CSE 167 (FA22) Computer Graphics: Colors

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Color Science

- Color science
- Inner product between functions
- Physical color
- Perceived color
- Color space and gamut
• Phenomenological description
• What causes color?
• How can we measure color?
• How can we describe color?
• How can we control color?
Discovering color (pigment)

- Artists studied color for millennia
- They knew that mixing 3 primary color creates a full range of hues
Discovering color (light)

- Isaac Newton experimented with prism
- White light contains all the “rainbow color”
- Colored light combines into white
- C. Huygens (1678), T. Young (1800): light are waves with wavelength around 380–760nm
- Young, Helmholtz, Maxwell: 3 primary color of light
Additive and subtractive color

Additive

Subtractive
Descriptions of color

**RGB**
- Controls the light bulbs in each pixel

**CMYK**
- Controls the ink in a printer
Descriptions of color

Hue, Saturation, Lightness/Brightness/Value (HSL, HSB, HSV)

- More intuitive
- Useful in design
The above description of color is based on what we perceive.

What is the objective and mathematical description of color?

We need to distinguish **physical color** & **perceived color**.
L² pairing between functions

- Color science
- Inner product between functions
- Physical color
- Perceived color
- Color space and gamut
Crash course on the $L^2$ pairing

**Recall.** The inner product (dot product) between two arrays

$$f = (f_0, f_1, \ldots, f_{n-1}) \in \mathbb{R}^n$$

$$g = (g_0, g_1, \ldots, g_{n-1}) \in \mathbb{R}^n$$

is the sum of elementwise product

$$\sum_{i=0}^{n-1} f_i g_i$$
Definition. The $L^2$ pairing between two functions

\[ f : [a, b] \to \mathbb{R} \]

\[ g : [a, b] \to \mathbb{R} \]

on an interval $[a, b]$ is the integral

\[ \int_a^b f(x)g(x) \, dx \]

• You can think of it as the inner product between functions.
• Here, we will view $f, g$ playing different roles.
Crash course on the $L^2$ pairing

$$\int_{a}^{b} f(x)g(x) \, dx$$

• Here, we will view $f$, $g$ playing different roles.
  ▶ $f(x)$ is assumed to be a **continuous function**.
  ▶ $g(x)$ can be a **distribution** (derivative of a possibly discontinuous function).
Crash course on the $L^2$ pairing

- $g(x)$ can be a **distribution** (derivative of a possibly discontinuous function).
  - Continuous functions are distribution
  - Integrable functions are distribution
  - Generalized functions such as the $\delta$-functions are distribution

**Definition.** The $\delta$-function $\delta_c(x)$ concentrated at $c \in [a, b]$ is the derivative of the step function jumping at $c$:

$$\delta_c(x) = \frac{d}{dx} H_c(x)$$

Where $H_c(x) = \begin{cases} 0, & x < c \\ 1, & x \geq c \end{cases}$
Crash course on the $L^2$ pairing

**Definition.** The $\delta$-function $\delta_c(x)$ concentrated at $c \in [a, b]$ is the derivative of the step function jumping at $c$:

\[
\delta_c(x) = \frac{d}{dx} H_c(x)
\]

\[
H_c(x) = \begin{cases} 
0, & x < c \\
1, & x \geq c
\end{cases}
\]

- Infinite impulse: $\delta_c(x) = \begin{cases} 
\infty, & x = c \\
0, & x \neq c
\end{cases}$

- Translation $\delta_c(x) = \delta_0(x - c)$

- $L^2$-pairing $\int_a^b f(x) \delta_c(x) \, dx = f(c)$
Physical color

- Color science
- Inner product between functions
- Physical color
- Perceived color
- Color space and gamut
Definition. A colored light is a nonnegative distribution over the spectral interval [380nm, 750nm] representing the radiance power for each wavelength.

- This is called the spectral power distribution.
- Single-spectral light is a $\delta$-function.
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- Blackbody radiation
Definition. A colored light is a nonnegative distribution over the spectral interval [380nm, 750nm] representing the radiance power for each wavelength.

- This is called the spectral power distribution.
- Single-spectral light is a $\delta$-function.
- Blackbody radiation
- Rayleigh scattering
Definition. Two colored lights $P_1(\lambda), P_2(\lambda)$ have the same chromatic color if they differ only by a scale

$$P_1(\lambda) = \alpha P_2(\lambda)$$

- The chromoticy lives in a projective space (infinite dimensional).
Perceived color

- Color science
- Inner product between functions
- Physical color
- **Perceived color**
- Color space and gamut
Vision

- Cone and rod photoreceptors in the retina turn light into neural signals.
- Rod is 100 times more sensitive to the cones, but slow response time. Rod is responsible for night vision (scotopic vision).
- Under sufficient illumination we use cone cells (photopic vision).
- It can take 10–30 minutes to switch between scotopic/photopic vision.
Color vision

- Under sufficient illumination we use cone cells (photopic vision).
- 90% of you have 3 types of cone cells with different sensitivity depending on the spectrum (long, middle, short).
- The sensitivity are continuous functions of the wavelength:

\[
\bar{\ell}(\lambda) \quad \bar{m}(\lambda) \quad \bar{s}(\lambda)
\]
Color vision

- The sensitivity are continuous functions of the wavelength:

\[ \ell(\lambda), m(\lambda), s(\lambda) \]

- When you watch a colored light \( P(\lambda) \), the strength of the neural signals from the 3 cones are the \( L^2 \)-pairings:

\[
L = \int_{380}^{750} \ell(\lambda)P(\lambda) \, d\lambda \\
M = \int_{380}^{750} m(\lambda)P(\lambda) \, d\lambda \\
S = \int_{380}^{750} s(\lambda)P(\lambda) \, d\lambda
\]
Definition. Two colored lights $P_1(\lambda)$, $P_2(\lambda)$ appear the same if the LMS values match

$$(L_1, M_1, S_1) = (L_2, M_2, S_2)$$

Definition. Two perceived colors $(L_1, M_1, S_1), (L_2, M_2, S_2)$ have the same chromatic color if $L_1 : M_1 : S_1 = L_2 : M_2 : S_2$

- The perceived chromaticities live on a 2-dimensional projective space.
- The colored light may be a combination of 3 monotone lights (RGB screen), but still appear the same as a natural colored light.
Color Space and Gamut

- Color science
- Inner product between functions
- Physical color
- Perceived color
- Color space and gamut
LMS color space

- Not all points in the 3D LMS space correspond to a color.
  - L,M,S are always nonnegative.
  - The source light is always a nonnegative distribution.
  - There is no light that would stimulate only the M cone cell but not the L cone cell.
  - All colored lights are mapped into the convex hull of the cone spanned by the spectral locus.
LMS color space

- Visualize the chromatic color on a 2D slice
LMS color space

- Visualize the chromatic color on a 2D slice

- Every artificial light that combines 3 lights must lie in the interior of a triangle.

- It is impossible to reproduce all colors by using 3 light bulbs.
LMS color space

- Visualize the chromatic color on a 2D slice

- Every artificial light that combines 3 lights must lie in the interior of a triangle.
- It is impossible to reproduce all colors by using 3 light bulbs.
- The standard RGB
Grassmann Law

- **Grassmann Law**

Given any triangle as the basis, say RGB, there exists functions \( \overline{r}(\lambda), \overline{g}(\lambda), \overline{b}(\lambda) \) such that any physical light \( P(\lambda) \) appear to have the same color as the point

\[
 rR + gG + bB
\]

with the coefficients computed by the pairing

\[
 r = \int \overline{r}(\lambda)P(\lambda) \, d\lambda \quad g = \int \overline{g}(\lambda)P(\lambda) \, d\lambda \\
 b = \int \overline{b}(\lambda)P(\lambda) \, d\lambda
\]
Grassmann Law

\[ rR + gG + bB \]

\[ r = \int \bar{r}(\lambda)P(\lambda) \, d\lambda \quad g = \int \bar{g}(\lambda)P(\lambda) \, d\lambda \]

\[ b = \int \bar{b}(\lambda)P(\lambda) \, d\lambda \]

- To determine \( \bar{r}, \bar{g}, \bar{b} \), take RGB as 3 known light bulbs, and take \( P(\lambda) = \delta_{\lambda_0}(\lambda) \)

- Tune the coefficients \( r, g, b \) in \( rR + gG + bB \) so that the synthetic light looks the same as \( P(\lambda) = \delta_{\lambda_0}(\lambda) \)

- Then we obtain \( \bar{r}(\lambda_0) = r, \quad \bar{g}(\lambda_0) = g, \quad \bar{b}(\lambda_0) = b \).

- We call \( \bar{r}, \bar{g}, \bar{b} \) the matching function.
Color matching experiment

- Primaries: Red (R), Green (G), Blue (B)
- Wavelengths: 700 nm, 546.1 nm, 435.8 nm

Equation:
\[ T(\lambda) = rR + gG + bB \]

Example:
\[ T(480) = -0.04R + 0.04G + 0.14B \]
Color matching experiment

![Diagram showing color matching experiment]

- **Primitives:** 700nm, 546.1nm, 435.8nm
- **White screen:**
  - Primaries: R, G, B
  - Matching field: 700nm
  - Test field: T(λ) 480nm
- **Test light:**
  - Monochromator

### Equation

\[
T(\lambda) = rR + gG + bB
\]

### Example

\[ T(480) = -0.04R + 0.04G + 0.14B \]
In 1931, CIE (International Commission on Illumination) proposed another basis based on 3 imaginary primaries X, Y, Z.

- Mixture of XYZ gives all color
- The corresponding matching functions $\bar{x}, \bar{y}, \bar{z}$ are linear transforms of $\bar{r}, \bar{g}, \bar{b}$
- $\bar{x}, \bar{y}, \bar{z}$ are nonnegative (equivalent to the first item above)
- $\bar{x}, \bar{y}, \bar{z}$ have equal areas under their curves.
- The y-coefficient $\int \bar{y}(\lambda)P(\lambda) \, d\lambda$ is defined to be the luminance.
CIE 1931 XYZ color space
• **Color gamut**: The range of chromaticities that can be produced by mixing primaries.

• **Real device gamut** are limited, can’t produce all chromaticities.

• **Gamut mapping**: Methods to approximate out-of-gamut chromaticities
  
  ▶ Typically involves desaturating source chromaticities until they fall within the gamut boundaries.
Color gamut

Various hardwares

Various specs
Color gamut

Wide gamut and
Hide dynamical range
Conversion between different bases

- XYZ, RGB are 3D vectors with respect to different bases
- Relation between them is recorded by a 3x3 matrix

\[
\begin{bmatrix}
X \\
Y \\
Z
\end{bmatrix} =
\begin{bmatrix}
0.4360747 & 0.3850649 & 0.1430804 \\
0.2225045 & 0.7168786 & 0.0606169 \\
0.0139322 & 0.0971045 & 0.7141733
\end{bmatrix}
\begin{bmatrix}
R \\
G \\
B
\end{bmatrix}
\]

\[
\begin{bmatrix}
R \\
G \\
B
\end{bmatrix} =
\begin{bmatrix}
3.1338561 & -1.6168667 & -0.4906146 \\
-0.9787684 & 1.9161415 & 0.0334540 \\
0.0719453 & -0.2289914 & 1.4052427
\end{bmatrix}
\begin{bmatrix}
X \\
Y \\
Z
\end{bmatrix}
\]
Conversion between different bases

- Different versions of RGB’s are given in the form of different matrix relating to XYZ

<table>
<thead>
<tr>
<th>RGB Working Space</th>
<th>Reference White</th>
<th>RGB to XYZ [M]</th>
<th>XYZ to RGB [M]⁻¹</th>
</tr>
</thead>
<tbody>
<tr>
<td>Adobe RGB (1998)</td>
<td>D50</td>
<td>0.6097559 0.2052401 0.1492240</td>
<td>1.9624274 -0.6105343 -0.3413404</td>
</tr>
<tr>
<td></td>
<td></td>
<td>0.3111242 0.6256560 0.0632197</td>
<td>-0.9787684 1.9161415 0.0334540</td>
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<tr>
<td></td>
<td></td>
<td>0.0194811 0.0608902 0.7448387</td>
<td>0.0286869 -0.1406752 1.3487655</td>
</tr>
<tr>
<td>AppleRGB</td>
<td>D50</td>
<td>0.4755678 0.3396722 0.1489800</td>
<td>2.8510695 -1.3605261 -0.4708281</td>
</tr>
<tr>
<td></td>
<td></td>
<td>0.2551812 0.6725693 0.0722496</td>
<td>-1.0927680 2.0348871 0.0227598</td>
</tr>
<tr>
<td></td>
<td></td>
<td>0.0184697 0.1133771 0.6933632</td>
<td>0.1027403 -0.2964984 1.4510659</td>
</tr>
<tr>
<td>Bruce RGB</td>
<td>D50</td>
<td>0.4941816 0.3204834 0.1495550</td>
<td>2.6502856 -1.2014485 -0.4289936</td>
</tr>
<tr>
<td></td>
<td></td>
<td>0.2521531 0.6844869 0.0633600</td>
<td>-0.9787684 1.9161415 0.0334400</td>
</tr>
<tr>
<td></td>
<td></td>
<td>0.0157886 0.0629304 0.7464909</td>
<td>0.0264570 -0.1361227 1.3458542</td>
</tr>
<tr>
<td>CIE RGB</td>
<td>D50</td>
<td>0.4868870 0.3062984 0.1710347</td>
<td>2.3638081 -0.8676030 -0.4988161</td>
</tr>
<tr>
<td></td>
<td></td>
<td>0.1746583 0.8247541 0.0005877</td>
<td>-0.5005940 1.3962369 0.1047562</td>
</tr>
<tr>
<td></td>
<td></td>
<td>-0.0012563 0.0169832 0.8094831</td>
<td>0.0141712 -0.0306400 1.2323842</td>
</tr>
<tr>
<td>NTSC RGB</td>
<td>D50</td>
<td>0.6343706 0.1852204 0.1446290</td>
<td>1.8464881 -0.5521299 -0.2766458</td>
</tr>
<tr>
<td></td>
<td></td>
<td>0.3109496 0.5915984 0.0974520</td>
<td>-0.9826630 2.0044755 -0.0690396</td>
</tr>
<tr>
<td></td>
<td></td>
<td>-0.0011817 0.0555518 0.7708399</td>
<td>0.0736477 -0.1453020 1.3018376</td>
</tr>
</tbody>
</table>
Device calibration

**Camera Calibration**

- **Camera RGBs** → **Camera Characterization**
  - Linearization
  - RGB-XYZ Transform:
    - 31.6908, 36.3301, 16.3398
    - 14.7594, 61.5971, 12.0728
    - -1.9633, 9.7871, 66.1084

- Original Scene

**Display Calibration**

- **Displayed RGBs** → **Display Characterization**
  - Gamma Correction
  - XYZ-RGB Transform:
    - 3.4569, -1.6633, -0.5588
    - -0.9453, 1.8581, 0.0628
    - 0.0460, -0.1536, 0.9879

- Visual Match to Original Scene
Next time

Transparency