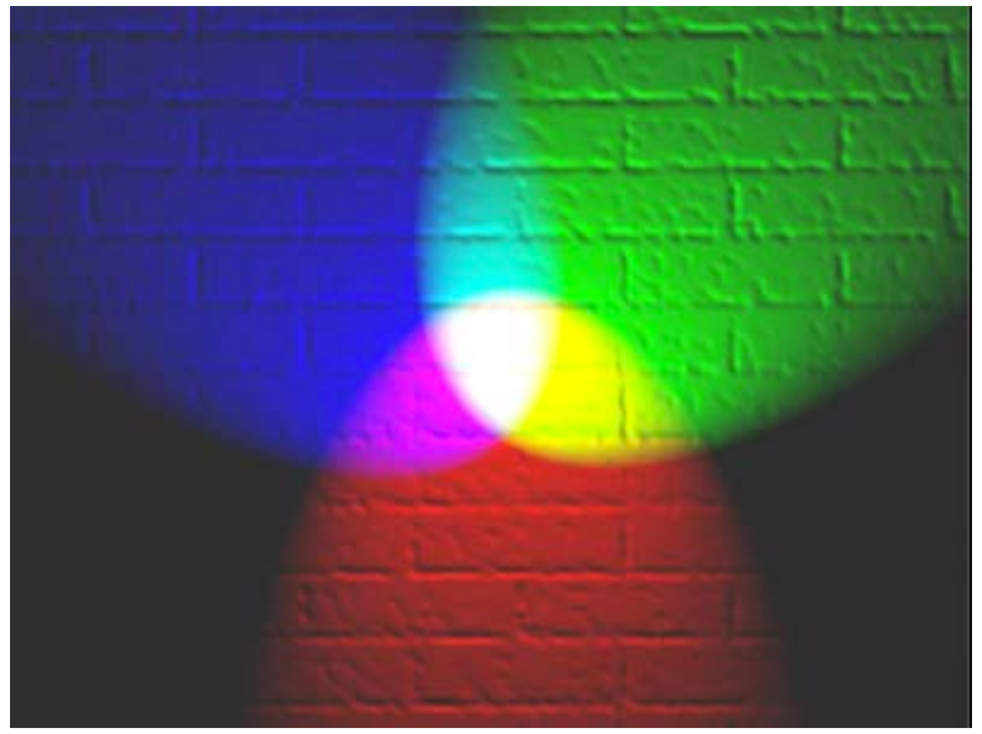



# Additive and subtractive color

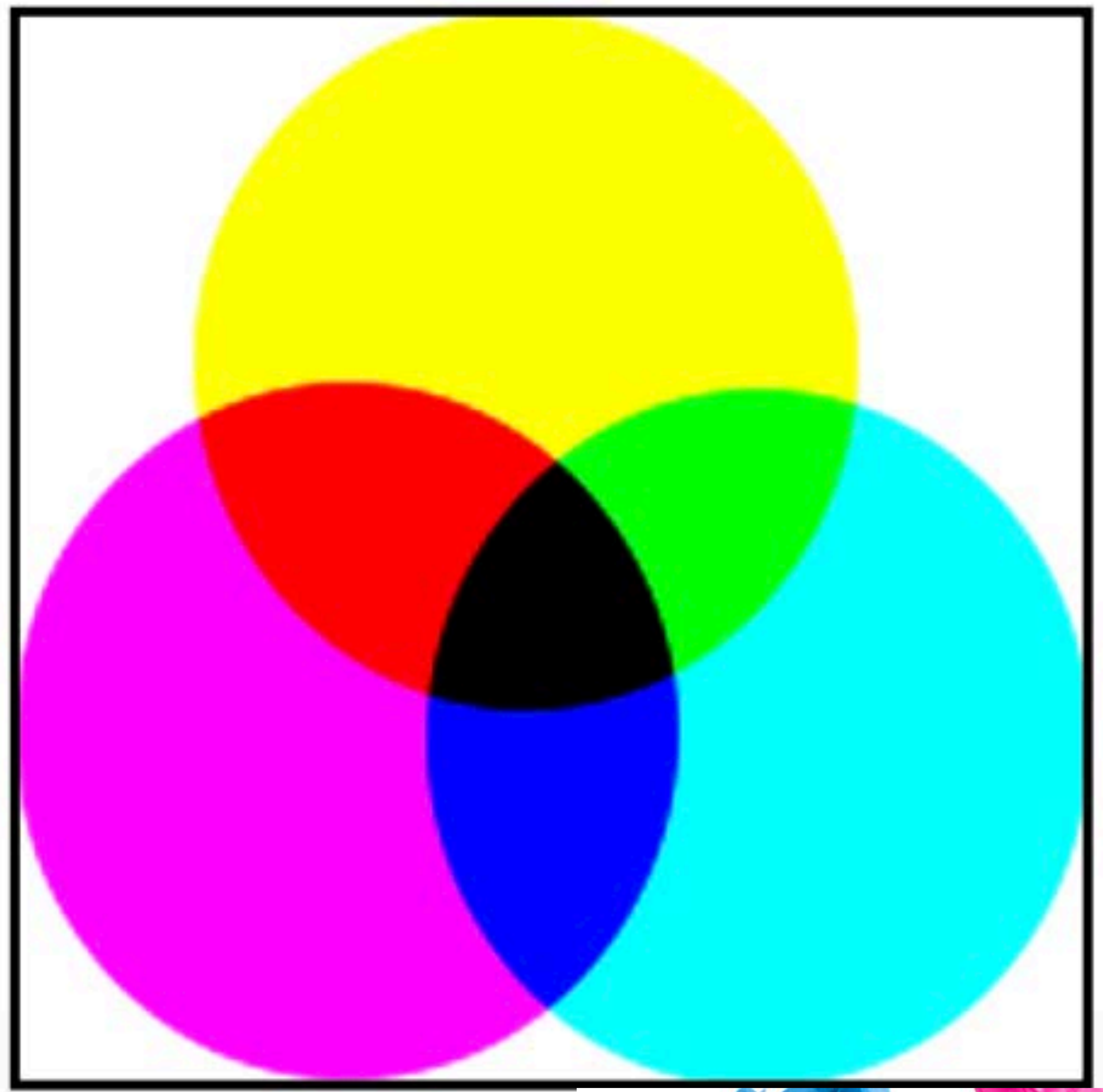
## Additive

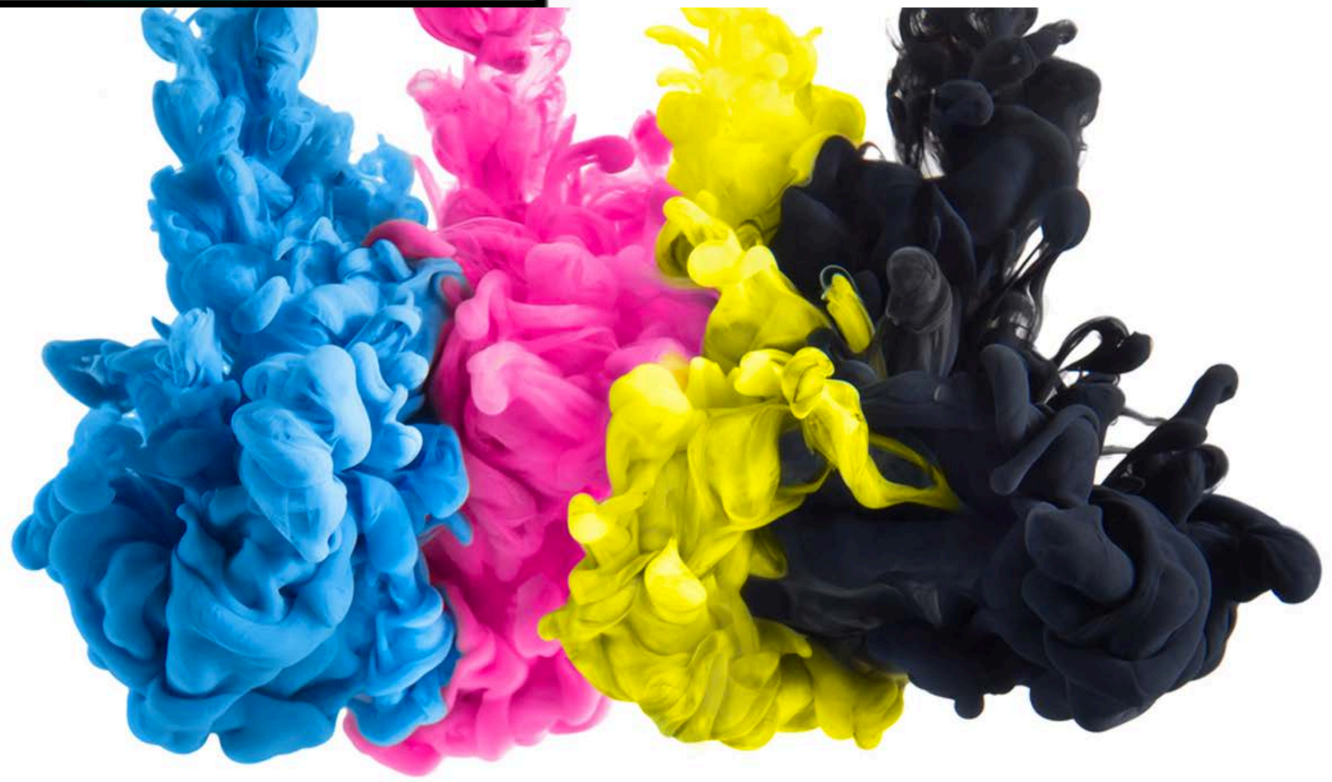






## Subtractive

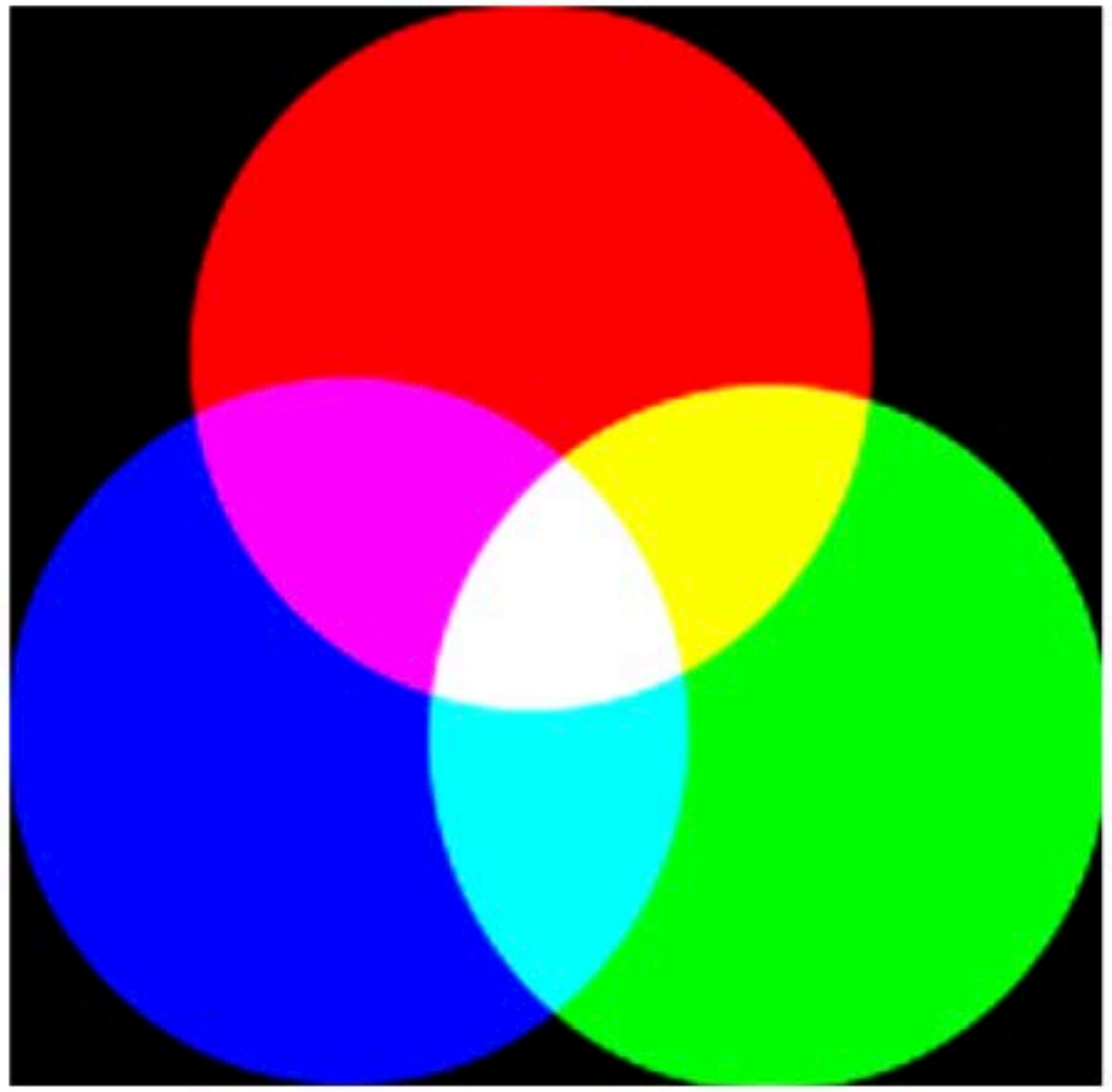


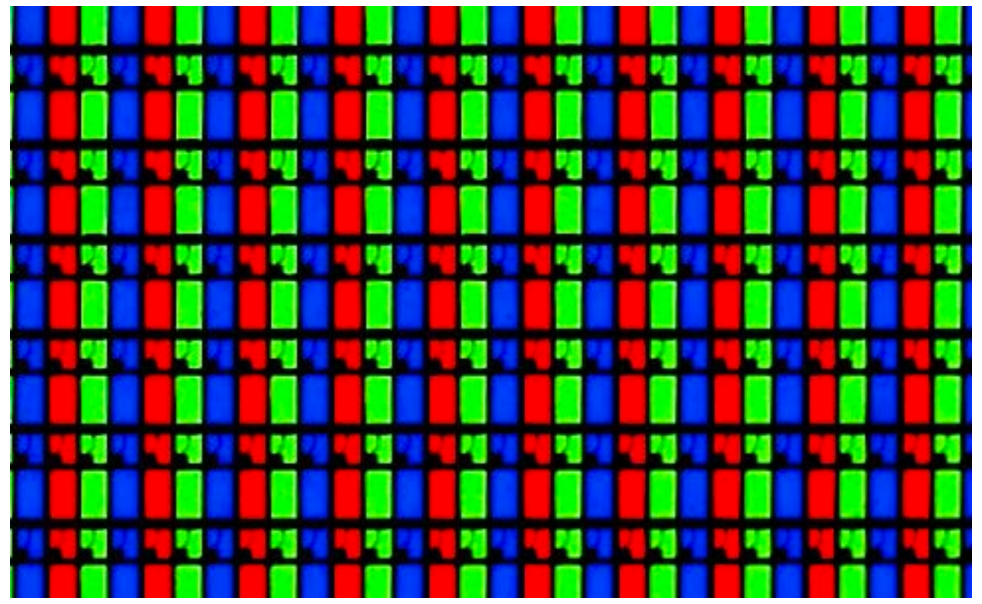


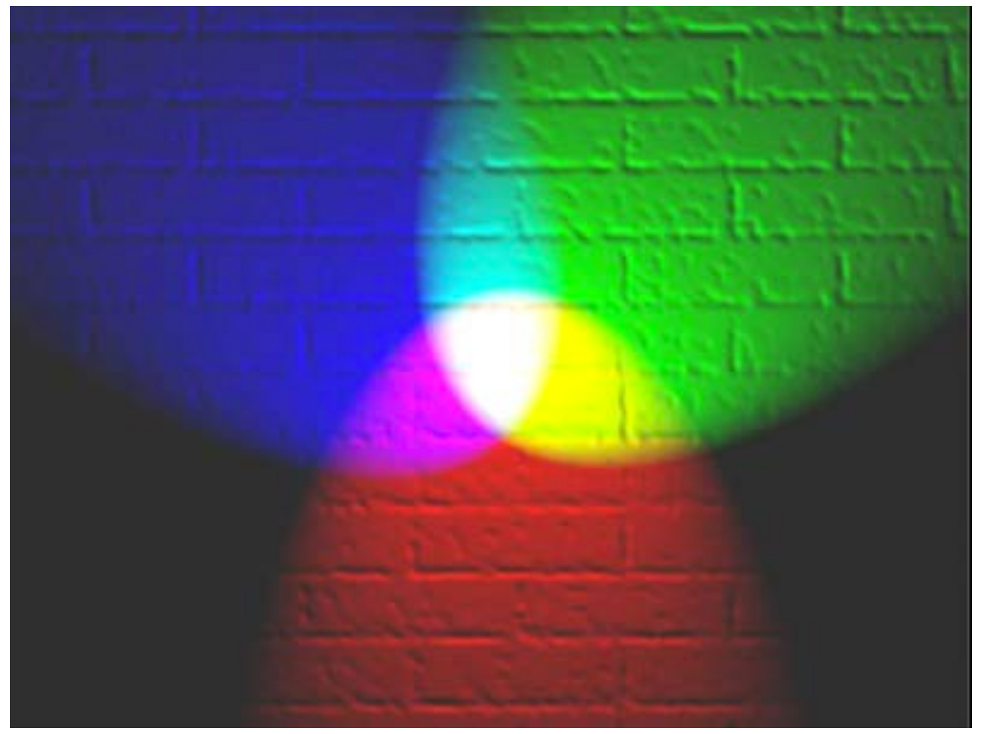
# Descriptions of color

## RGB

- Controls the light bulbs in each pixel

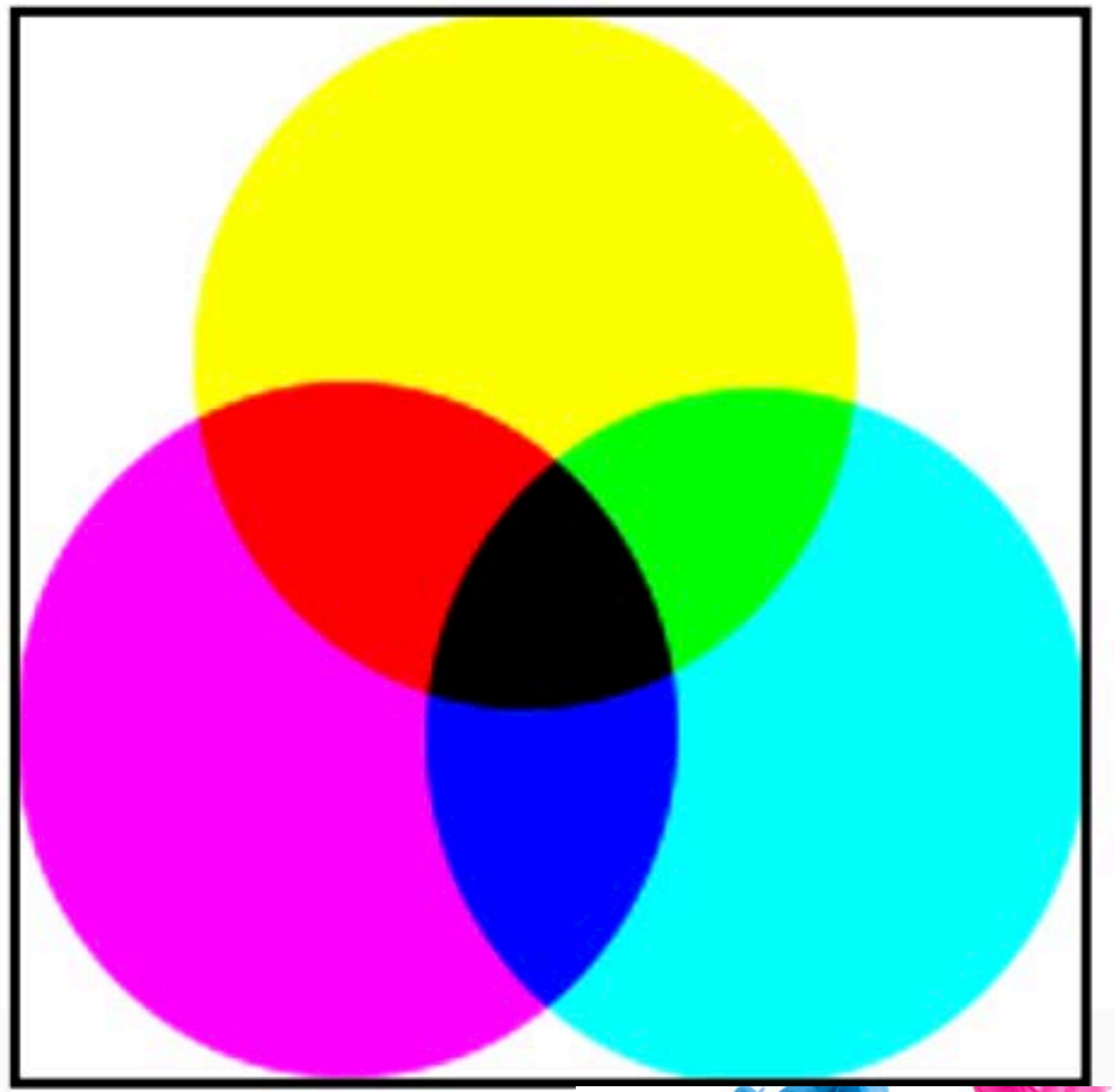






## CMYK

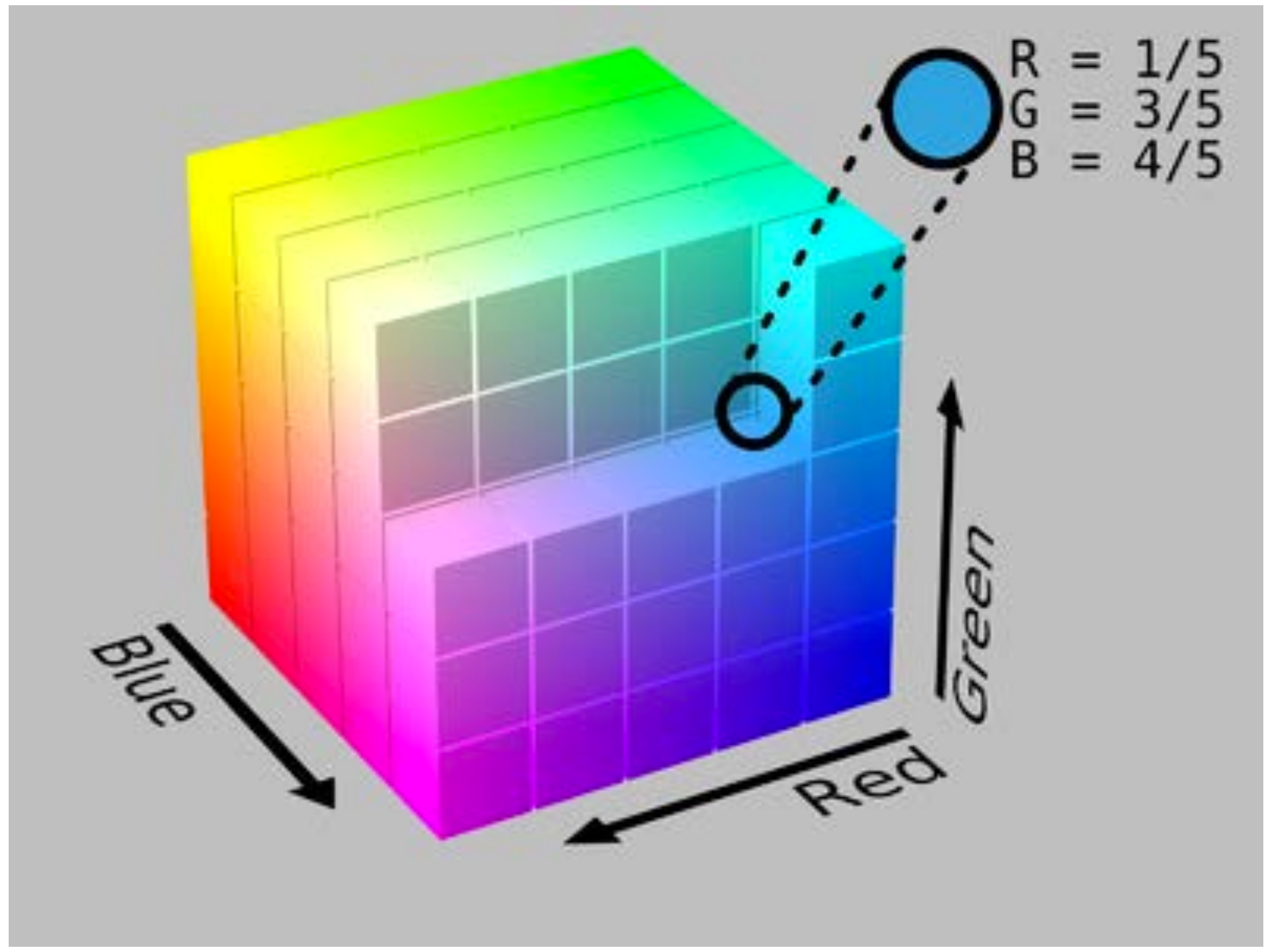
- Controls the ink in a printer



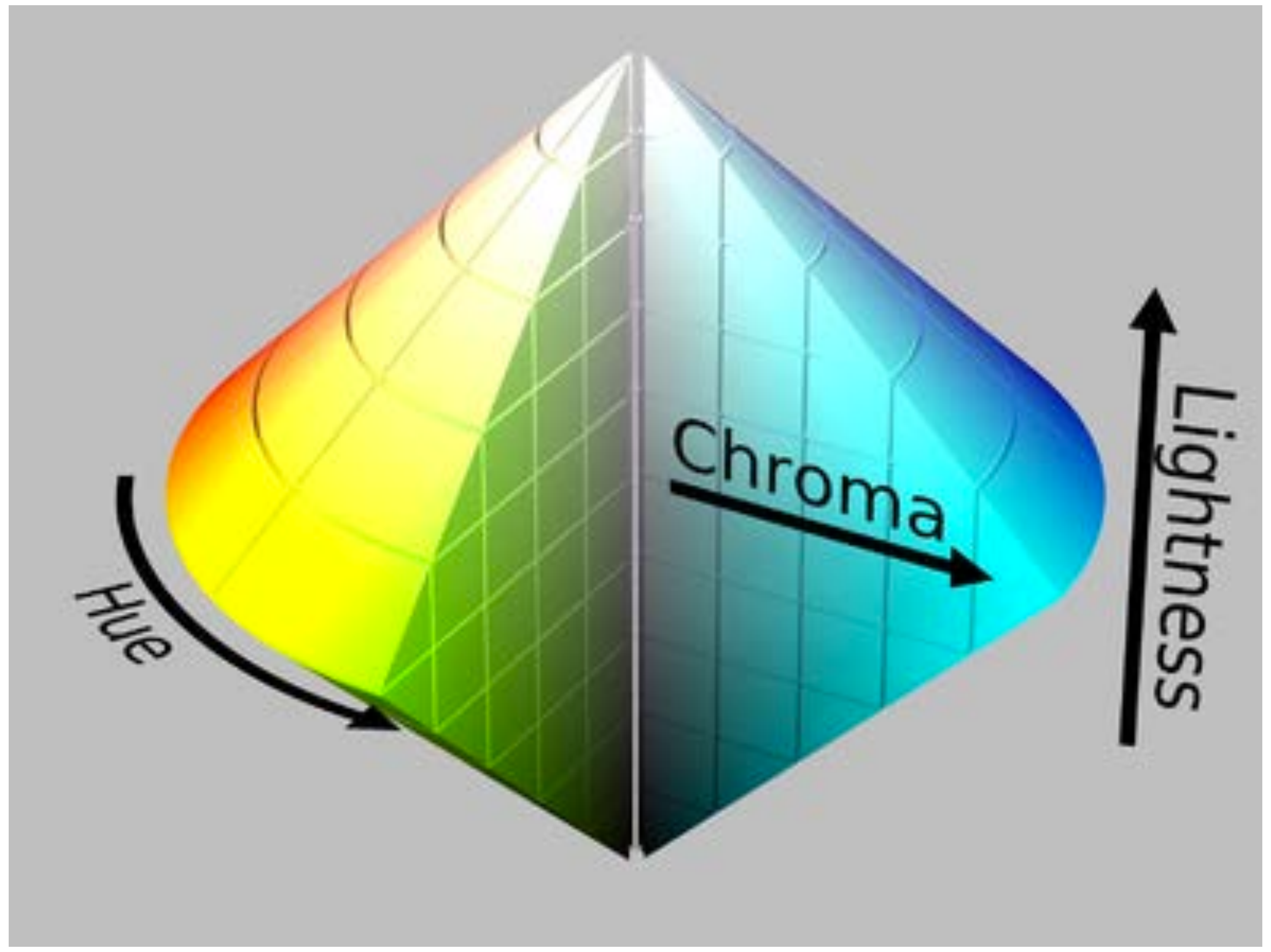
# Descriptions of color

## Hue, Saturation, Lightness/Brightness/Value (HSL, HSB, HSV)

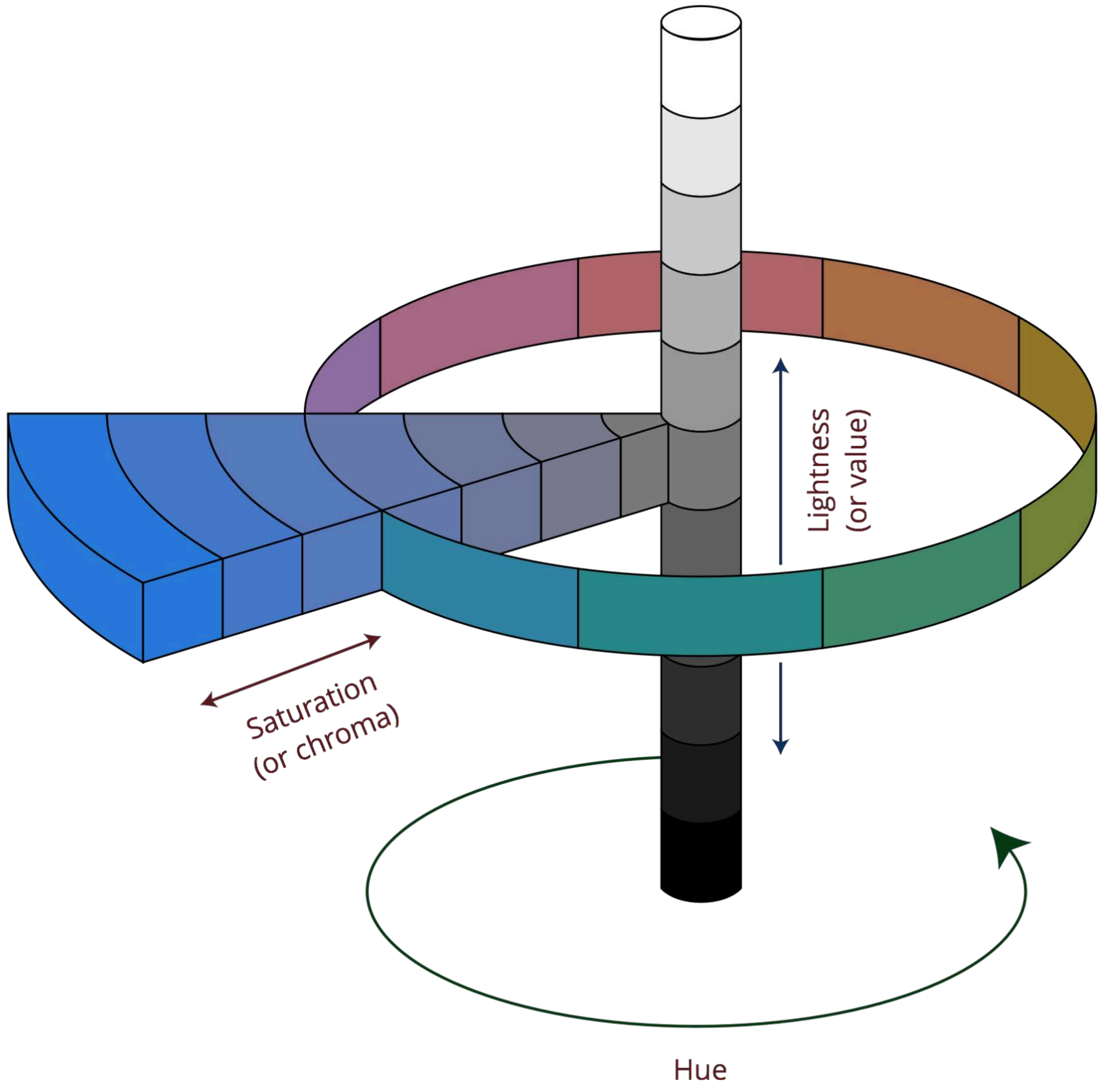
- More intuitive
- Useful in design




RGB




HSL



# Real Story

- The above description of color is based on what we perceive.
- What is the objective and mathematical description of color?
- We need to distinguish **physical color** & **perceived color**.

# $L^2$ pairing between functions

- Color science
- Inner product between functions
- Physical color
- Perceived color
- Color space and gamut

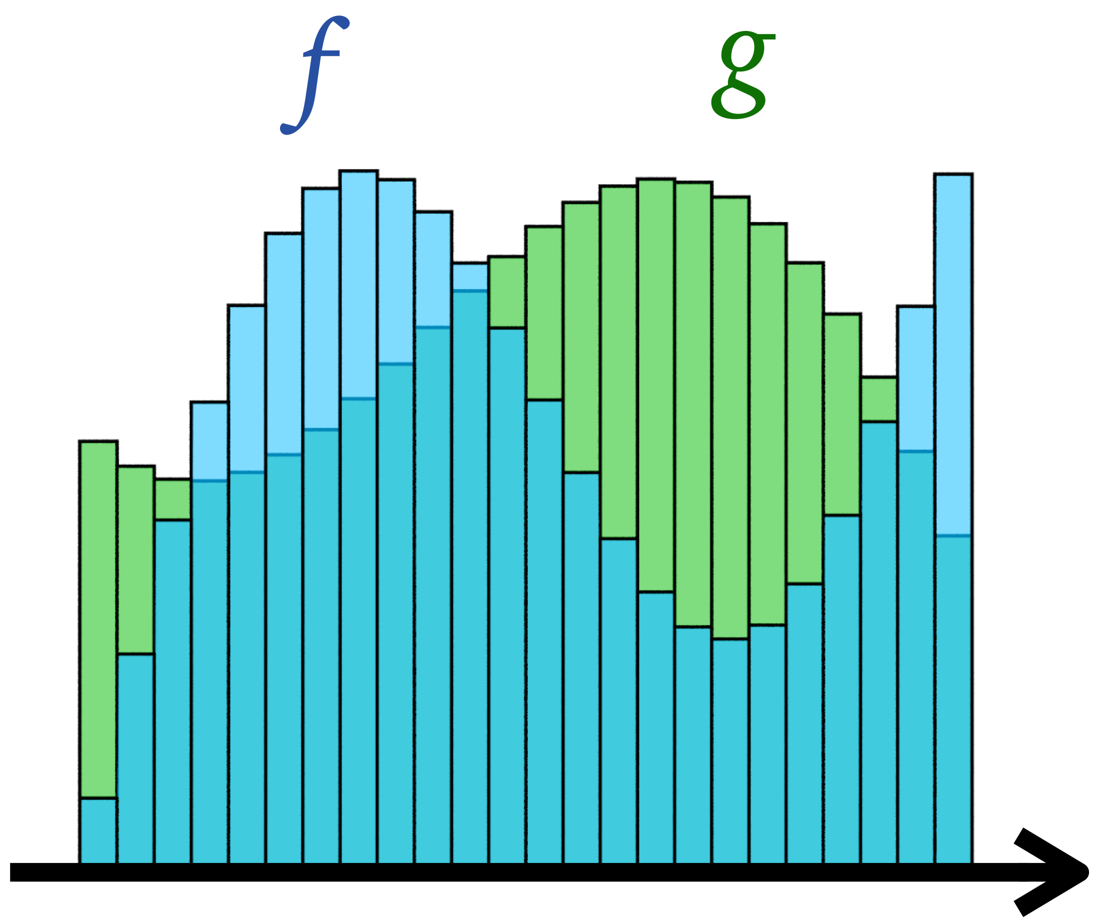
# Crash course on the $L^2$ pairing

**Recall.** The inner product (dot product) between two arrays

$$f = (f_0, f_1, \ldots, f_{n-1}) \in \mathbb{R}^n$$
$$g = (g_0, g_1, \ldots, g_{n-1}) \in \mathbb{R}^n$$

is the sum of elementwise product

$$\sum_{i=0}^{n-1} f_i g_i$$



# Crash course on the $L^2$ pairing

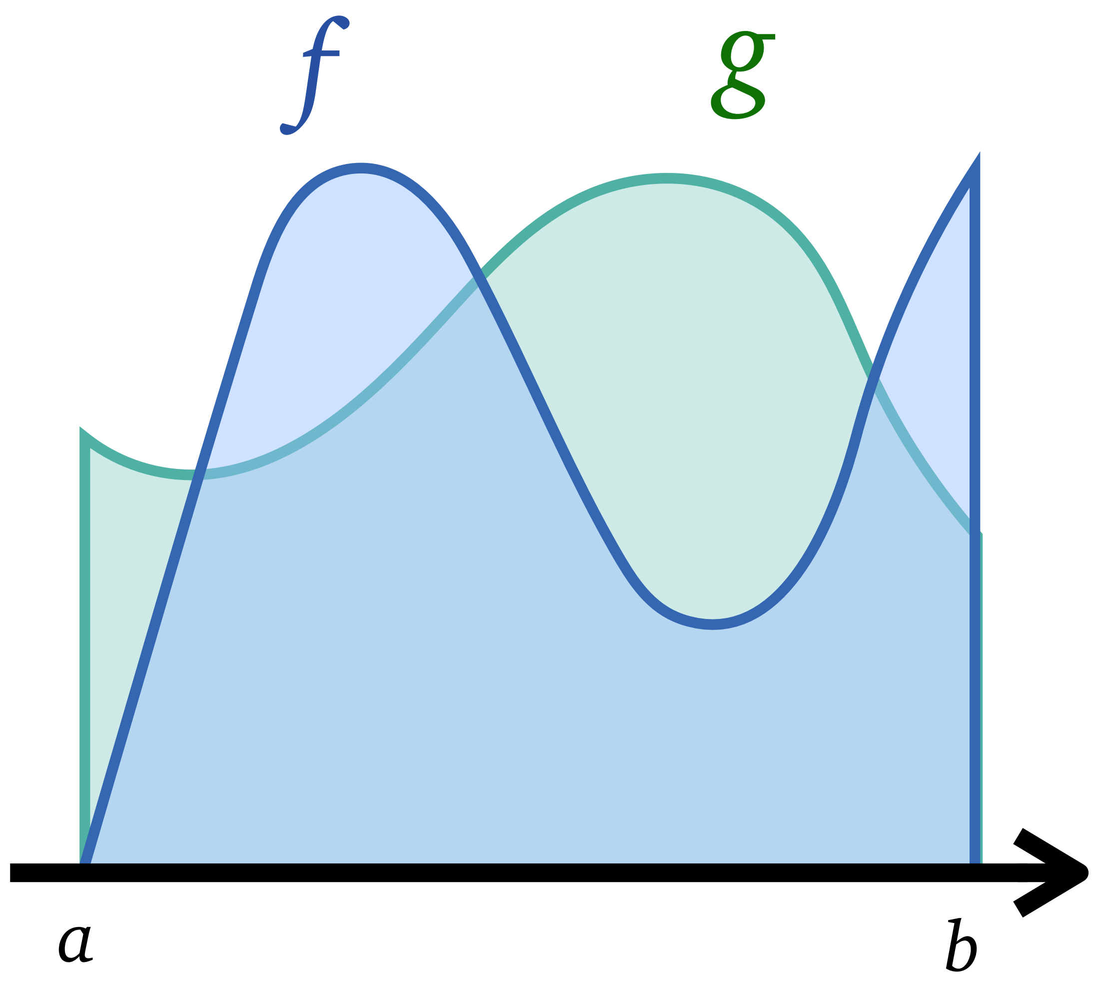
**Definition.** The $L^2$ pairing between two functions

$$f : [a, b] \to \mathbb{R}$$

$$g : [a, b] \to \mathbb{R}$$

on an interval $[a, b]$ is the integral

$$\int_a^b f(x) g(x)\, dx$$




- You can think of it as the inner product between functions.
- Here, we will view $f, g$ playing different roles.

# Crash course on the $L^2$ pairing

$$\int_a^b f(x)g(x)\,dx$$

sensor signal

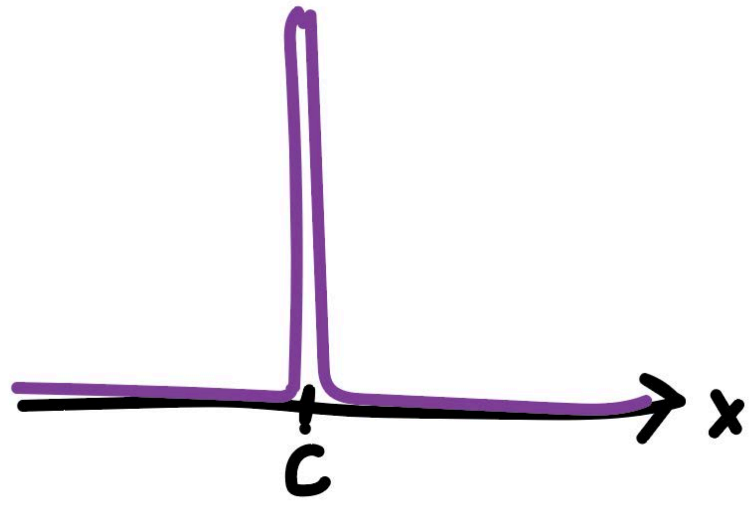
- Here, we will view $f, g$ playing different roles.
  - $f(x)$ is assumed to be a **continuous function**.
  - $g(x)$ can be a **distribution** (derivative of a possibly discontinuous function).

# Crash course on the $L^2$ pairing

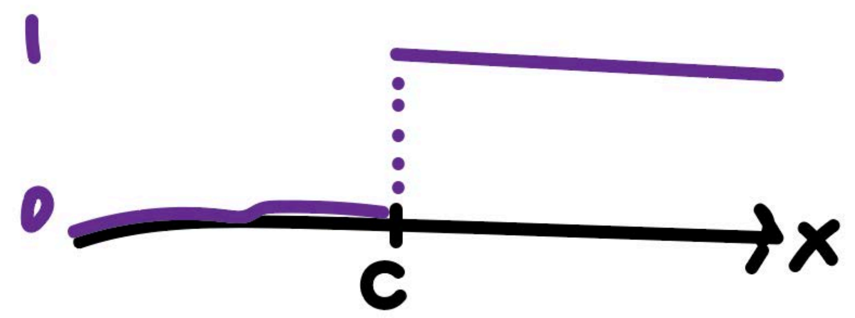
- $g(x)$ can be a **distribution** (derivative of a possibly discontinuous function).
  - Continuous functions are distribution
  - Integrable functions are distribution
  - Generalized functions such as the $\delta$-functions are distribution

**Definition.** The $\delta$-function $\delta_c(x)$ concentrated at $c \in [a, b]$ is the derivative of the step function jumping at $c$:

$$\delta_c(x) = \frac{d}{dx} H_c(x)$$




$$H_c(x) = \begin{cases} 0, & x < c \\ 1, & x \geq c \end{cases}$$



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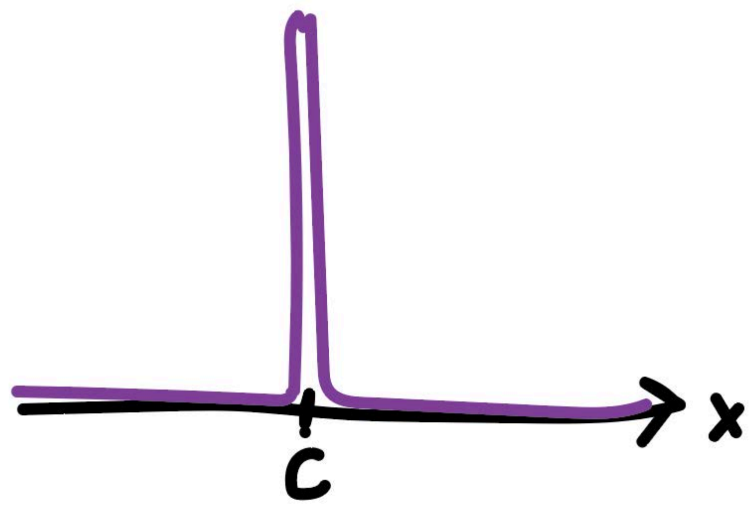


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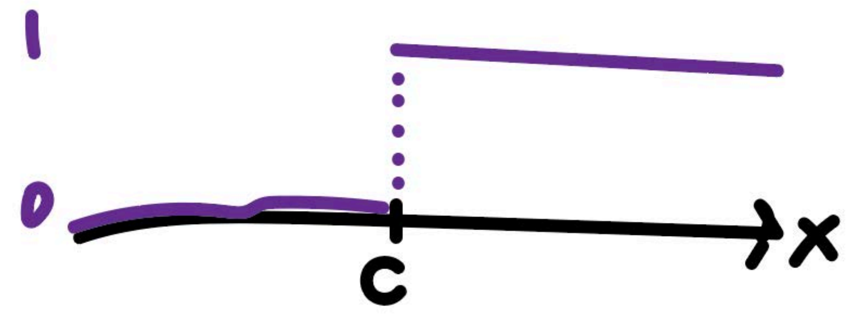



- Infinite impulse: $\delta_c(x) = \begin{cases} \infty, & x = c \\ 0, & x \neq c \end{cases}$
- Translation $\delta_c(x) = \delta_0(x - c)$
- $L^2$ -pairing $\displaystyle\int_a^b f(x)\delta_c(x)\, dx = f(c)$

# Physical color

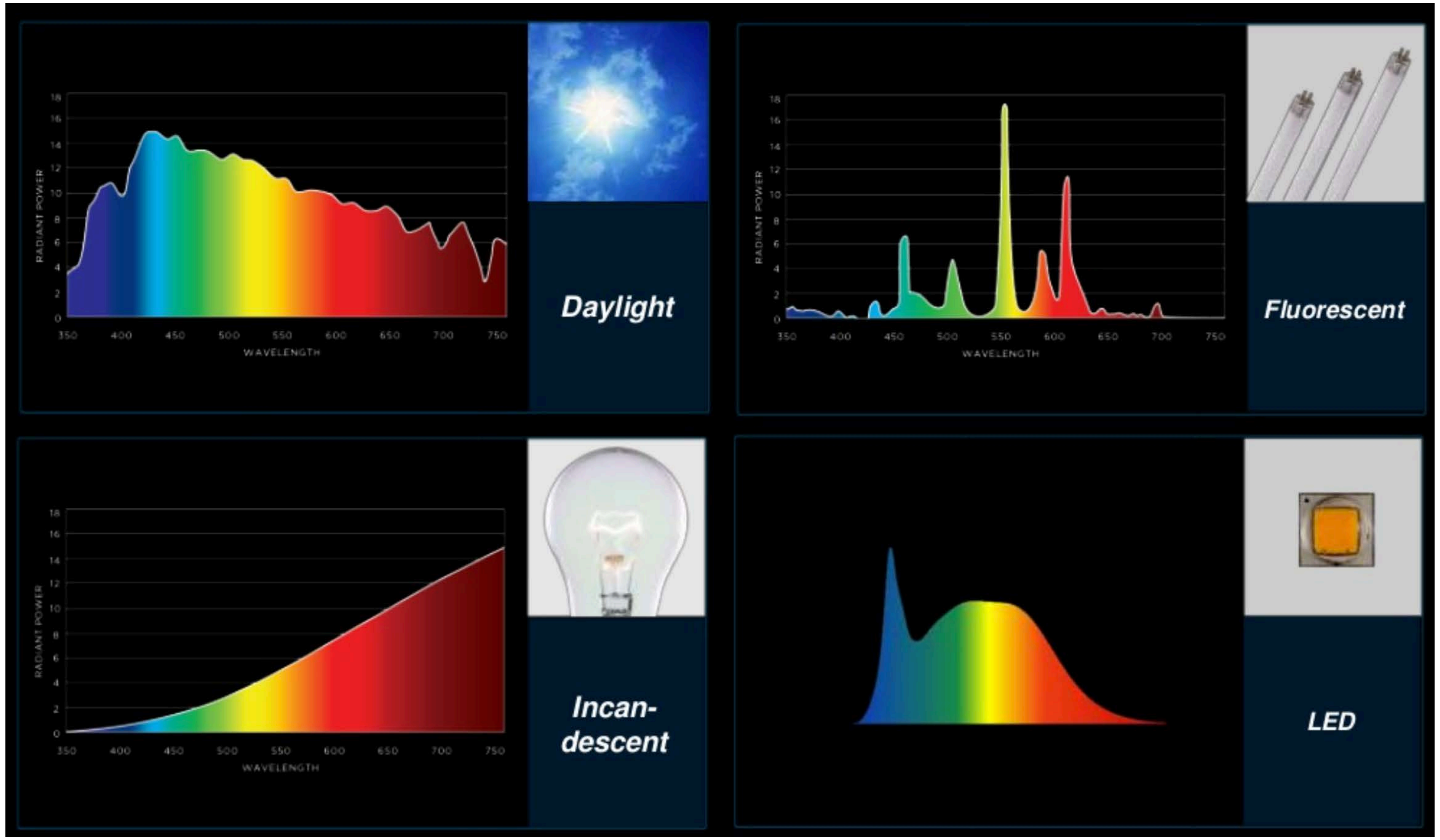
- Color science
- Inner product between functions
- Physical color
- Perceived color
- Color space and gamut

# Physical colored light

**Definition.** A **colored light** is a nonnegative distribution over the spectral interval [380nm, 750nm] representing the radiance power for each wavelength.

- This is called the **spectral power distribution.**

- Single-spectral light is a $\delta$-function.



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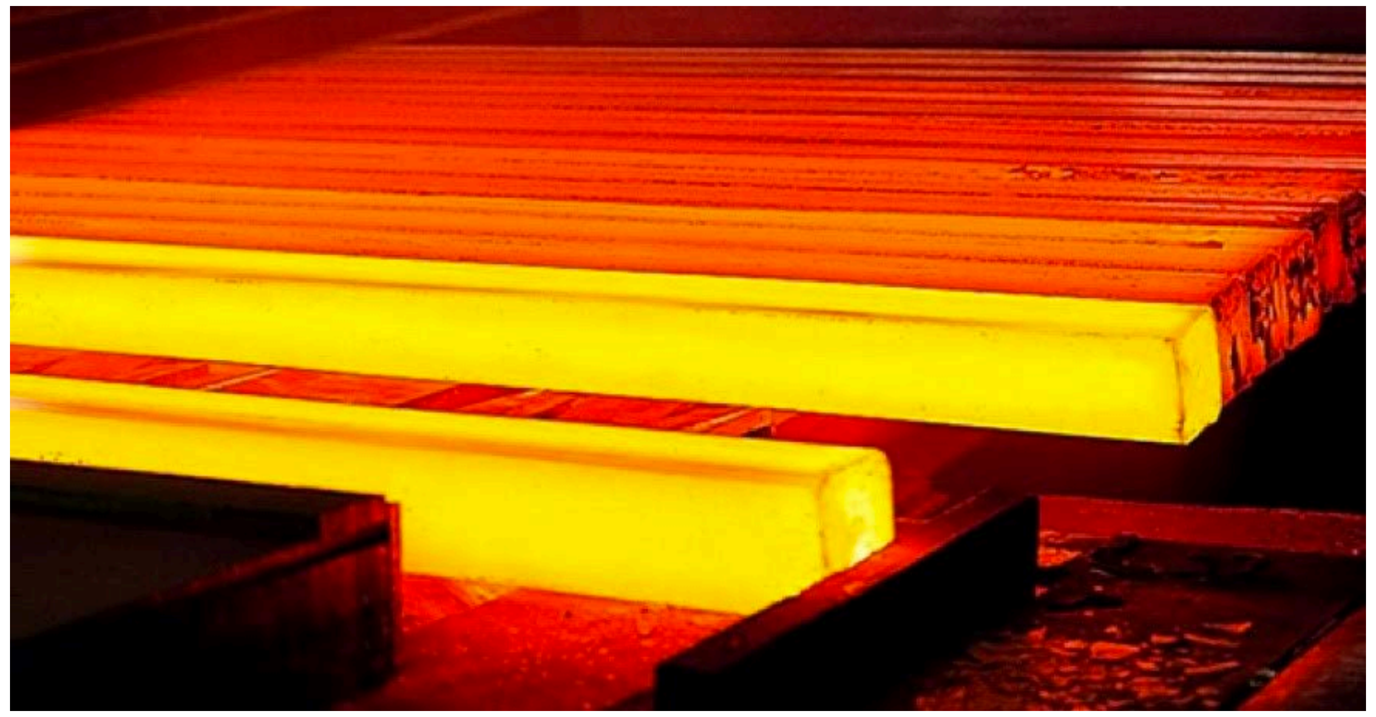
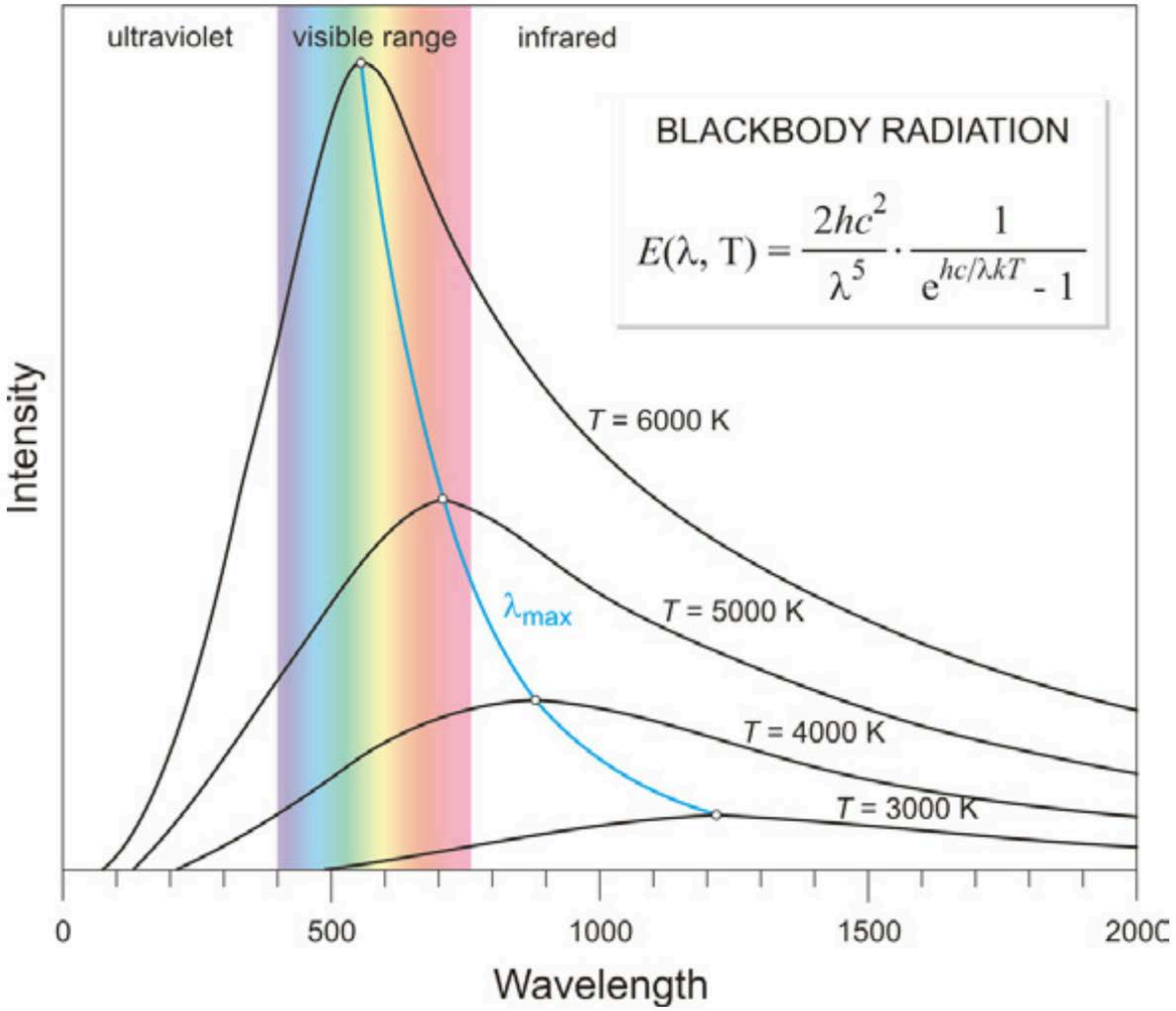
- Blackbody radiation





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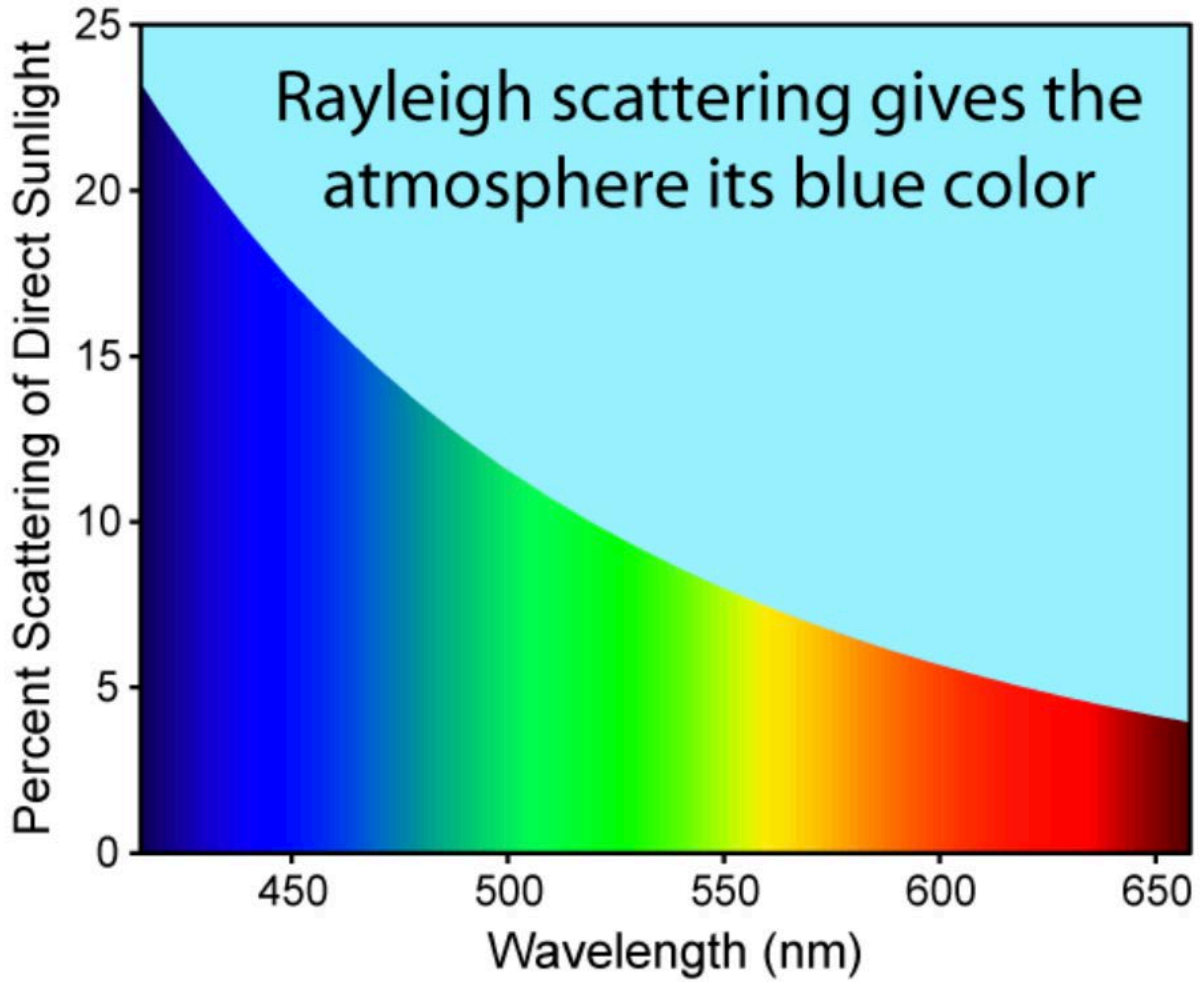
- This is called the **spectral power distribution.**
- Single-spectral light is a $\delta$-function.
- Blackbody radiation
- Rayleigh scattering



# Physical colored light

**Definition.** Two colored lights $P_1(\lambda)$, $P_2(\lambda)$ have the same chromatic color if they differ only by a scale

$$P_1(\lambda) = \alpha P_2(\lambda)$$

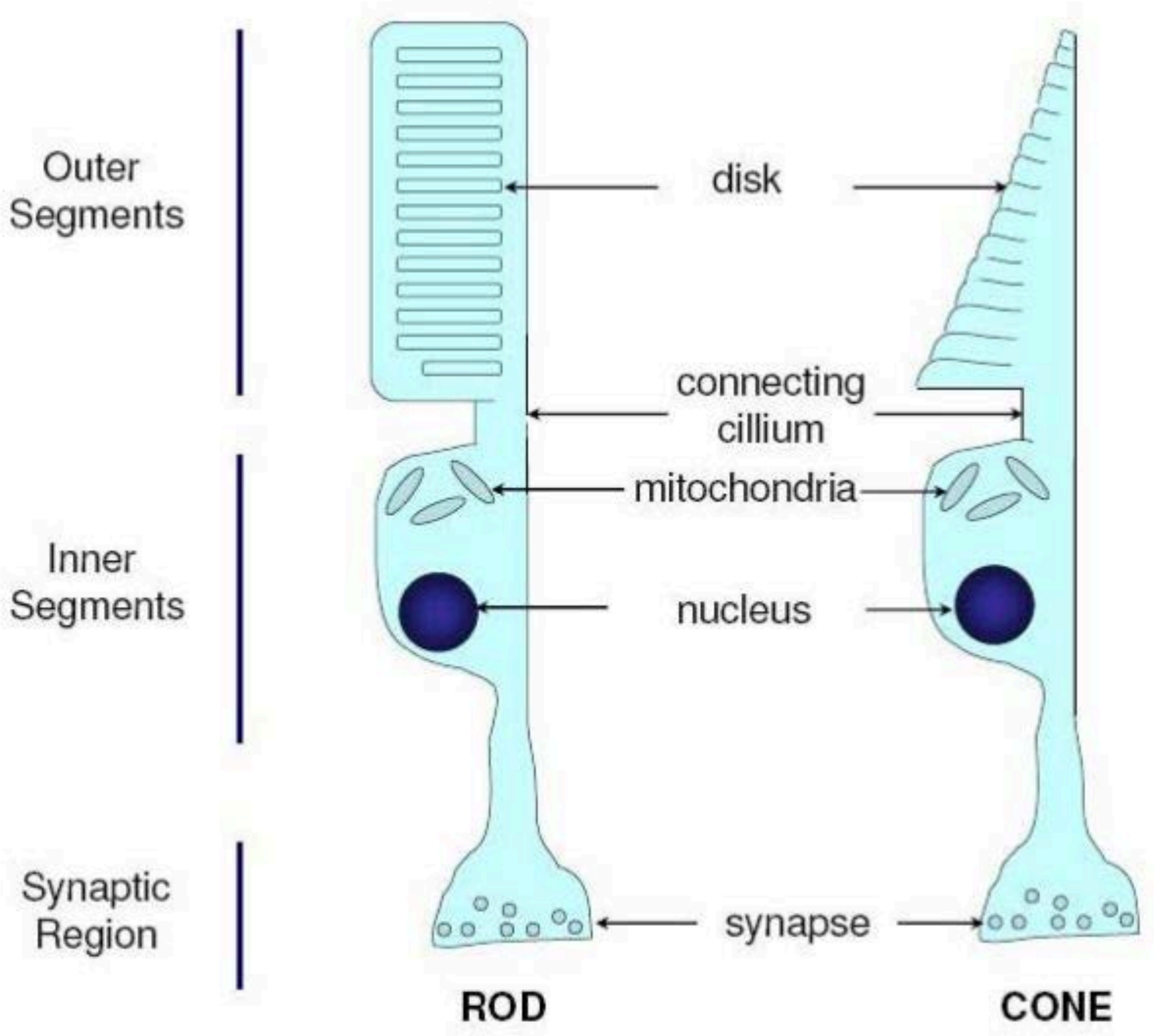
- The **chromoticity** lives in a projective space (infinite dimensional).

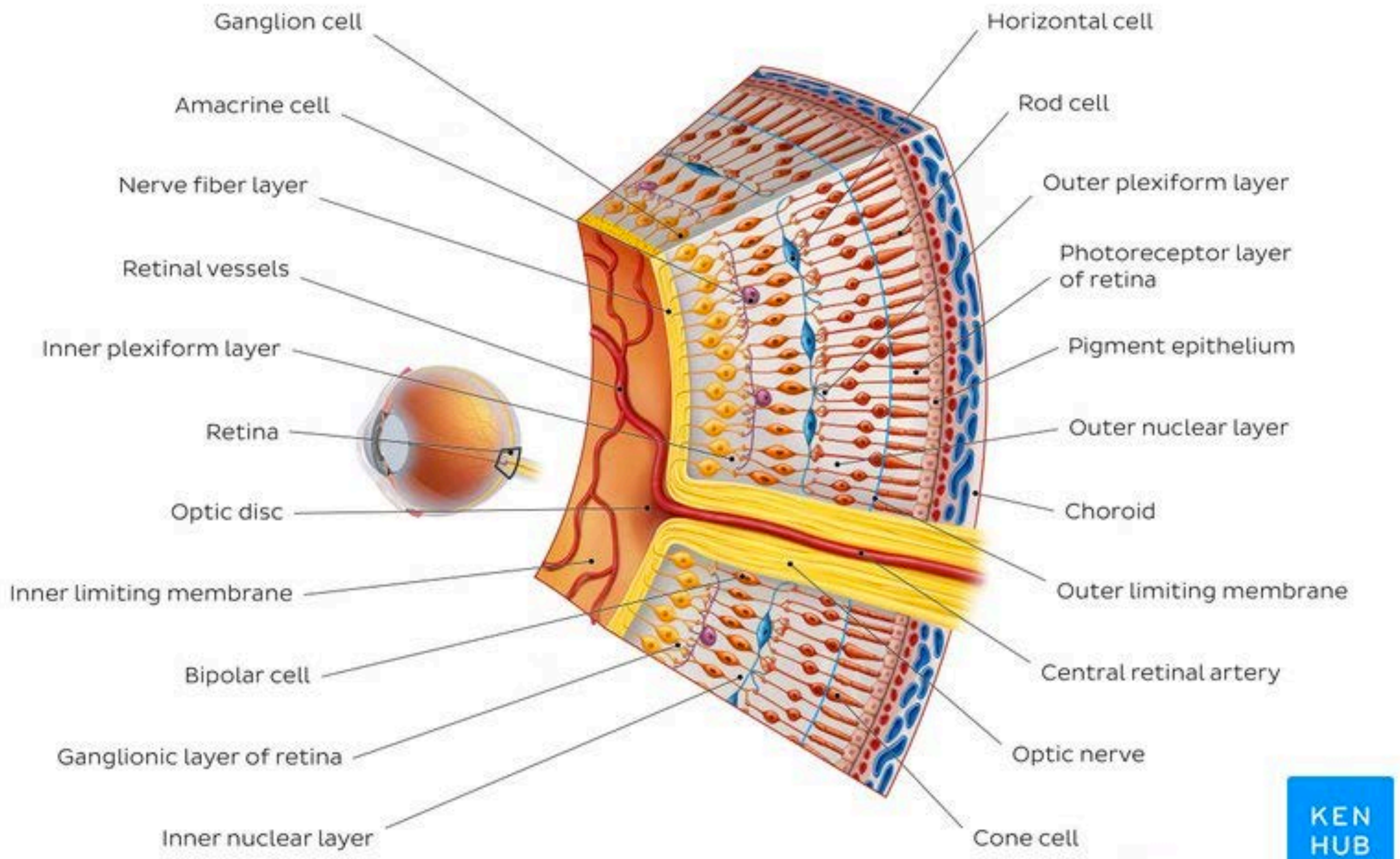
# Perceived color

- Color science
- Inner product between functions
- Physical color
- Perceived color
- Color space and gamut

# Vision

- Cone and rod photoreceptors in the retina turn light into neural signals.
- Rod is 100 times more sensitive to the cones, but slow response time. Rod is responsible for night vision (scotopic vision).
- Under sufficient illumination we use cone cells (photopic vision).
- It can take 10–30 minutes to switch between scotopic/photopic vision.

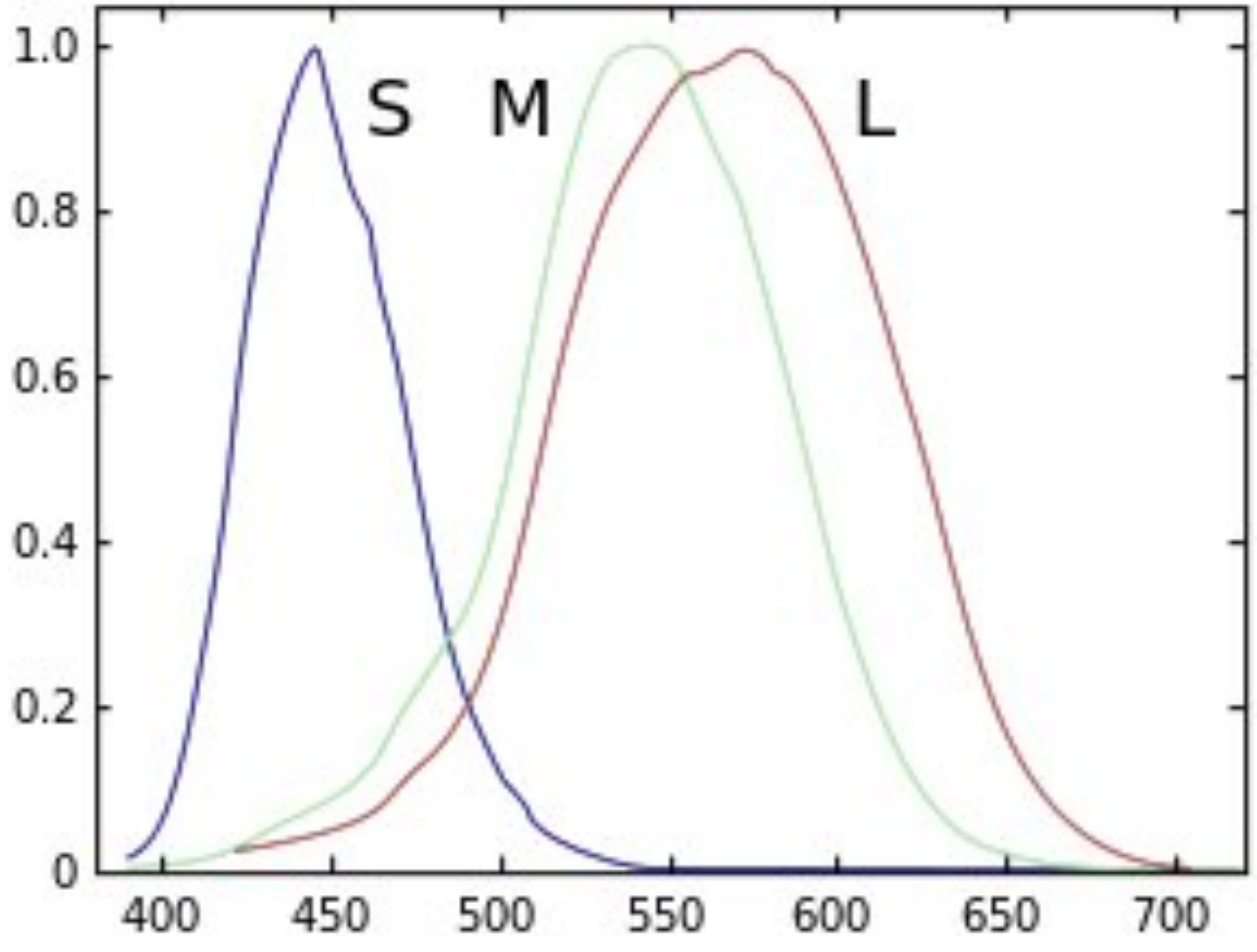


# Color vision

- Under sufficient illumination we use cone cells (photopic vision).
- 90% of you have 3 types of cone cells with different sensitivity depending on the spectrum (long, middle, short).
- The sensitivity are continuous functions of the wavelength:

$$\bar{\ell}(\lambda) \quad \bar{m}(\lambda) \quad \bar{s}(\lambda)$$



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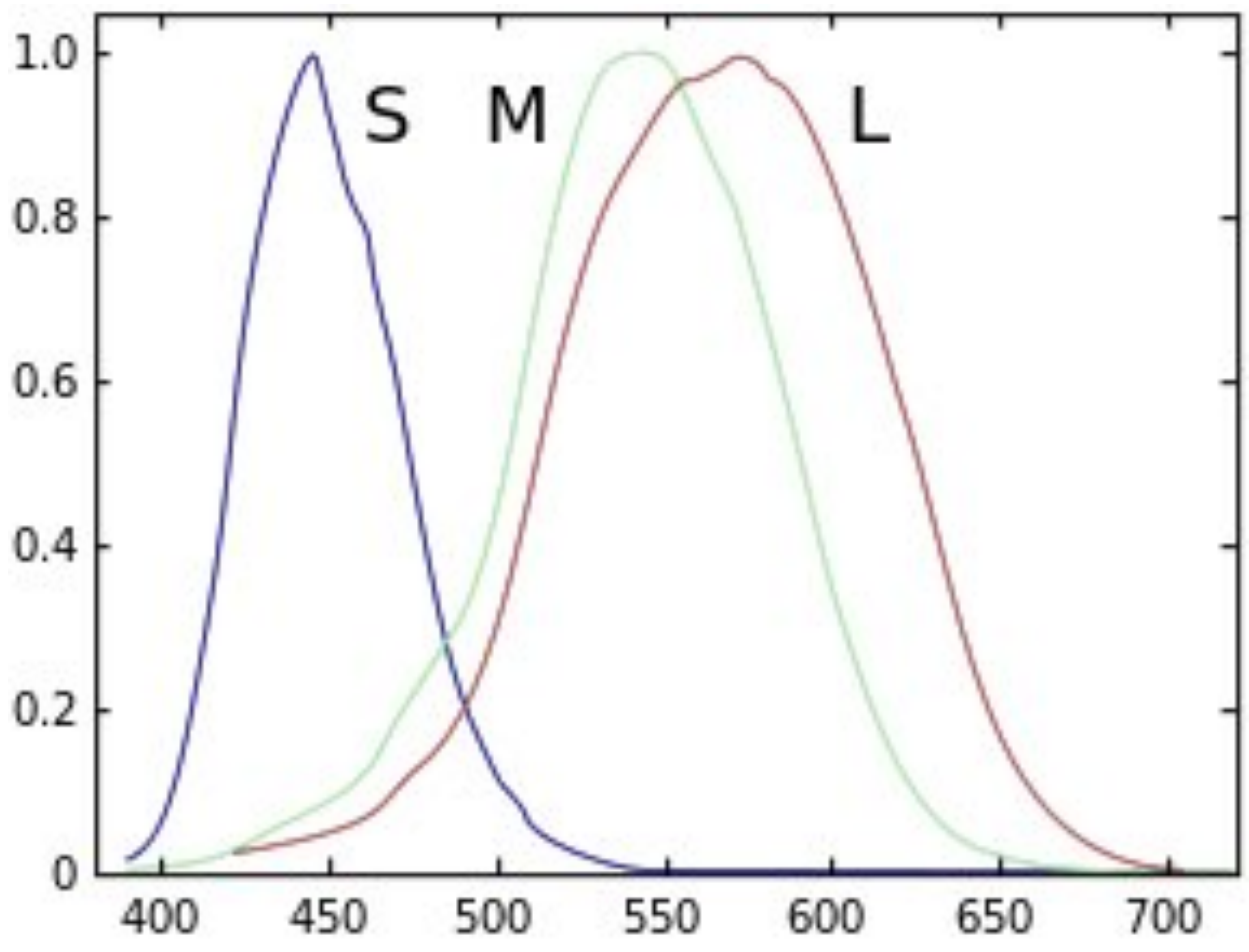
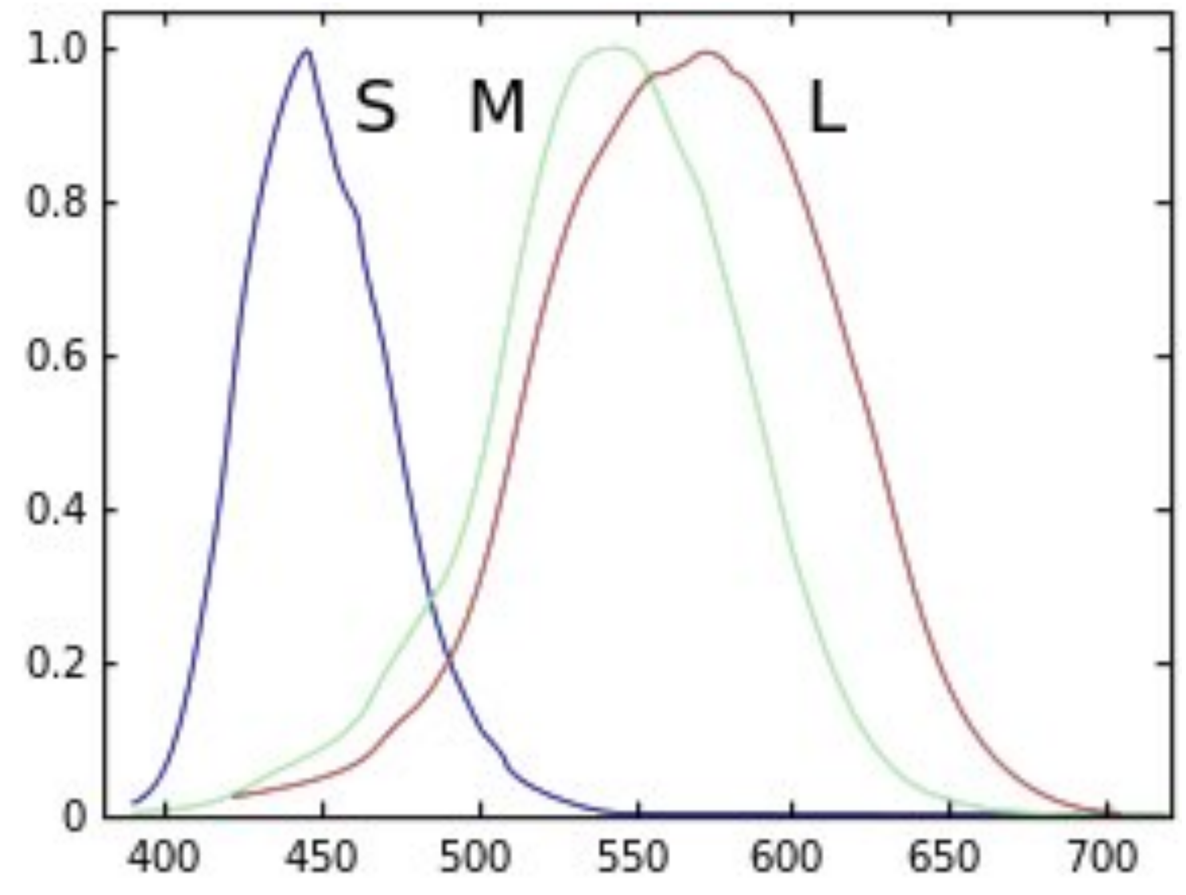



- When you watch a colored light $P(\lambda)$ , the strength of the neural signals from the 3 cones are the $L^2$-pairings:

$$L = \int_{380}^{750} \bar{\ell}(\lambda)P(\lambda)\,d\lambda \qquad M = \int_{380}^{750} \bar{m}(\lambda)P(\lambda)\,d\lambda \qquad S = \int_{380}^{750} \bar{s}(\lambda)P(\lambda)\,d\lambda$$

# Color vision

$$L = \int_{380}^{750} \bar{\ell}(\lambda)P(\lambda)\,d\lambda \qquad M = \int_{380}^{750} \overline{m}(\lambda)P(\lambda)\,d\lambda \qquad S = \int_{380}^{750} \bar{s}(\lambda)P(\lambda)\,d\lambda$$




**Definition.** Two colored lights $P_1(\lambda)$, $P_2(\lambda)$ ***appear the same*** if the LMS values match

$$(L_1, M_1, S_1) = (L_2, M_2, S_2)$$

**Definition.** Two perceived colors $(L_1, M_1, S_1), (L_2, M_2, S_2)$ have the same chromatic color if $L_1 : M_1 : S_1 = L_2 : M_2 : S_2$

- The perceived chromaticities live on a 2-dimensional projective space.
- The colored light may be a combination of 3 monotone lights (RGB screen), but still appear the same as a natural colored light.

# Color Space and Gamut

- Color science
- Inner product between functions
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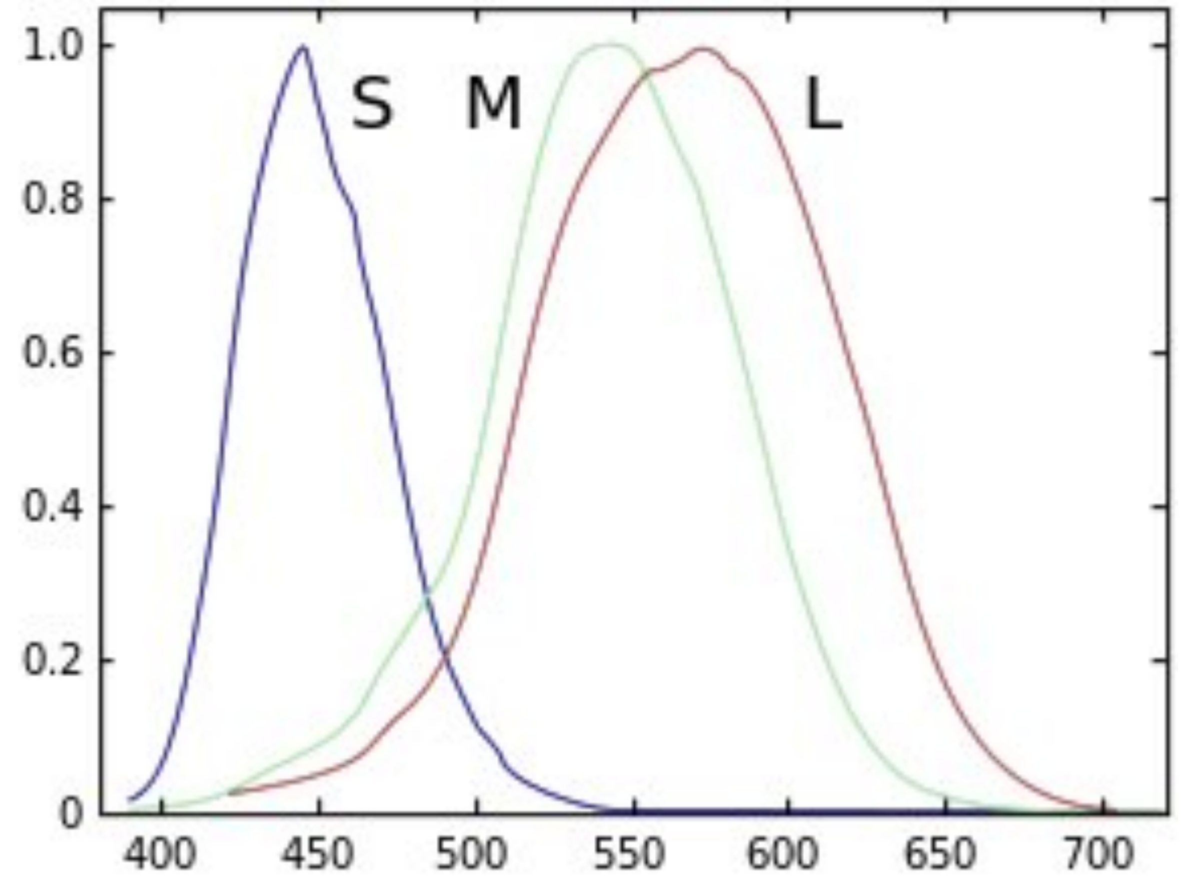
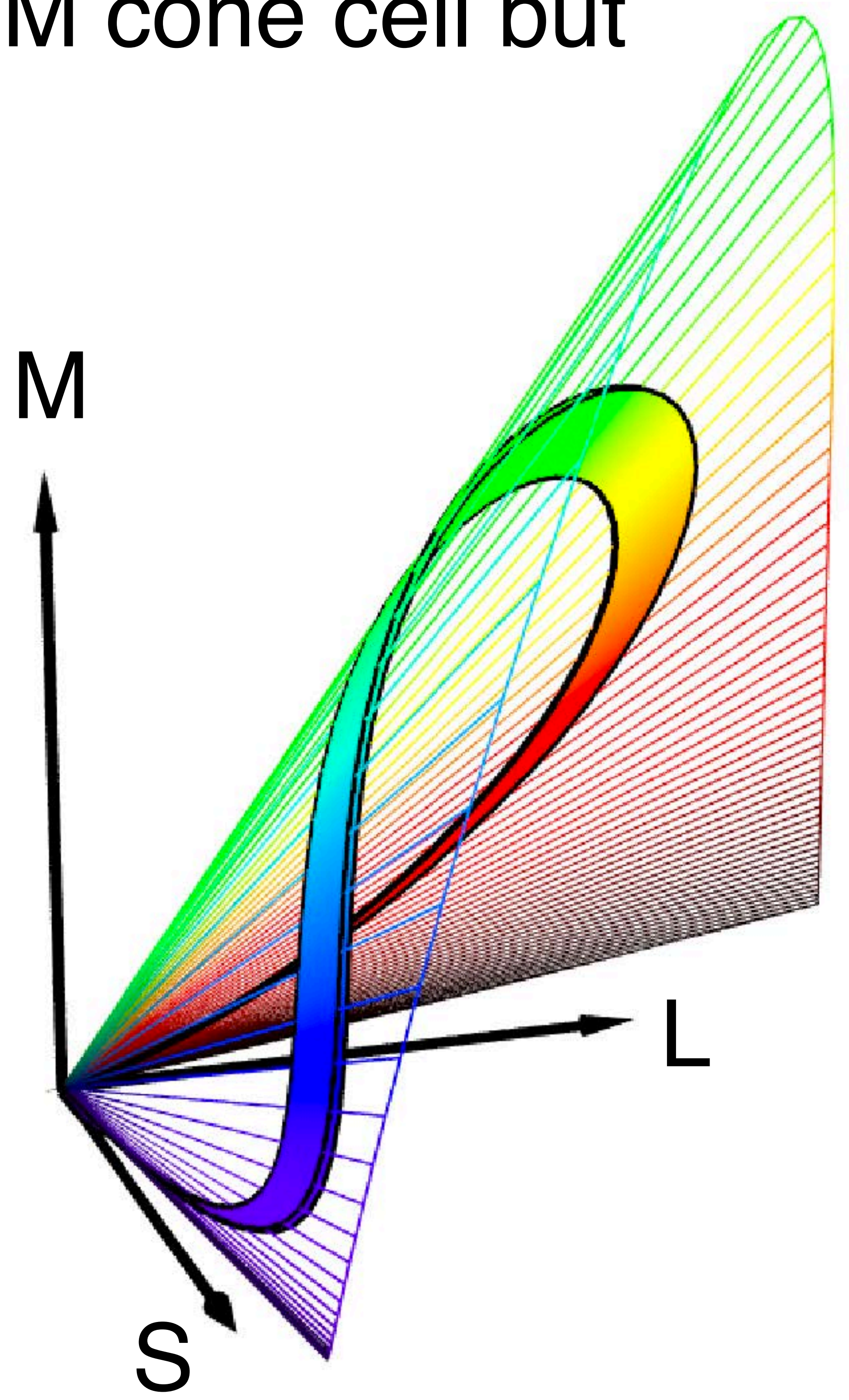
# LMS color space

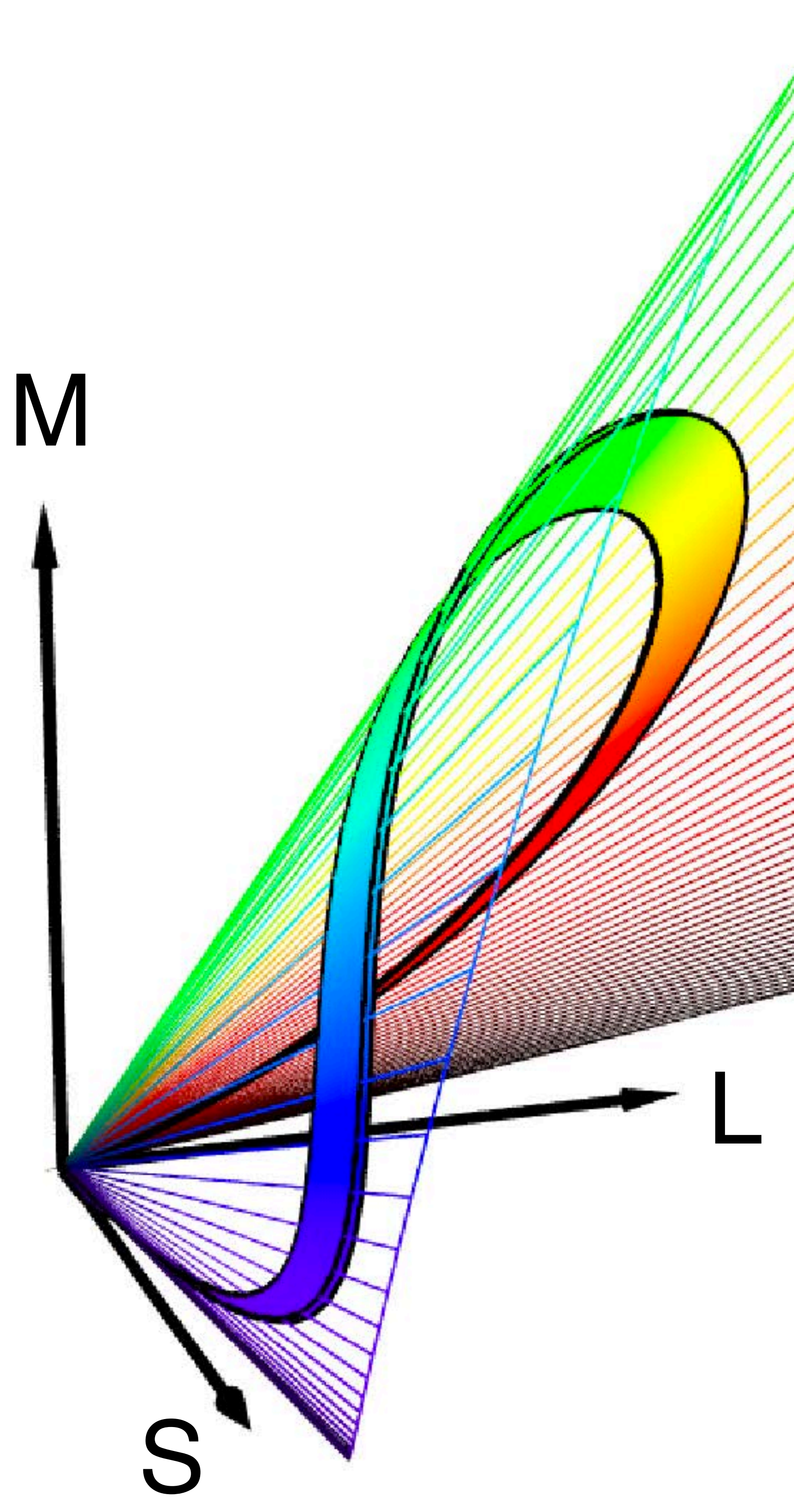
- Not all points in the 3D LMS space correspond to a color.
  - L,M,S are always nonnegative.
  - The source light is always a nonnegative distribution.
  - There is no light that would stimulate only the M cone cell but not the L cone cell.
  - All colored lights are mapped into the convex hull of the cone spanned by the spectral locus.

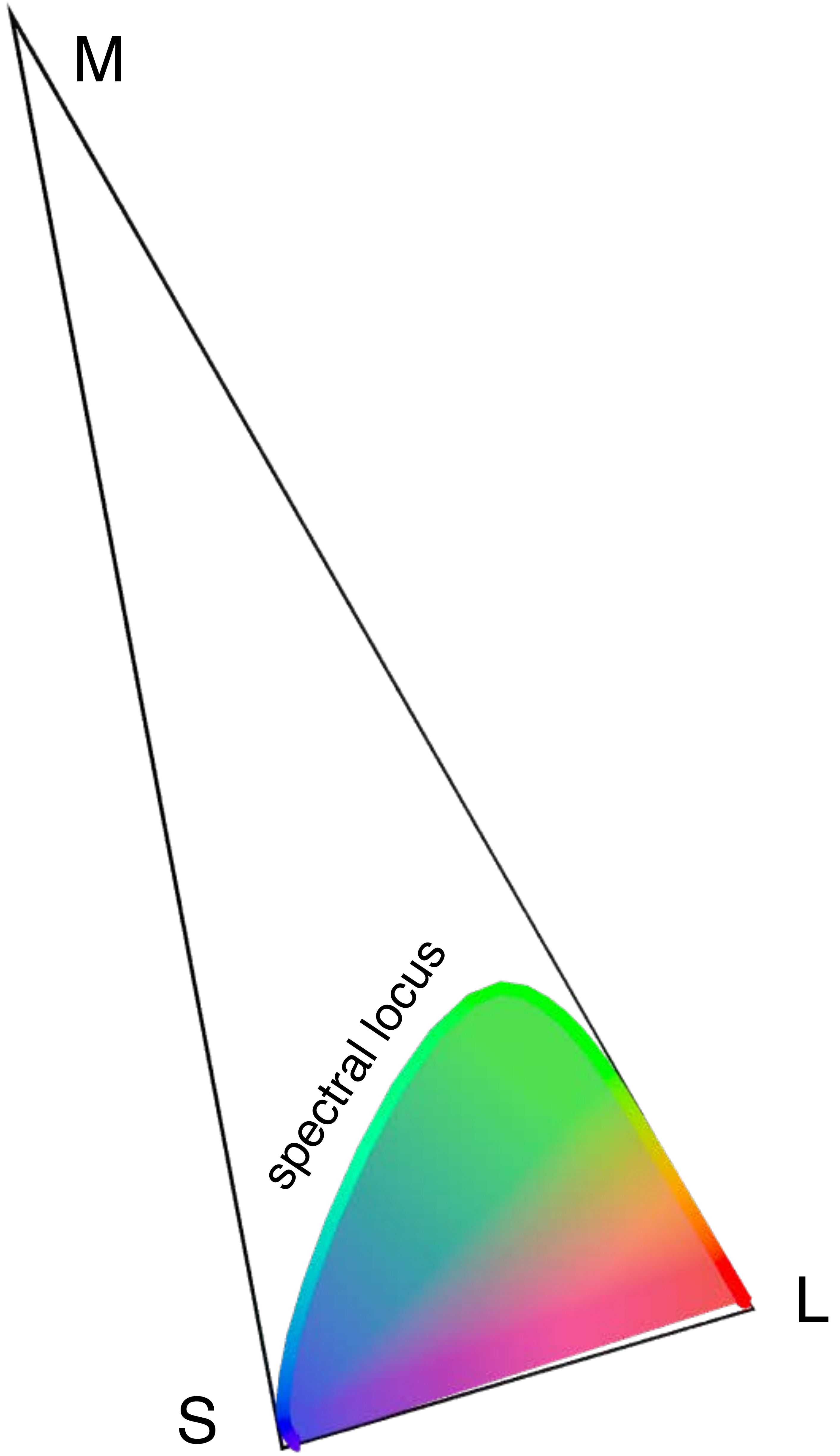


# LMS color space

- Visualize the chromatic color on a 2D slice



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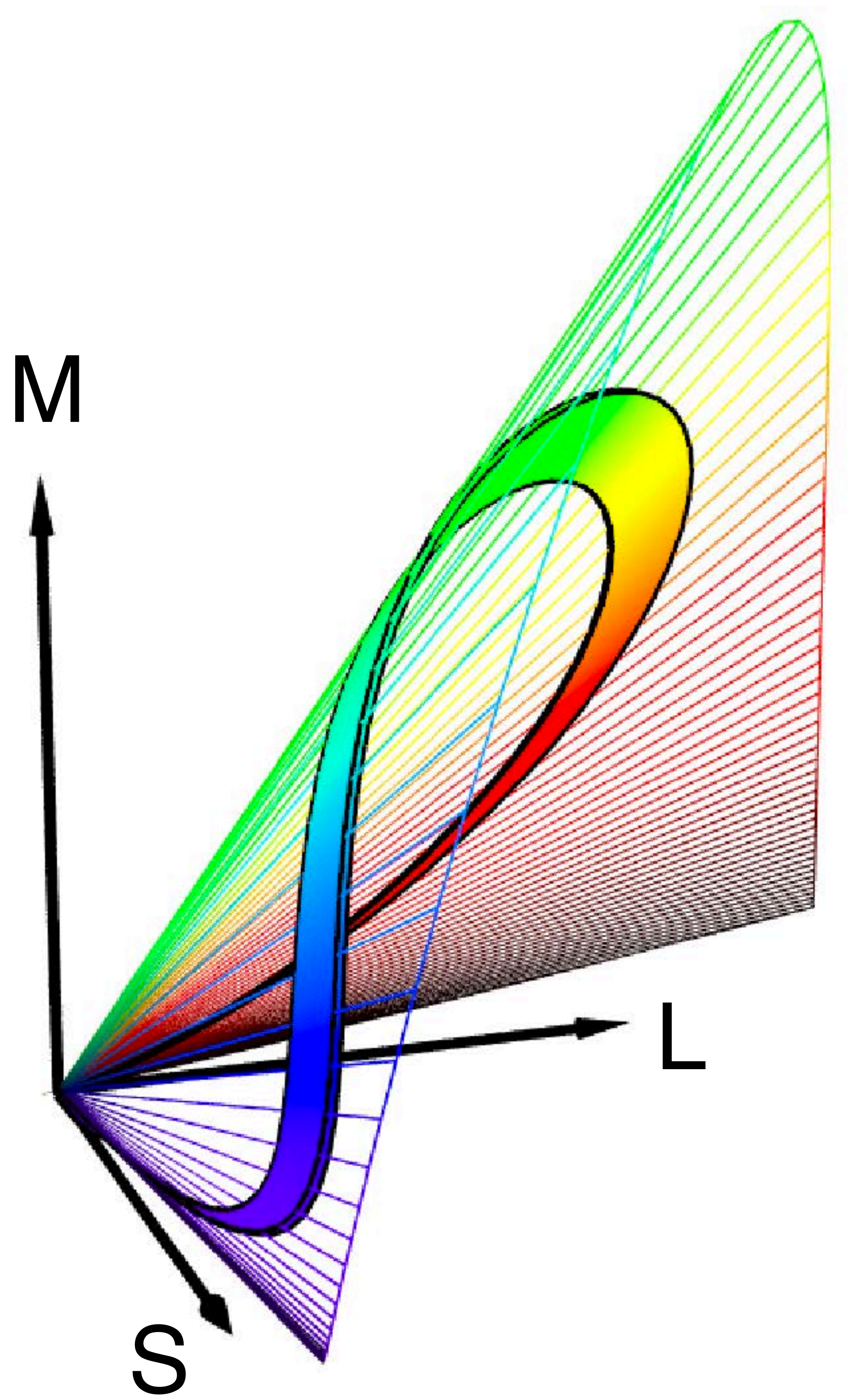
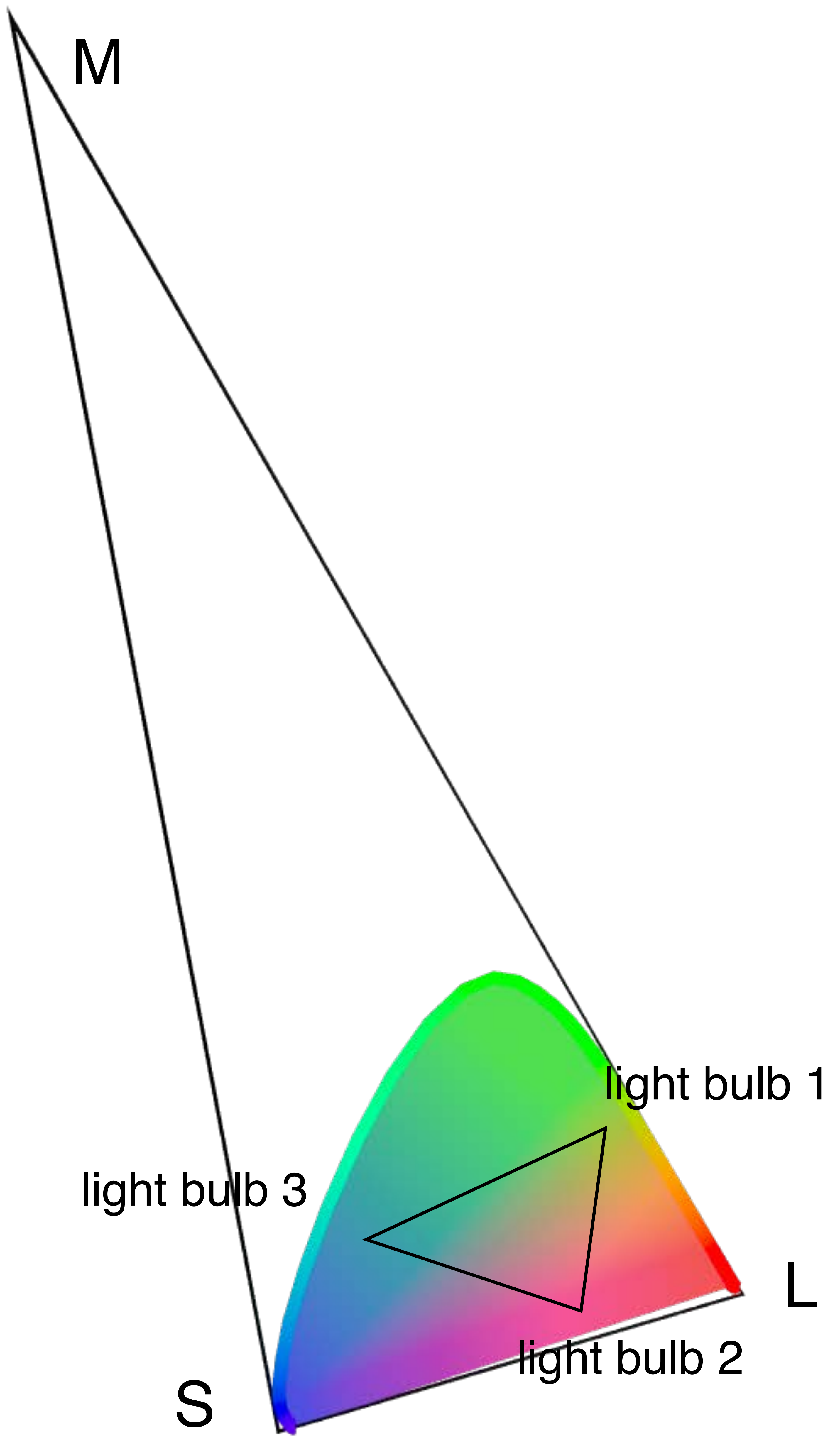



- Every artificial light that combines 3 lights must lie in the interior of a triangle.
- It is impossible to reproduce all colors by using 3 light bulbs.



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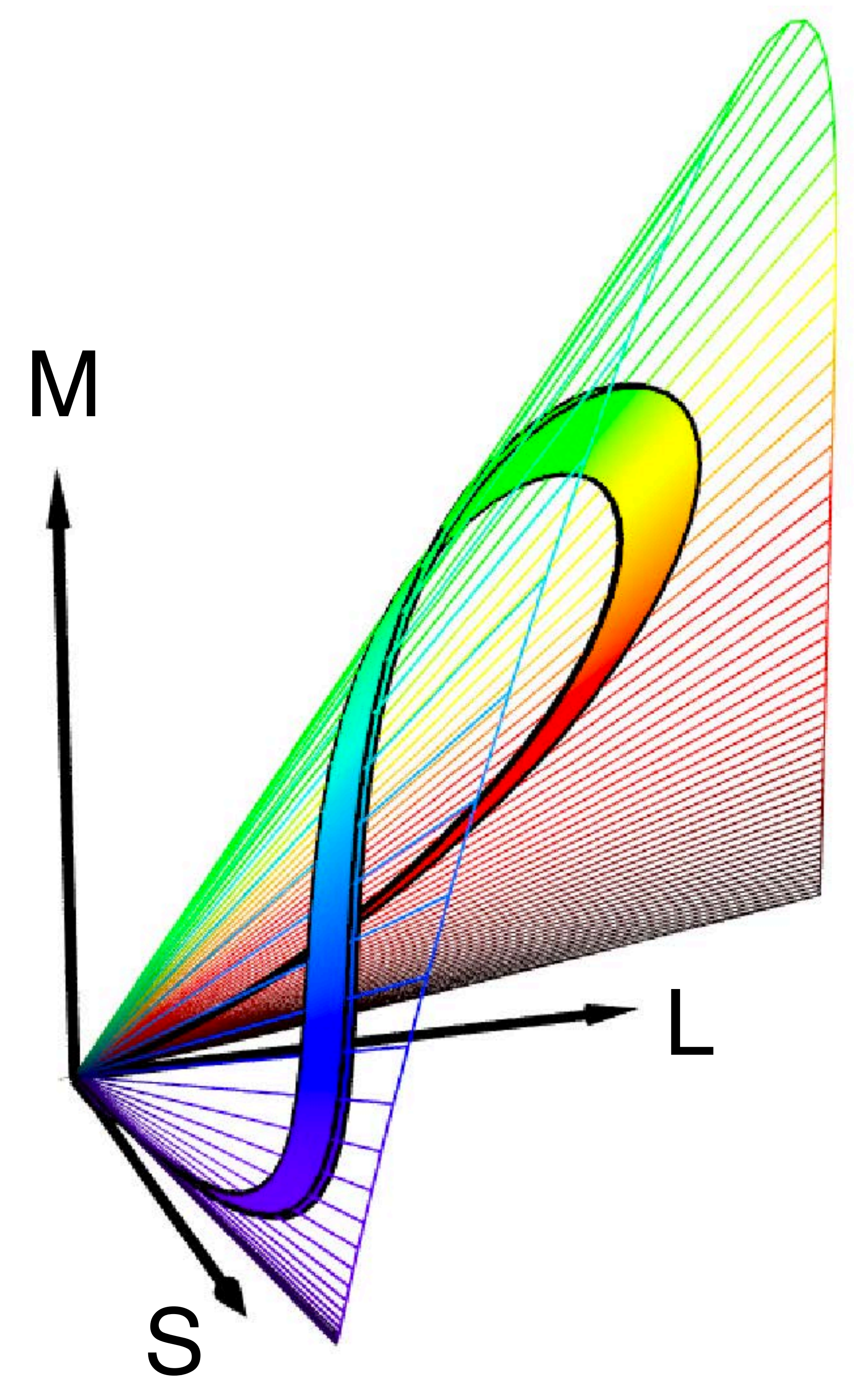
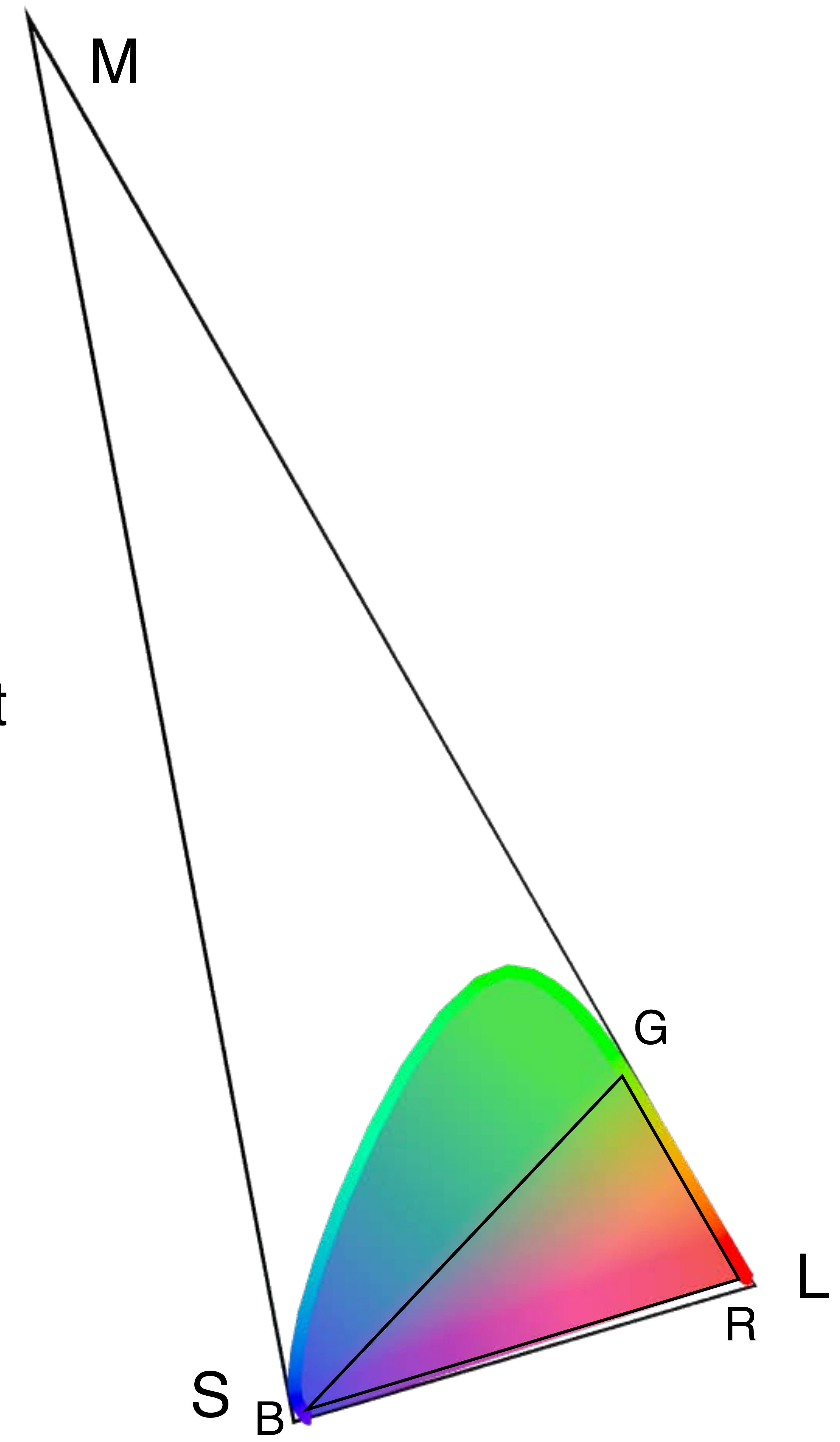



- Every artificial light that combines 3 lights must lie in the interior of a triangle.
- It is impossible to reproduce all colors by using 3 light bulbs.
- The standard RGB



# Grassmann Law

- **Grassmann Law**

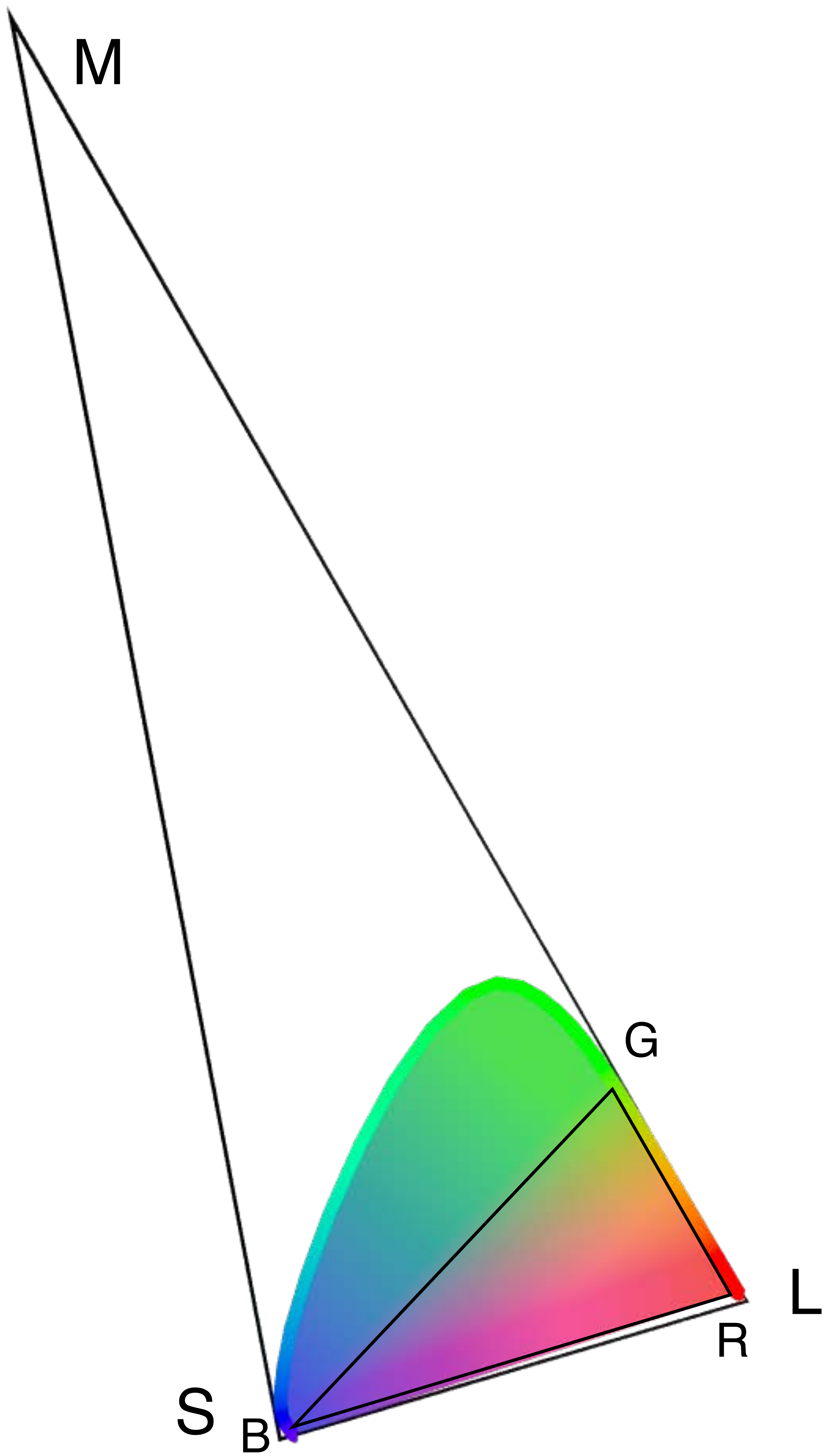
  Given any triangle as the basis, say RGB, there exists functions $\overline{r}(\lambda), \overline{g}(\lambda), \overline{b}(\lambda)$ such that any physical light $P(\lambda)$ appear to have the same color as the point

$$rR + gG + bB$$

  with the coefficients computed by the pairing

$$r = \int \overline{r}(\lambda)P(\lambda)\,d\lambda \qquad g = \int \overline{g}(\lambda)P(\lambda)\,d\lambda$$

$$b = \int \overline{b}(\lambda)P(\lambda)\,d\lambda$$



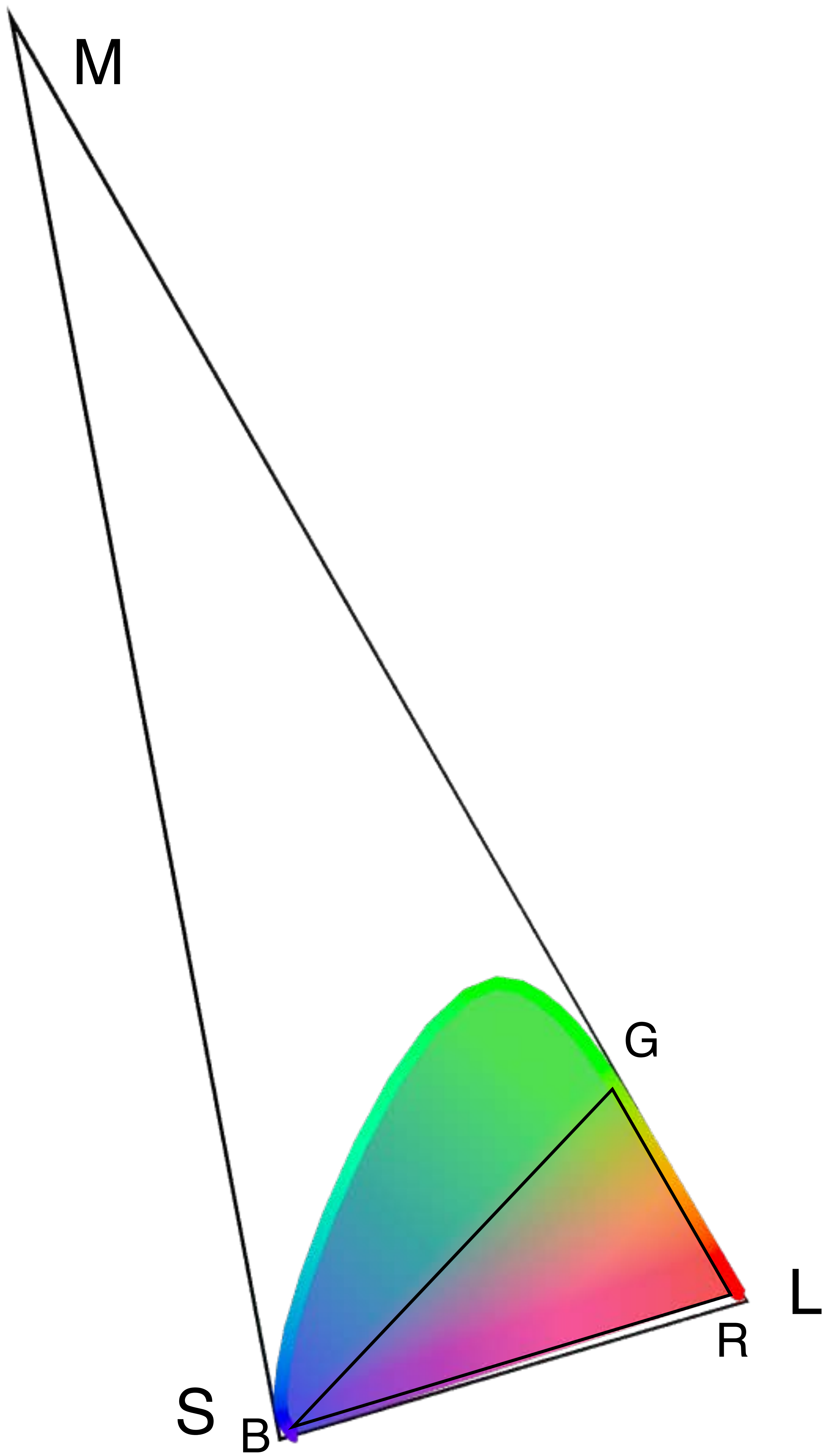
# Grassmann Law

$$rR + gG + bB$$

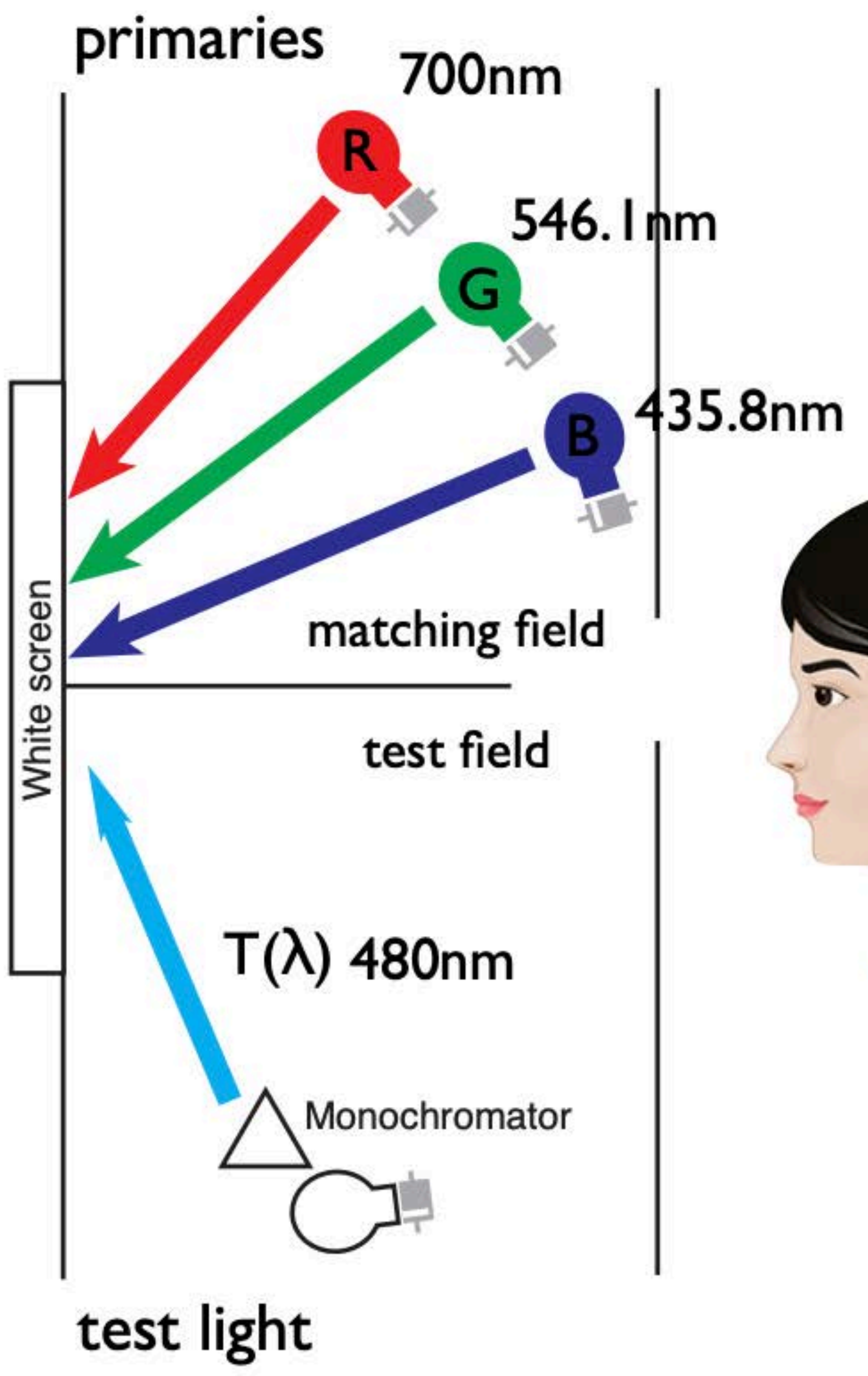
$$r = \int \bar{r}(\lambda)P(\lambda)\,d\lambda \qquad g = \int \bar{g}(\lambda)P(\lambda)\,d\lambda$$

$$b = \int \bar{b}(\lambda)P(\lambda)\,d\lambda$$

- To determine $\bar{r}, \bar{g}, \bar{b}$, take $RGB$ as 3 known light bulbs, and take $P(\lambda) = \delta_{\lambda_0}(\lambda)$
- Tune the coefficients r, g, b in $rR + gG + bB$ so that the synthetic light looks the same as $P(\lambda) = \delta_{\lambda_0}(\lambda)$
- Then we obtain $\bar{r}(\lambda_0) = r,\ \ \bar{g}(\lambda_0) = g,\ \ \bar{b}(\lambda_0) = b.$
- We call $\bar{r}, \bar{g}, \bar{b}$ the matching function.

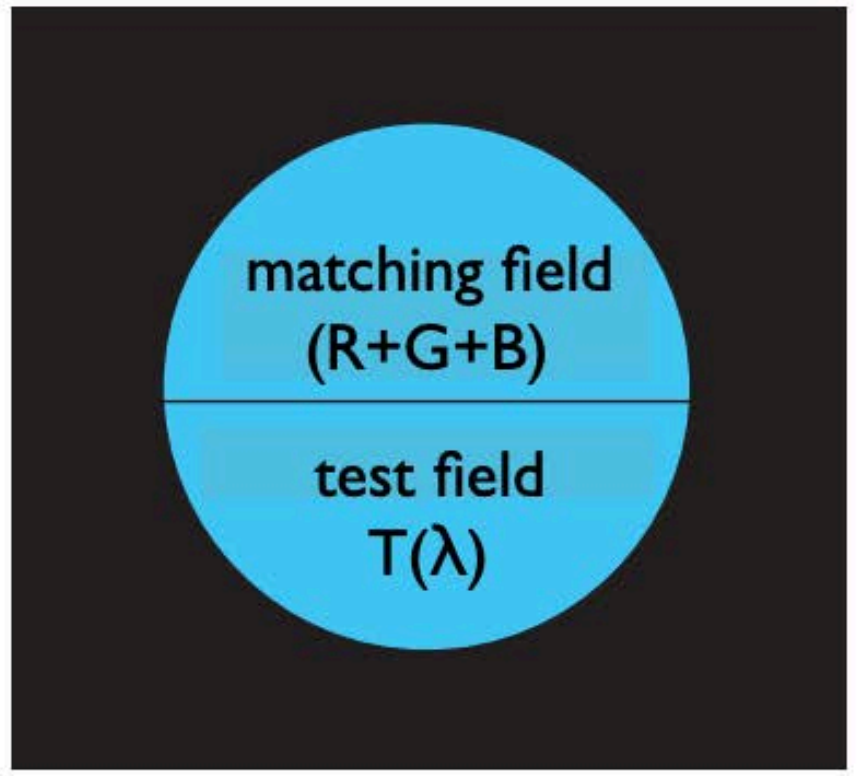


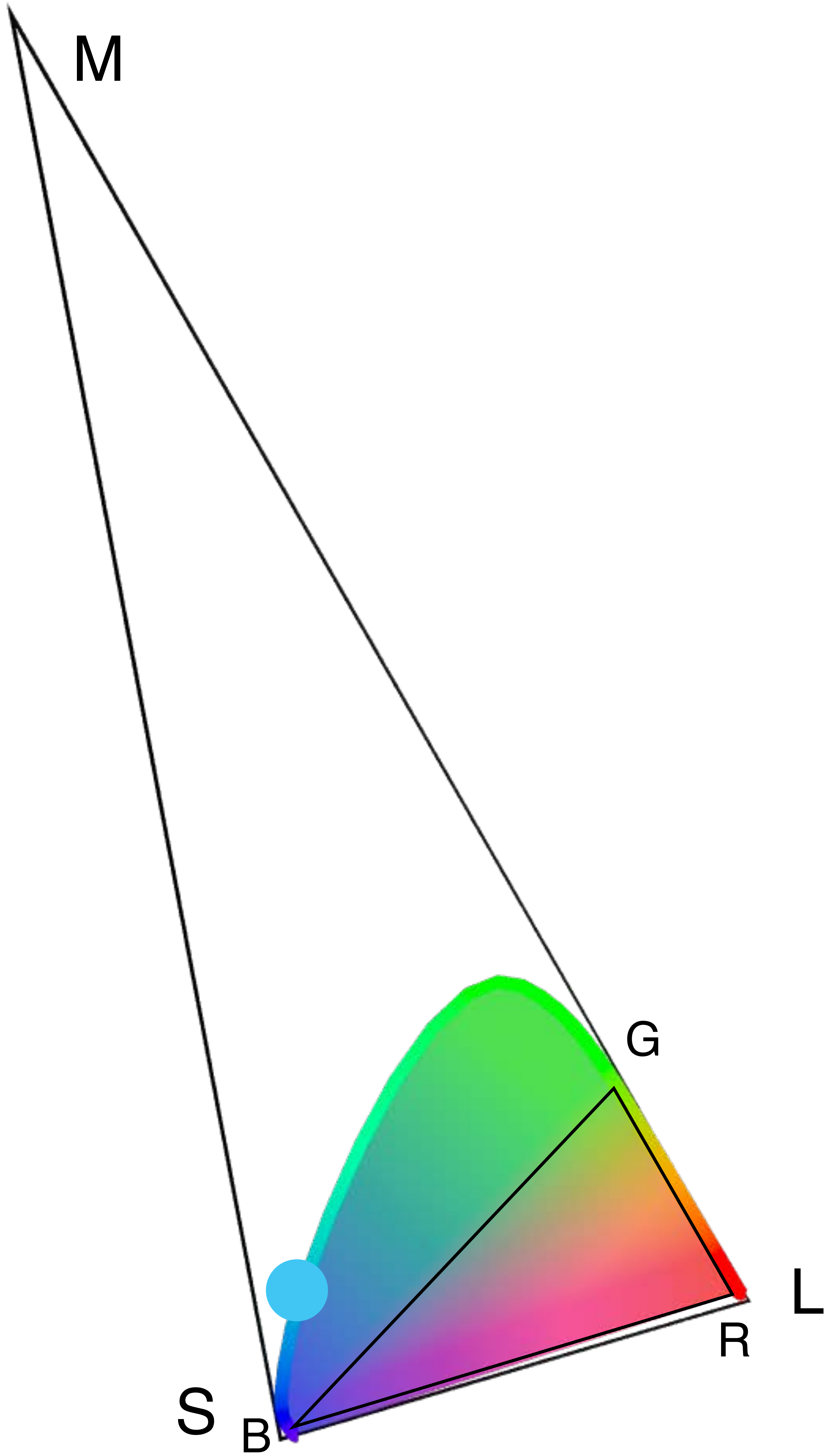
# Color matching experiment




T(λ) = rR+gG+bB

T(480)=-0.04R+0.04G+0.14B



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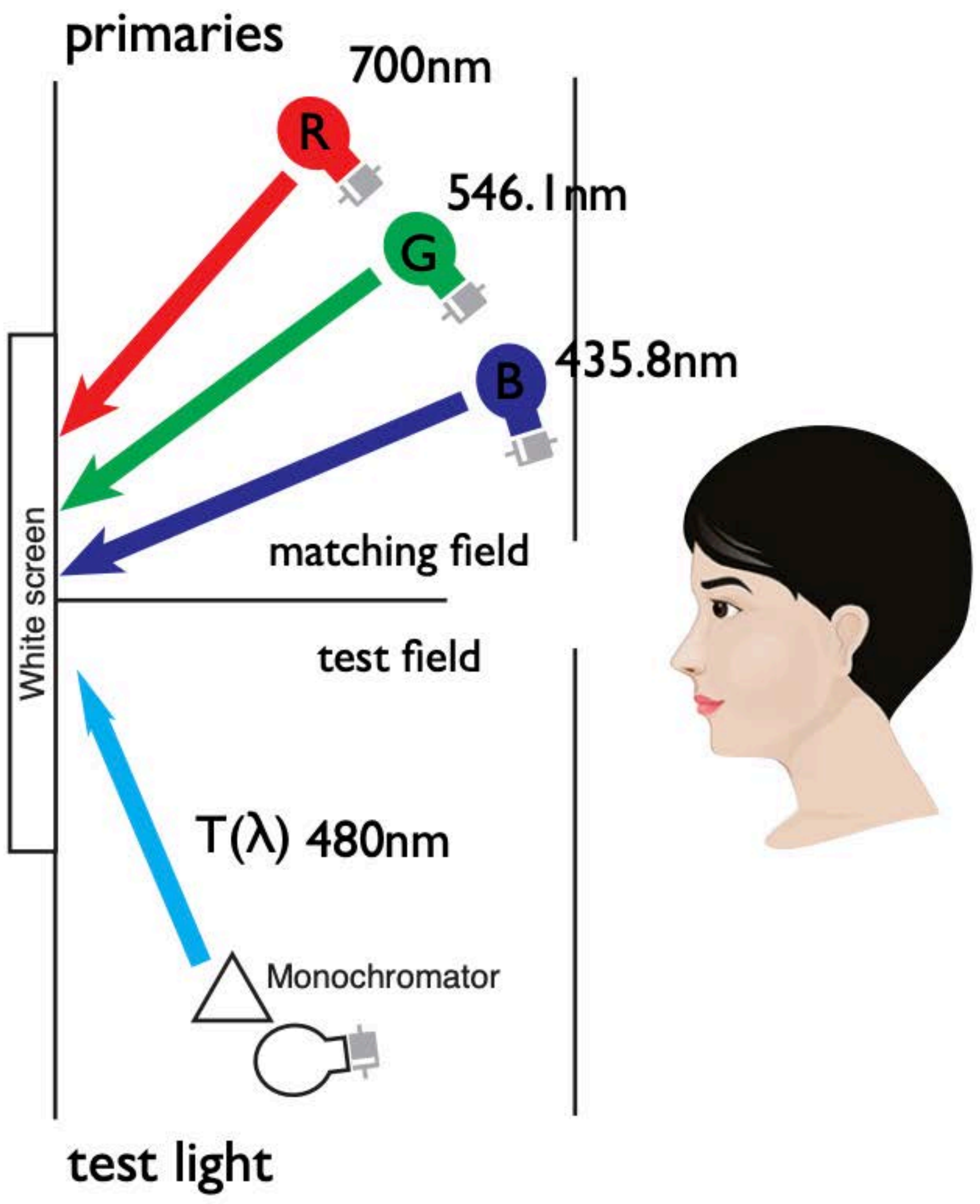


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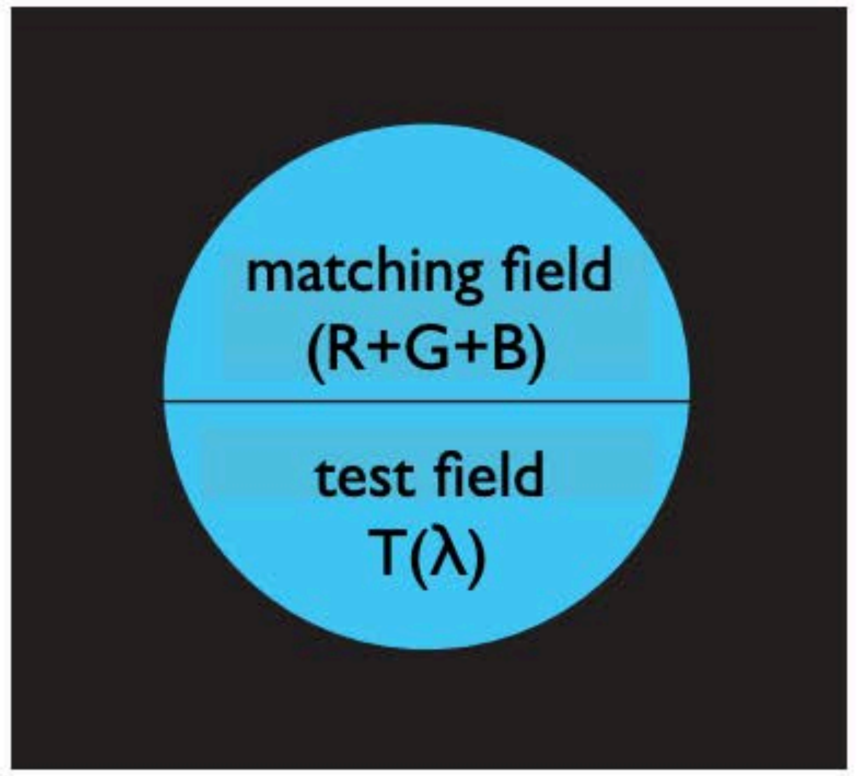
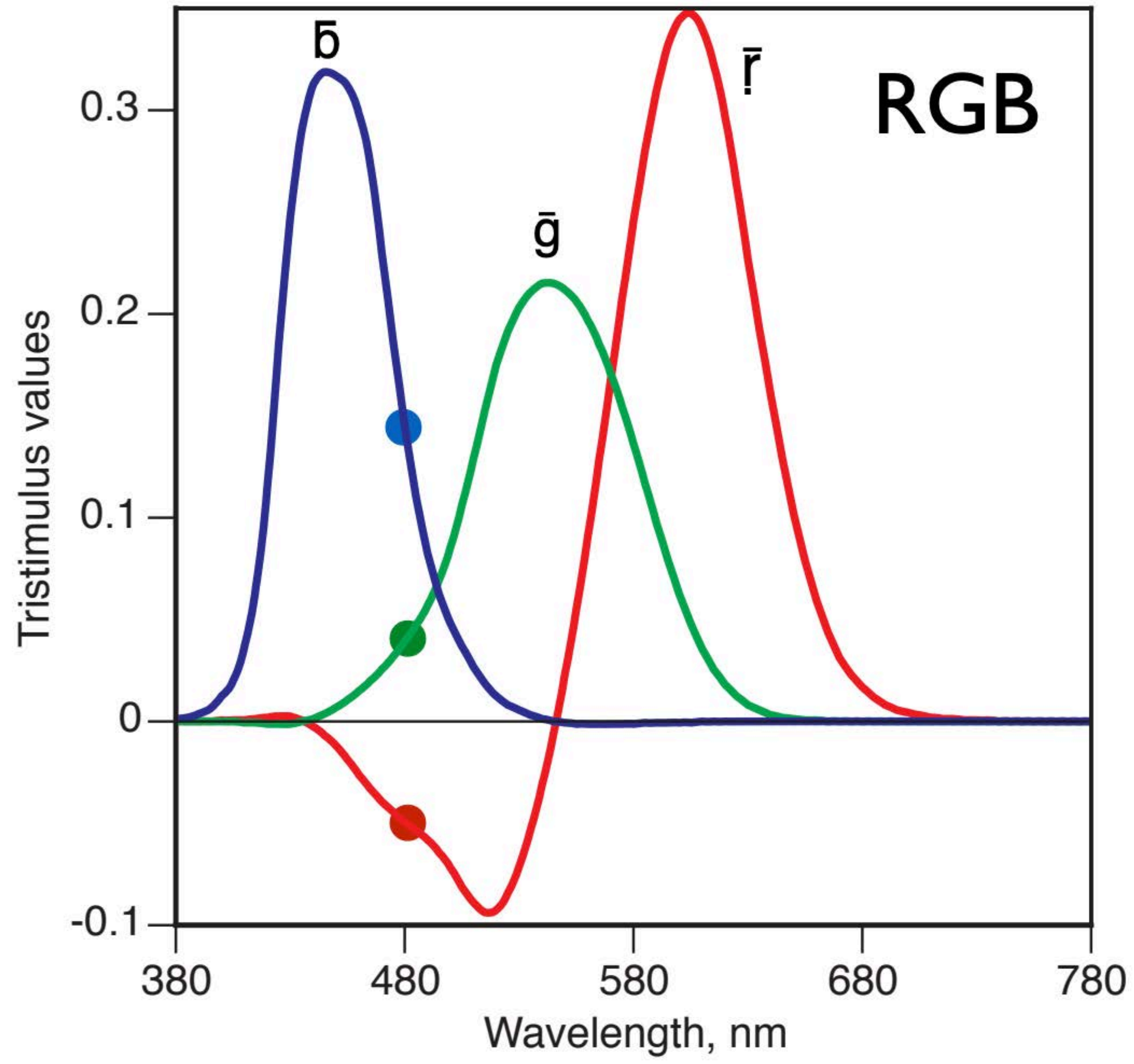
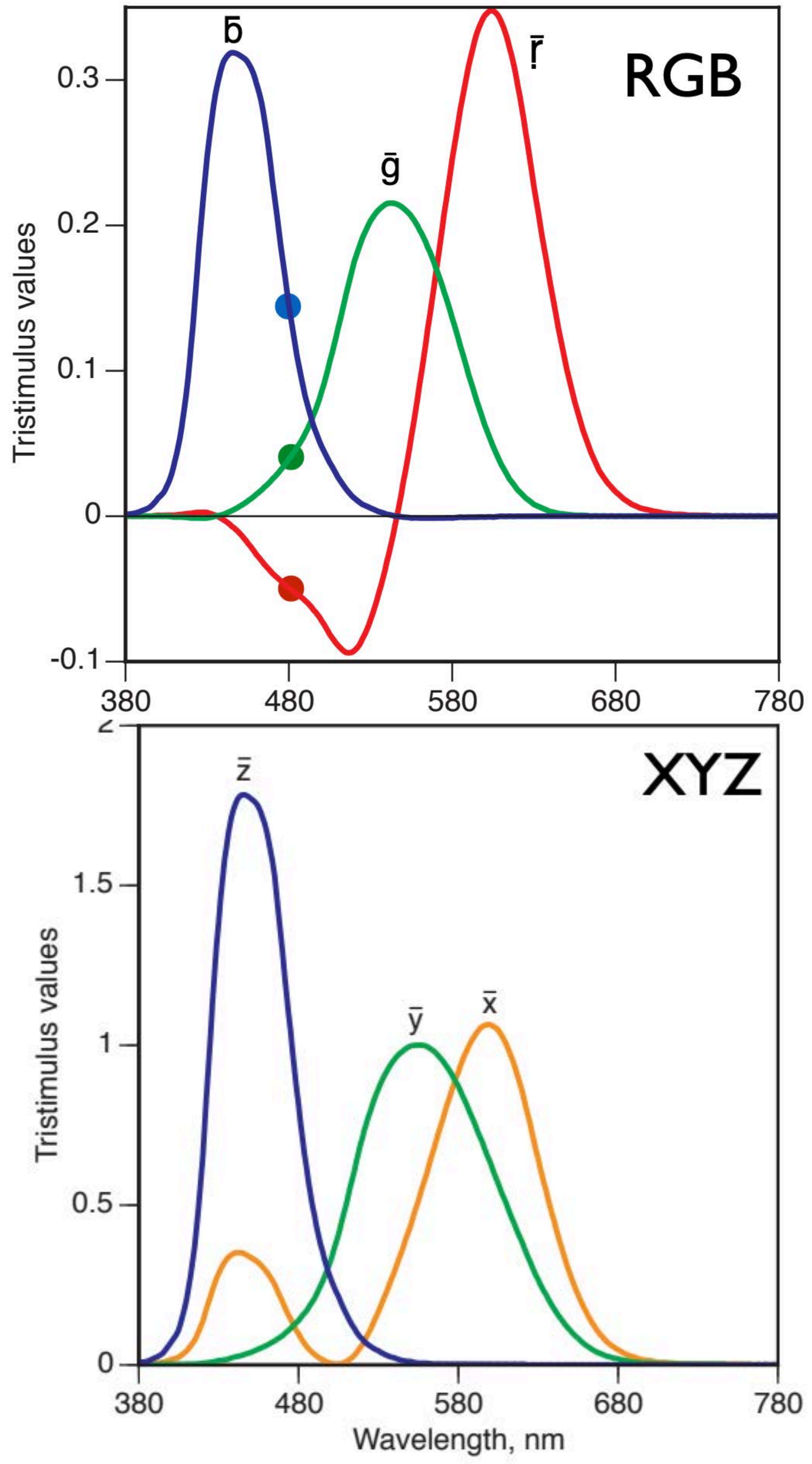
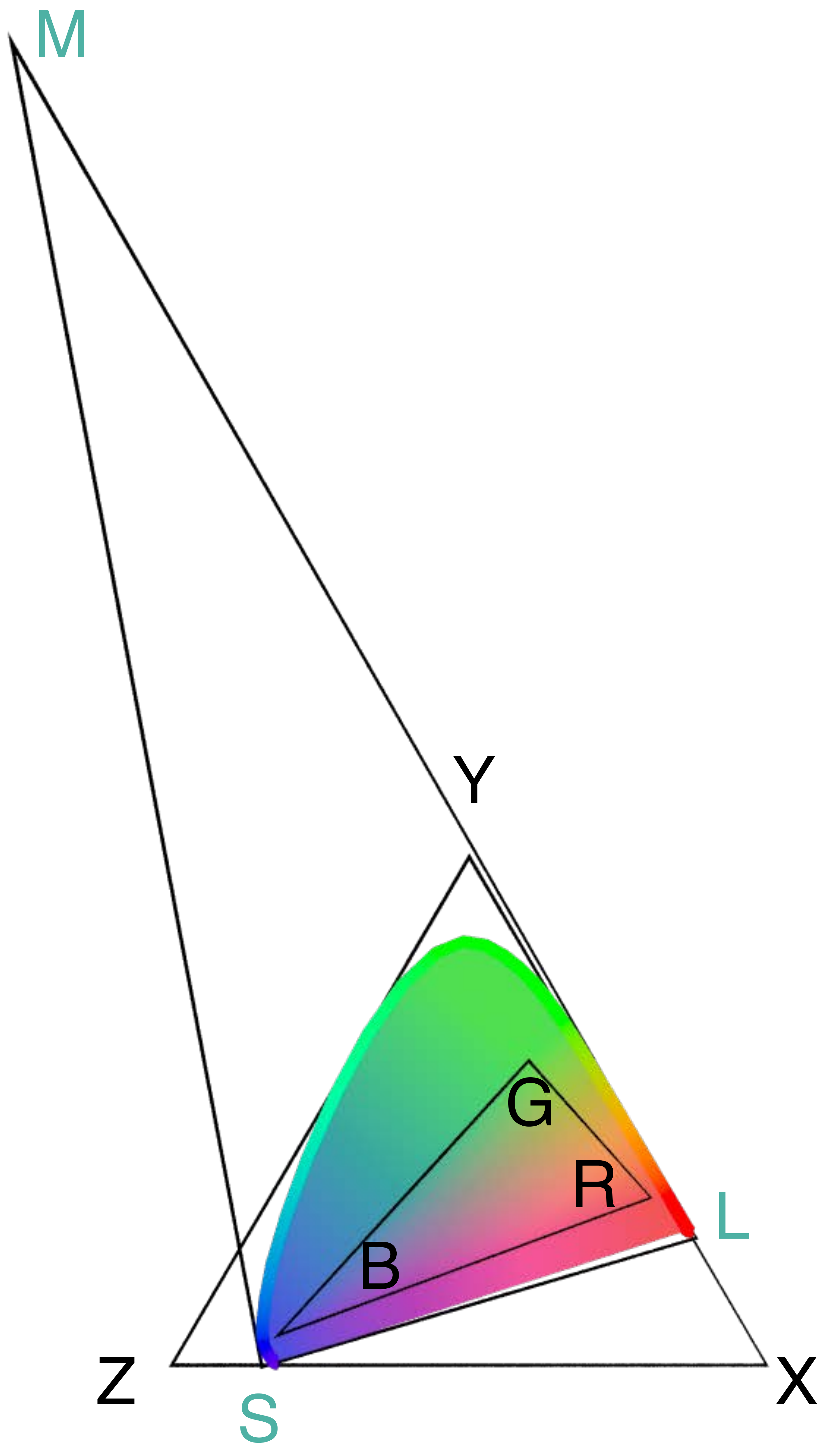


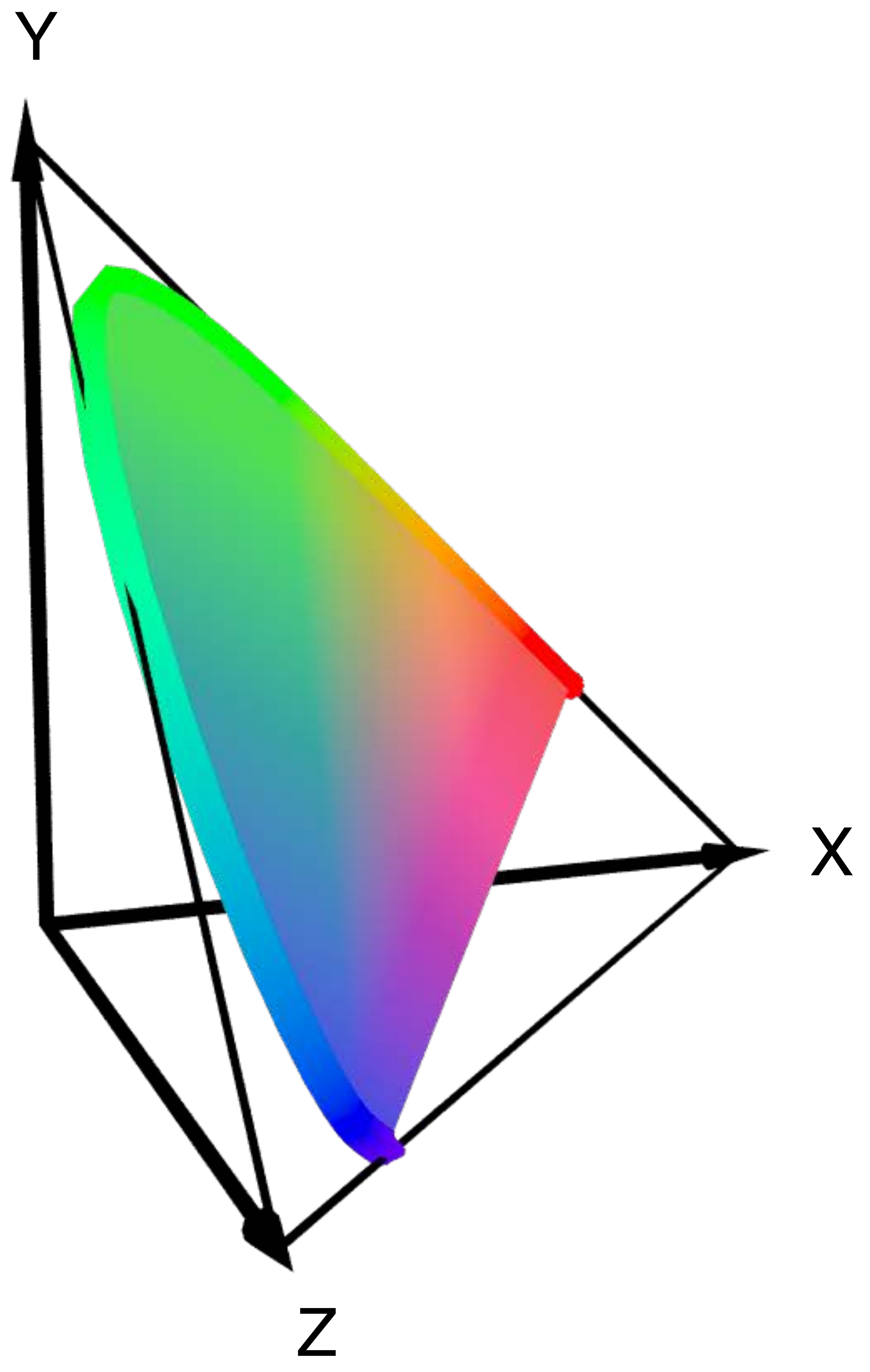
# CIE 1931 XYZ color space

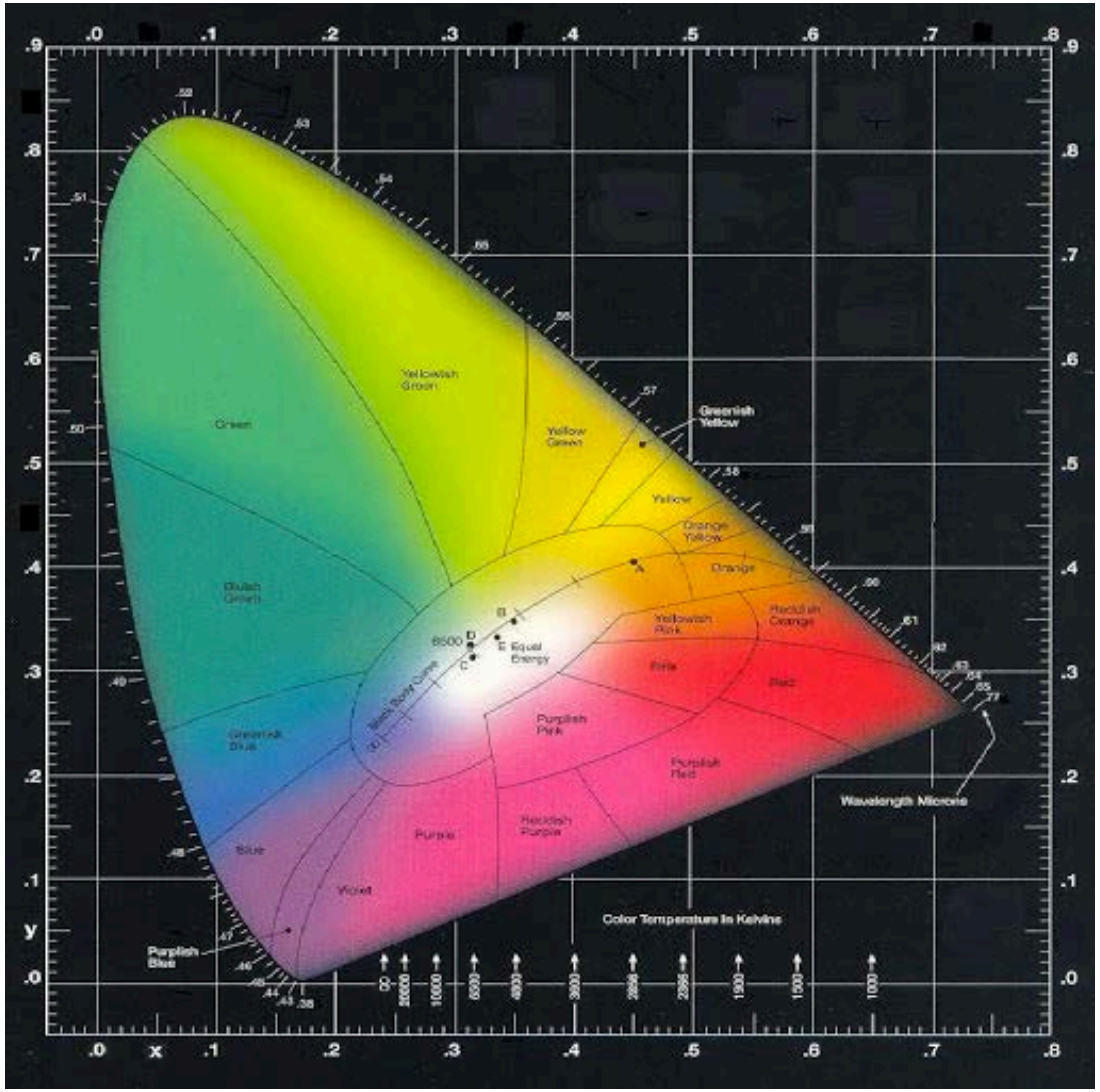
- In 1931, CIE (International Commission on Illumination) proposed another basis based on 3 imaginary primaries X, Y, Z.
  - Mixture of XYZ gives all color
  - The corresponding matching functions $\bar{x}, \bar{y}, \bar{z}$ are linear transforms of $\bar{r}, \bar{g}, \bar{b}$
  - $\bar{x}, \bar{y}, \bar{z}$ are nonnegative (equivalent to the first item above)
  - $\bar{x}, \bar{y}, \bar{z}$ have equal areas under their curves.
  - The y-coefficient $\int \bar{y}(\lambda)P(\lambda)\,d\lambda$ is defined to be the **luminance**.



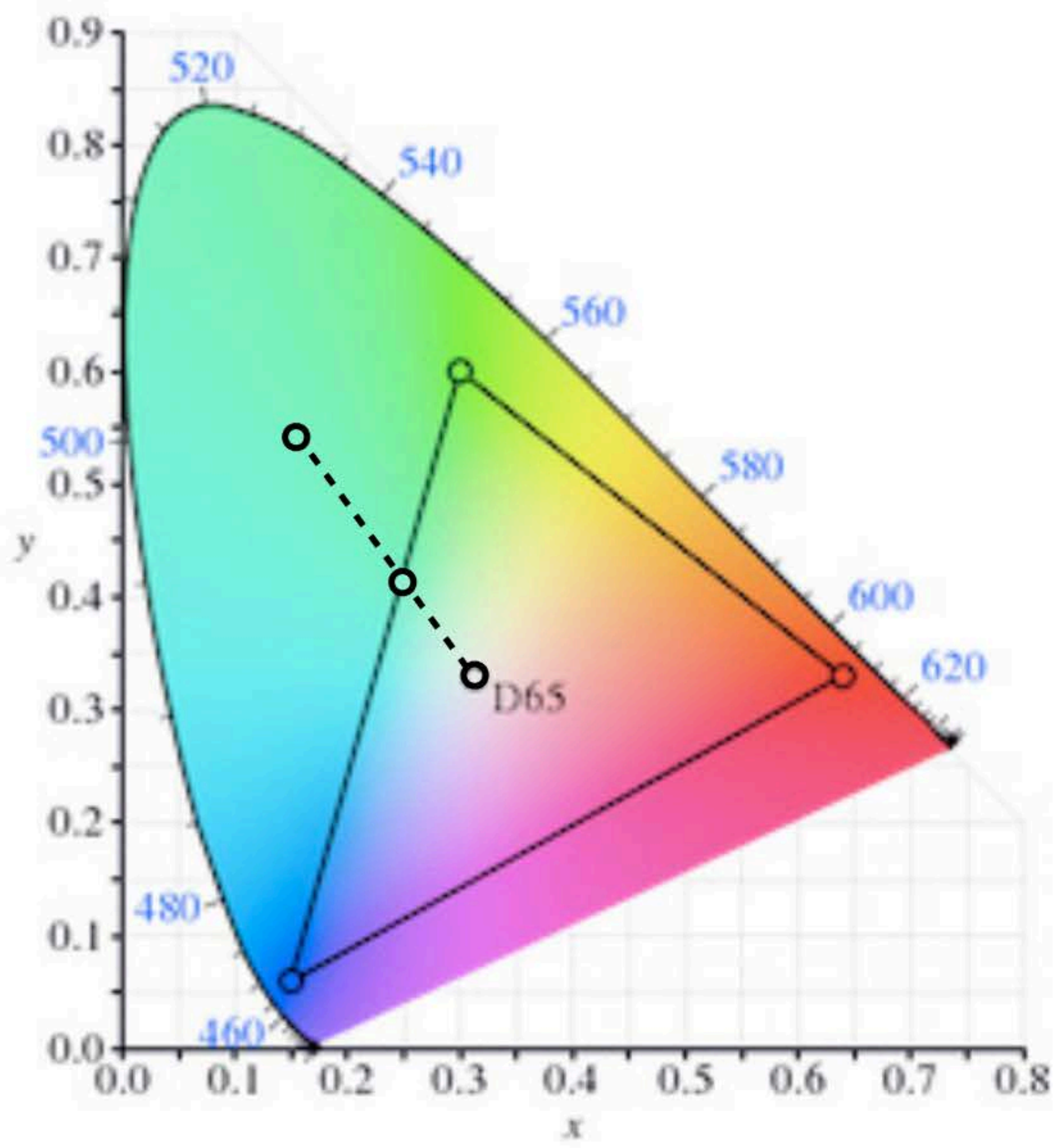
# CIE 1931 XYZ color space



# Color gamut

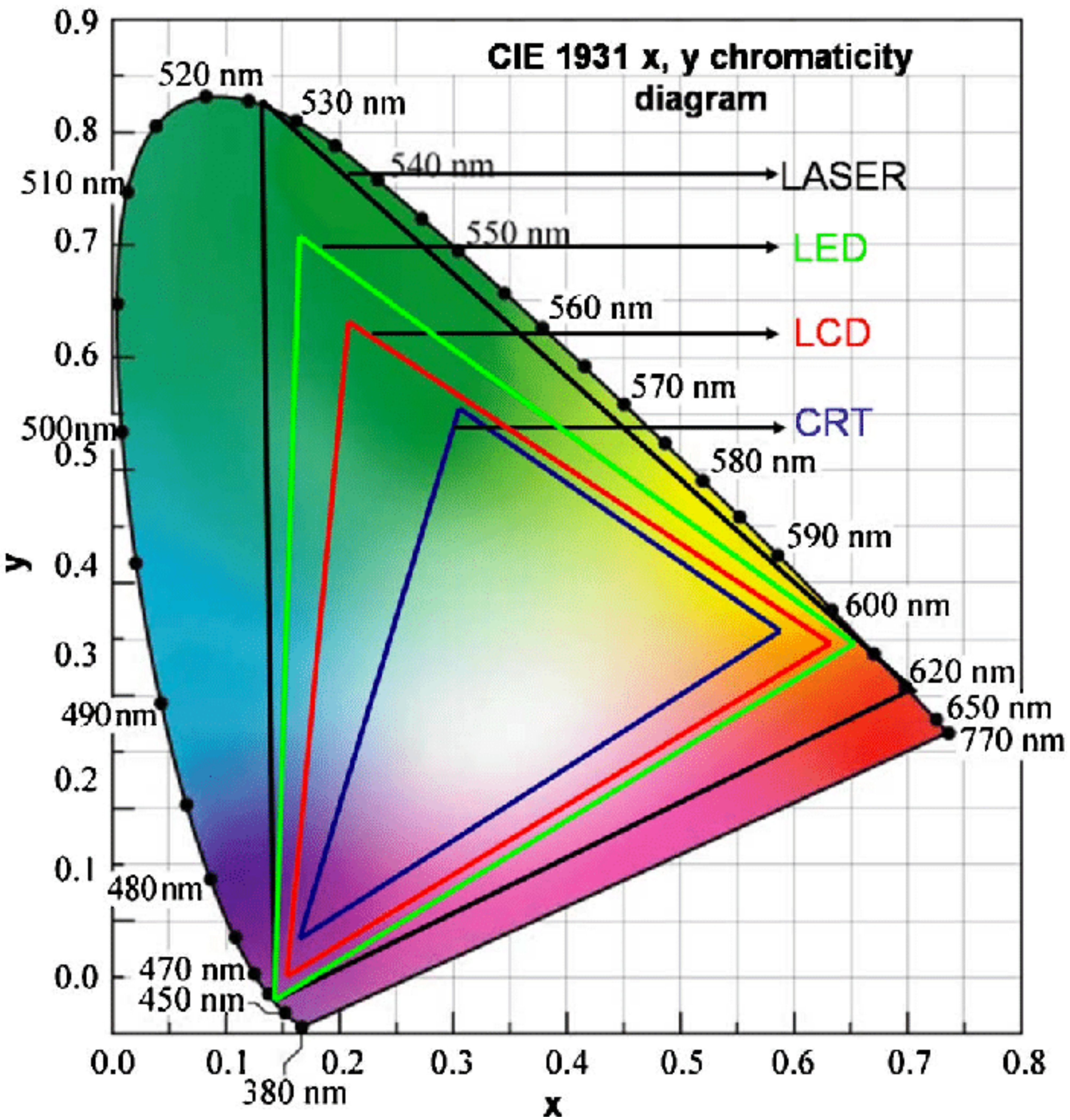
- Color gamut: The range of chromaticities that can be produced by mixing primaries.
- Real device gamut are limited, can't produce all chromaticities.
- Gamut mapping: Methods to approximate out-of-gamut chromaticities
    - Typically involves desaturating source chromaticities until they fall within the gamut boundaries.



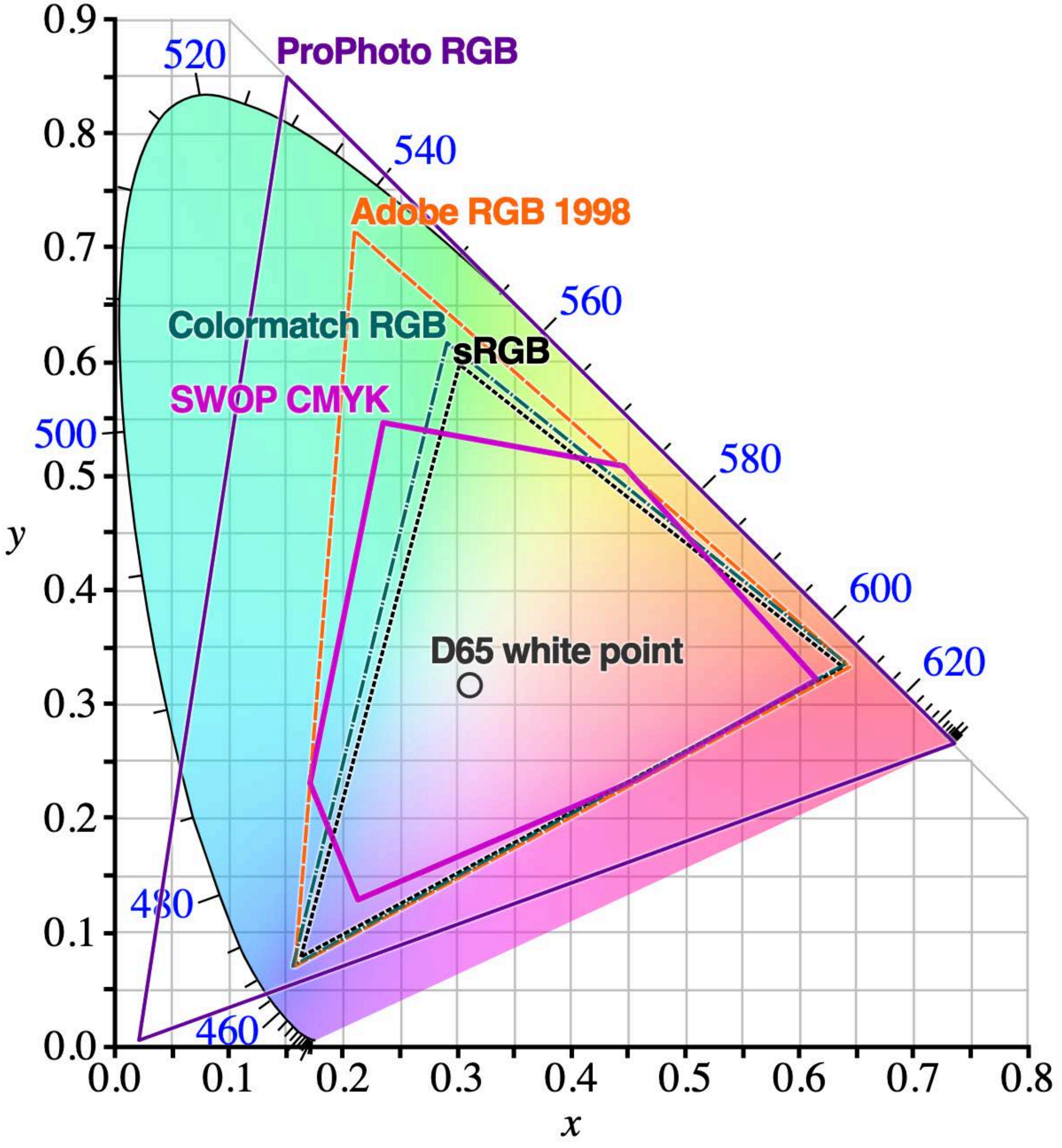

sRGB Specification

| Primary | x | y |
|---|---|---|
| Red | 0.6400 | 0.3300 |
| Green | 0.3000 | 0.6000 |
| Blue | 0.1500 | 0.0600 |
| **White Point** | 0.3127 | 0.3290 |

# Color gamut

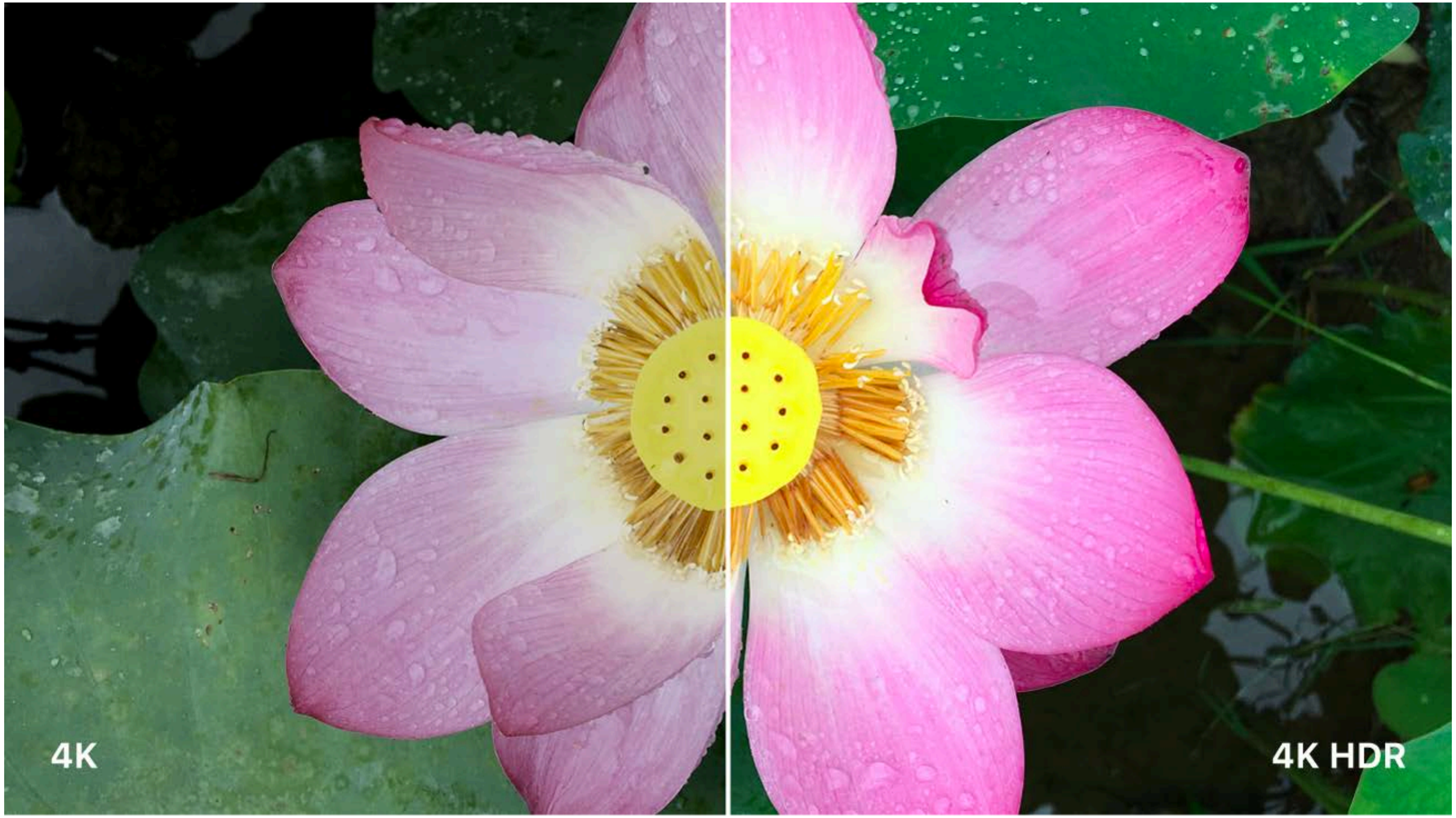



Various hardwares




Various specs

# Color gamut



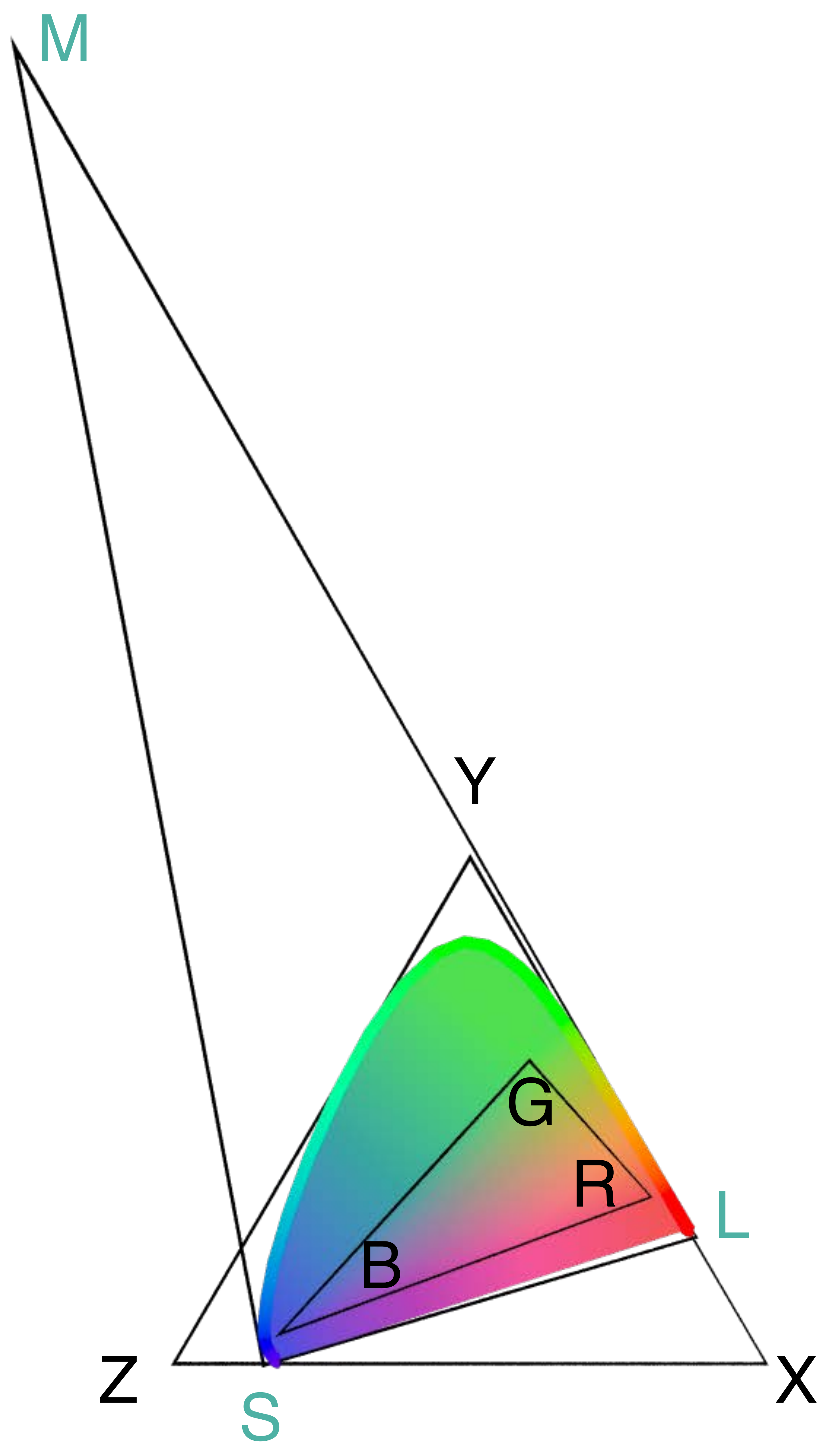

Wide gamut and
Hide dynamical range

# Conversion between different bases

- XYZ, RGB are 3D vectors with respect to different bases
- Relation between them is recorded by a 3x3 matrix

$$\begin{bmatrix} X \\ Y \\ Z \end{bmatrix} = \begin{bmatrix} 0.4360747 & 0.3850649 & 0.1430804 \\ 0.2225045 & 0.7168786 & 0.0606169 \\ 0.0139322 & 0.0971045 & 0.7141733 \end{bmatrix} \begin{bmatrix} R \\ G \\ B \end{bmatrix}$$

$$\begin{bmatrix} R \\ G \\ B \end{bmatrix} = \begin{bmatrix} 3.1338561 & -1.6168667 & -0.4906146 \\ -0.9787684 & 1.9161415 & 0.0334540 \\ 0.0719453 & -0.2289914 & 1.4052427 \end{bmatrix} \begin{bmatrix} X \\ Y \\ Z \end{bmatrix}$$

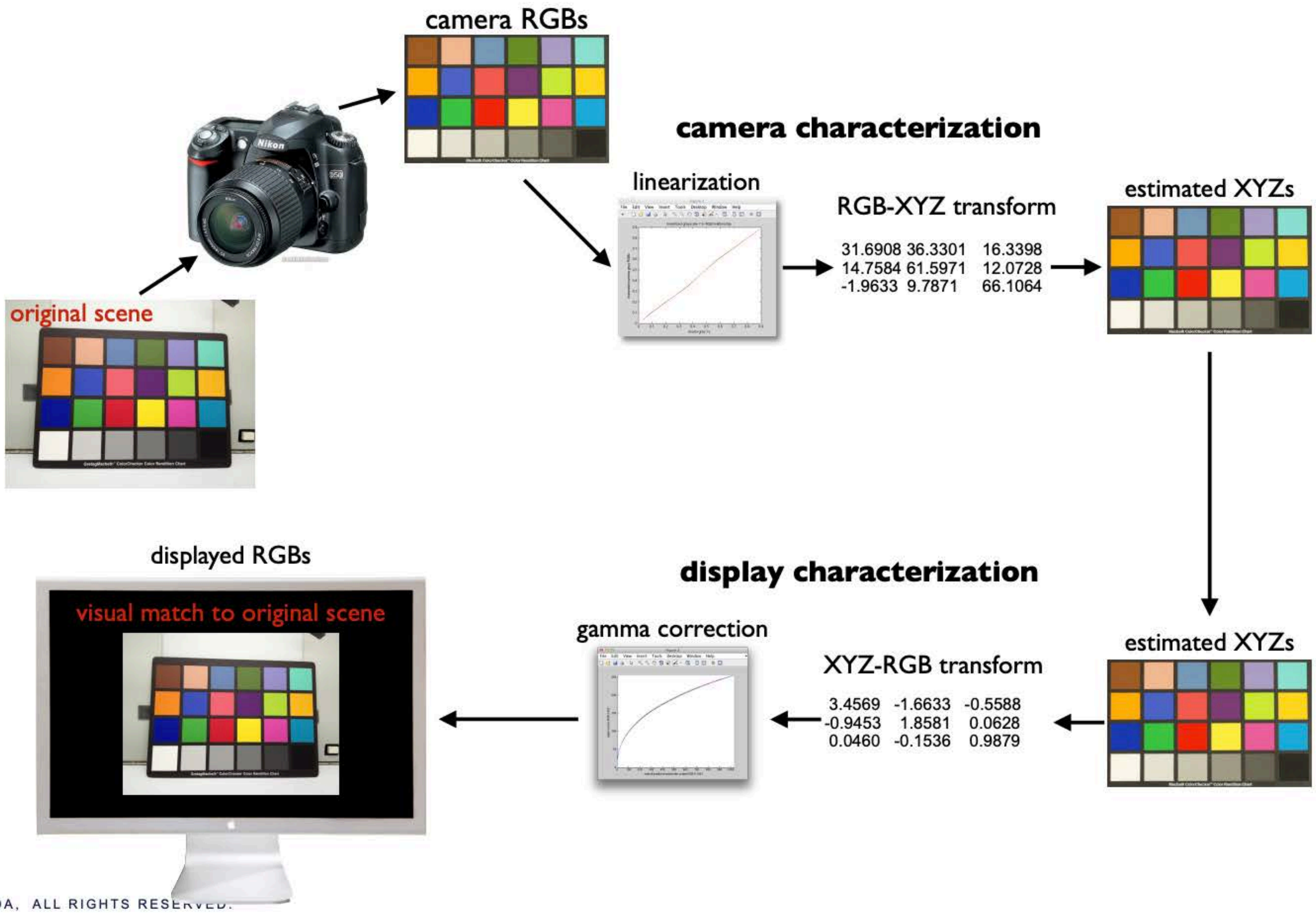


# Conversion between different bases

- Different versions of RGB's are given in the form of different matrix relating to XYZ

| RGB Working Space | Reference White | RGB to XYZ [M] | XYZ to RGB $[M]^{-1}$ |
|---|---|---|---|
| Adobe RGB (1998) | D50 | 0.6097559 0.2052401 0.1492240<br>0.3111242 0.6256560 0.0632197<br>0.0194811 0.0608902 0.7448387 | 1.9624274 -0.6105343 -0.3413404<br>-0.9787684 1.9161415 0.0334540<br>0.0286869 -0.1406752 1.3487655 |
| AppleRGB | D50 | 0.4755678 0.3396722 0.1489800<br>0.2551812 0.6725693 0.0722496<br>0.0184697 0.1133771 0.6933632 | 2.8510695 -1.3605261 -0.4708281<br>-1.0927680 2.0348871 0.0227598<br>0.1027403 -0.2964984 1.4510659 |
| Bruce RGB | D50 | 0.4941816 0.3204834 0.1495550<br>0.2521531 0.6844869 0.0633600<br>0.0157886 0.0629304 0.7464909 | 2.6502856 -1.2014485 -0.4289936<br>-0.9787684 1.9161415 0.0334540<br>0.0264570 -0.1361227 1.3458542 |
| CIE RGB | D50 | 0.4868870 0.3062984 0.1710347<br>0.1746583 0.8247541 0.0005877<br>-0.0012563 0.0169832 0.8094831 | 2.3638081 -0.8676030 -0.4988161<br>-0.5005940 1.3962369 0.1047562<br>0.0141712 -0.0306400 1.2323842 |
| NTSC RGB | D50 | 0.6343706 0.1852204 0.1446290<br>0.3109496 0.5915984 0.0974520<br>-0.0011817 0.0555518 0.7708399 | 1.8464881 -0.5521299 -0.2766458<br>-0.9826630 2.0044755 -0.0690396<br>0.0736477 -0.1453020 1.3018376 |

# Device calibration



# Next time

Transparency