CSE 167 (FA21)
Computer Graphics:
Transformation Recap

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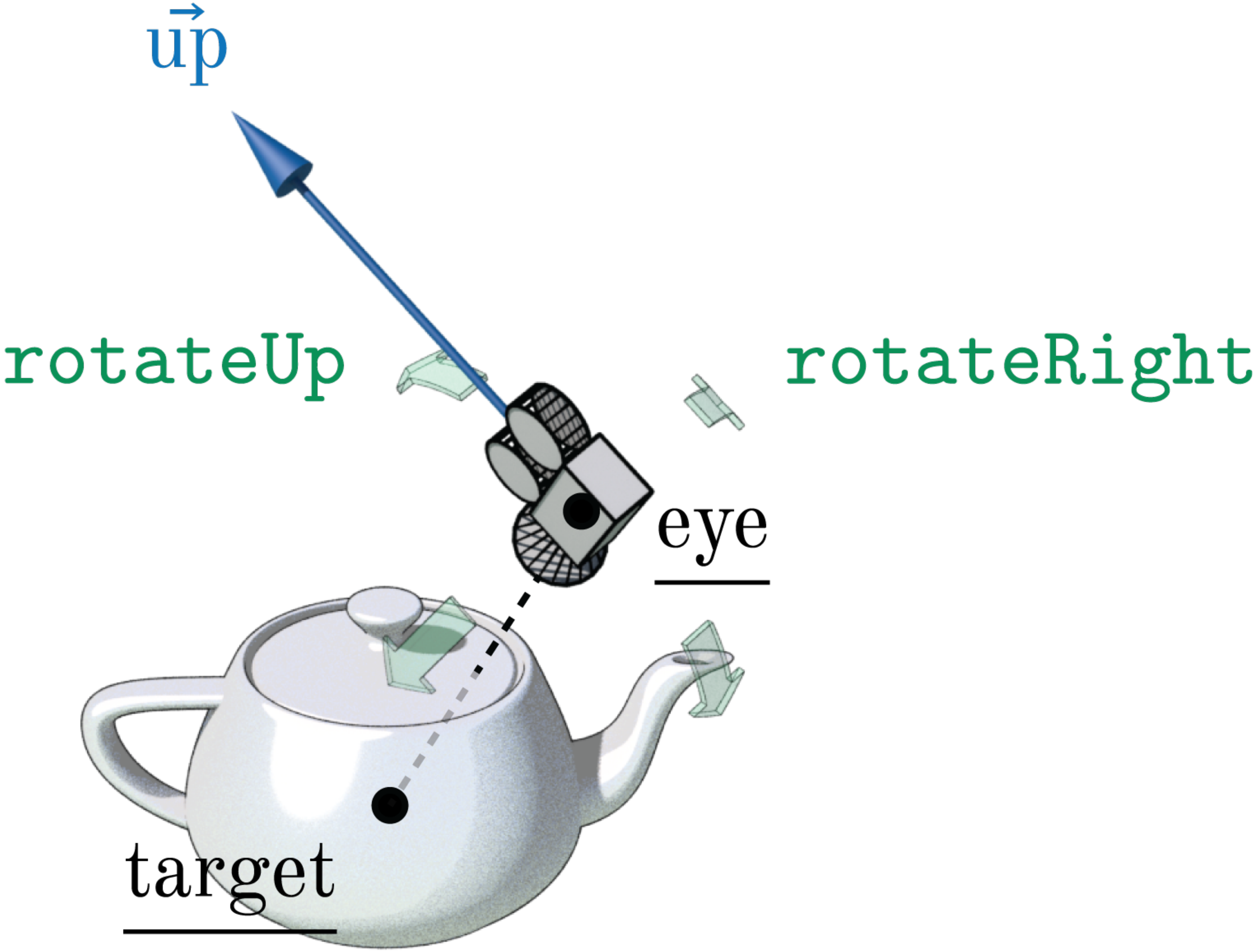
Camera::rotateRight, rotateUp



Positions and Displacements

Points/positions

$$\begin{bmatrix} p_x \\ p_y \\ p_z \\ 1 \end{bmatrix}$$

Vectors/displacements

$$\begin{bmatrix} v_x \\ v_y \\ v_z \\ 0 \end{bmatrix}$$

Linear transformation on vectors

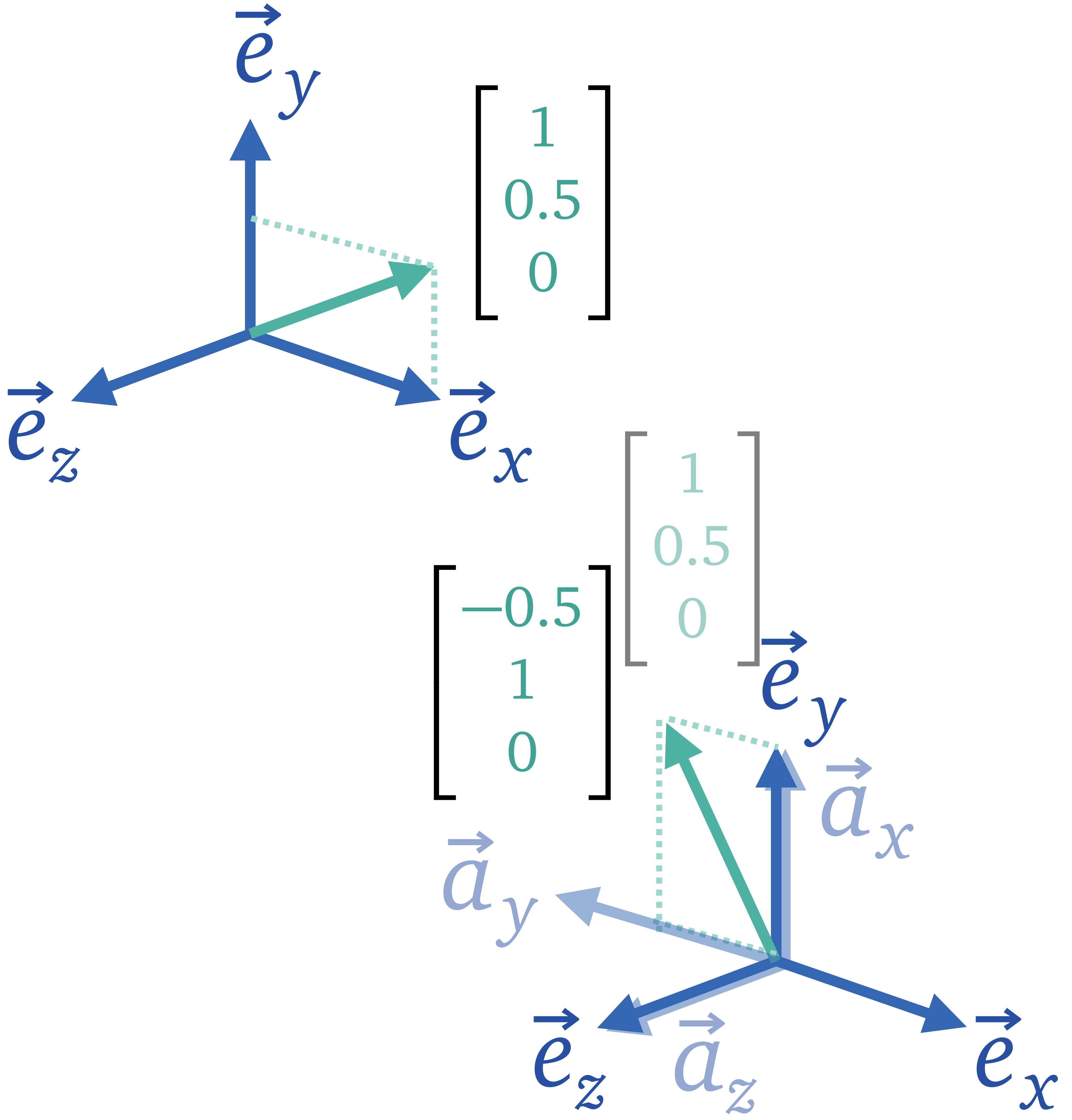
- Linear transformations are applied to vectors.
- In 3D, we don't need the 4th homogeneous coordinate. Just apply a 3x3 matrix to a 3D vector.

$$\begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix} \begin{bmatrix} v_x \\ v_y \\ v_z \end{bmatrix} = \begin{bmatrix} a_{11}v_x + a_{12}v_y + a_{13}v_z \\ a_{21}v_x + a_{22}v_y + a_{23}v_z \\ a_{31}v_x + a_{32}v_y + a_{33}v_z \end{bmatrix}$$

Linear transformation on vectors

- Linear transformations are applied to vectors.
- In 3D, we don't need the 4th homogeneous coordinate. Just apply a 3x3 matrix to a 3D vector.

$$\begin{bmatrix} 0 & -1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} v_x \\ v_y \\ v_z \end{bmatrix} = \begin{bmatrix} -v_y \\ v_x \\ v_z \end{bmatrix}$$



Affine transformation on positions

- Affine transformations are applied to points.
- We need the 4th homogeneous coordinate to handle translations.

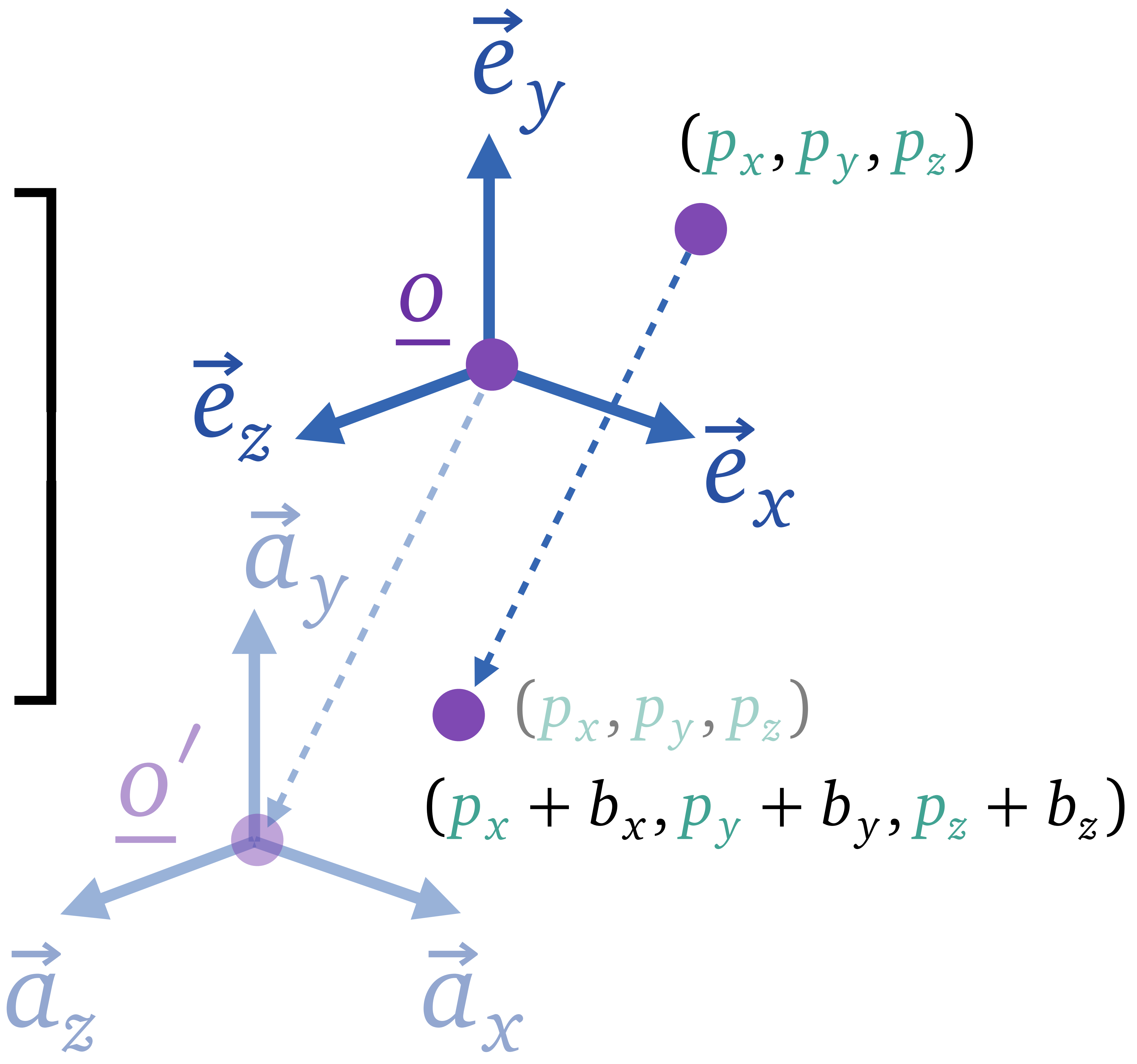
$$\begin{bmatrix} a_{11} & a_{12} & a_{13} & b_x \\ a_{21} & a_{22} & a_{23} & b_y \\ a_{31} & a_{32} & a_{33} & b_z \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} p_x \\ p_y \\ p_z \\ 1 \end{bmatrix} = \begin{bmatrix} a_{11}p_x + a_{12}p_y + a_{13}p_z + b_x \\ a_{21}p_x + a_{22}p_y + a_{23}p_z + b_y \\ a_{31}p_x + a_{32}p_y + a_{33}p_z + b_z \\ 1 \end{bmatrix}$$

Affine transformation on positions

- Affine transformations are applied to points.
- We need the 4th homogeneous coordinate to handle translations.

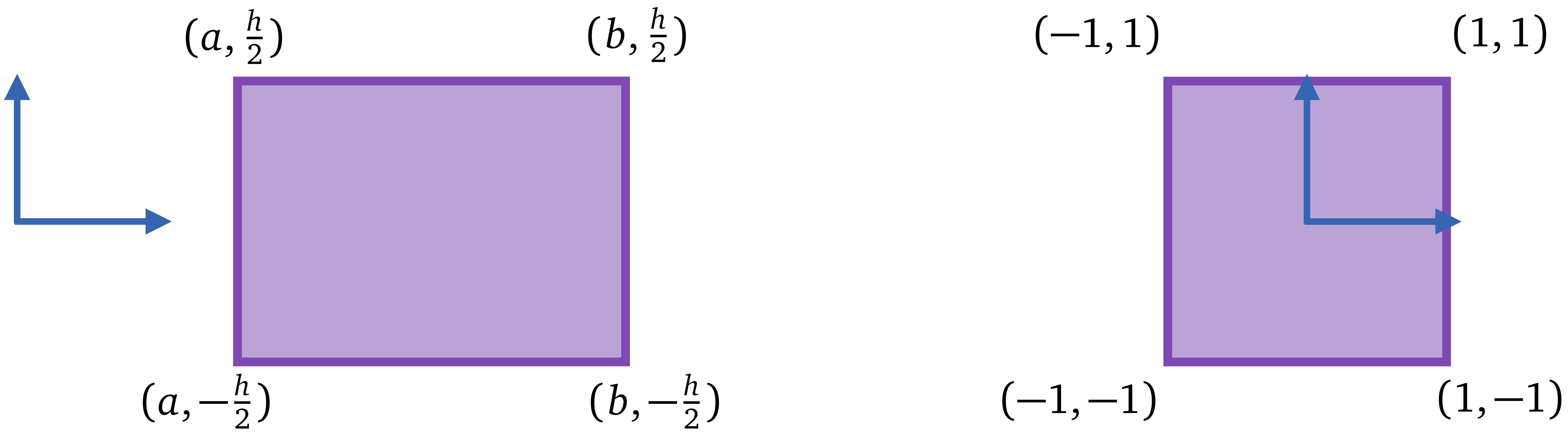
(pure translation)

$$\begin{bmatrix} 1 & & & b_x \\ & 1 & & b_y \\ & & 1 & b_z \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} p_x \\ p_y \\ p_z \\ 1 \end{bmatrix} = \begin{bmatrix} p_x + b_x \\ p_y + b_y \\ p_z + b_z \\ 1 \end{bmatrix}$$



Affine transformation on positions

- Exercise: Find the transformation that transforms the following 2D box to a square centered at the origin



Induced transformation

Induced transformation

Induced transformation

- If all positions of a geometric object is transformed by an affine transformation

$$\begin{bmatrix} a_{11} & a_{12} & a_{13} & b_x \\ a_{21} & a_{22} & a_{23} & b_y \\ a_{31} & a_{32} & a_{33} & b_z \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} p_x \\ p_y \\ p_z \\ 1 \end{bmatrix} = \begin{bmatrix} a_{11}p_x + a_{12}p_y + a_{13}p_z + b_x \\ a_{21}p_x + a_{22}p_y + a_{23}p_z + b_y \\ a_{31}p_x + a_{32}p_y + a_{33}p_z + b_z \\ 1 \end{bmatrix}$$

- Then all displacement vectors are transformed by the upper-left 3x3 block matrix

$$\begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix} \begin{bmatrix} v_x \\ v_y \\ v_z \end{bmatrix} = \begin{bmatrix} a_{11}v_x + a_{12}v_y + a_{13}v_z \\ a_{21}v_x + a_{22}v_y + a_{23}v_z \\ a_{31}v_x + a_{32}v_y + a_{33}v_z \end{bmatrix}$$

Induced transformation

- If all positions of a geometric object is transformed by an affine transformation

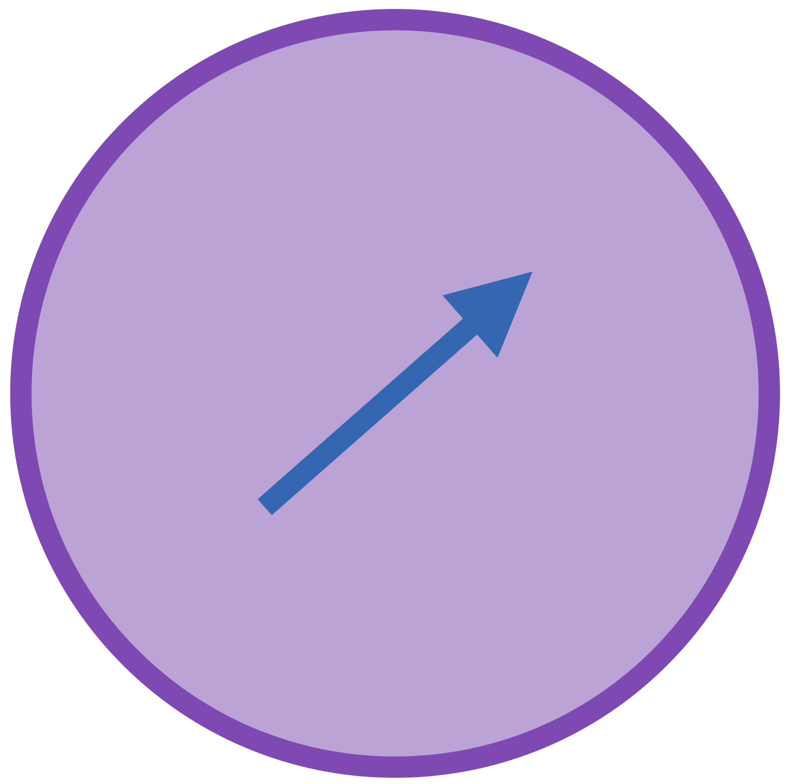
$$\begin{bmatrix} a_{11} & a_{12} & a_{13} & b_x \\ a_{21} & a_{22} & a_{23} & b_y \\ a_{31} & a_{32} & a_{33} & b_z \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} p_x \\ p_y \\ p_z \\ 1 \end{bmatrix} = \begin{bmatrix} a_{11}p_x + a_{12}p_y + a_{13}p_z + b_x \\ a_{21}p_x + a_{22}p_y + a_{23}p_z + b_y \\ a_{31}p_x + a_{32}p_y + a_{33}p_z + b_z \\ 1 \end{bmatrix}$$

- How about normal vectors?

Induced Transformation on Vectors

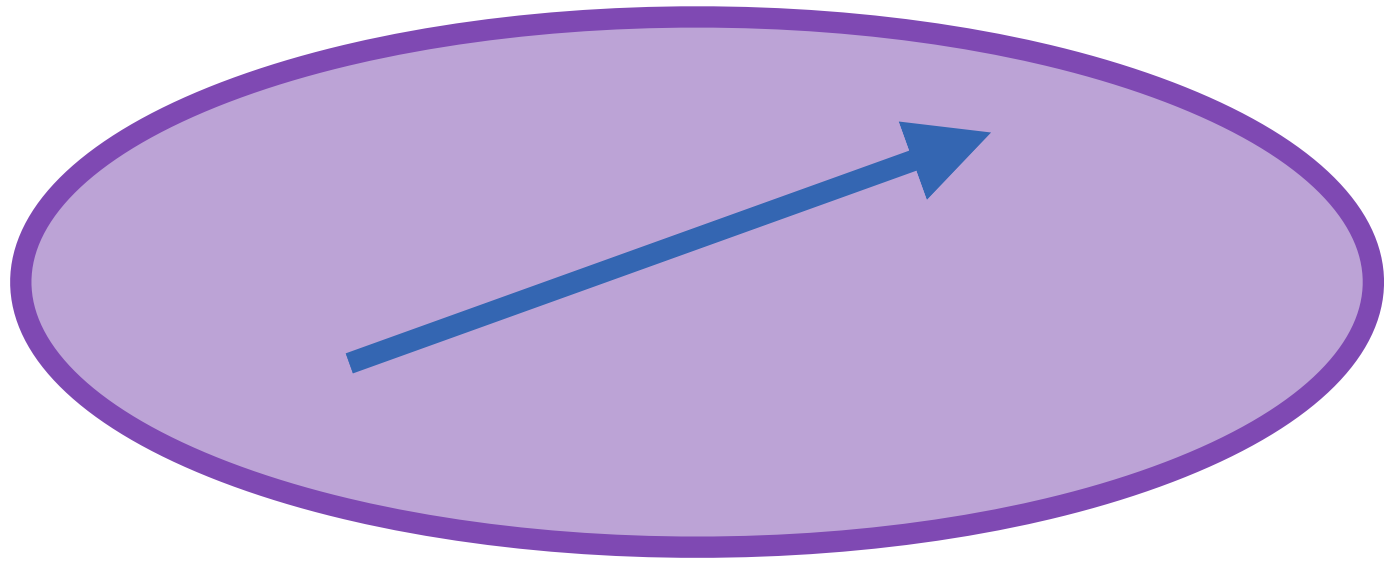
- Suppose we have an affine transformation on positions

$$\begin{bmatrix} \mathbf{p} \\ 1 \end{bmatrix} \mapsto \begin{bmatrix} \mathbf{A} & \mathbf{b} \\ \mathbf{0} & 1 \end{bmatrix} \begin{bmatrix} \mathbf{p} \\ 1 \end{bmatrix}$$



- Then displacement vectors will transform by

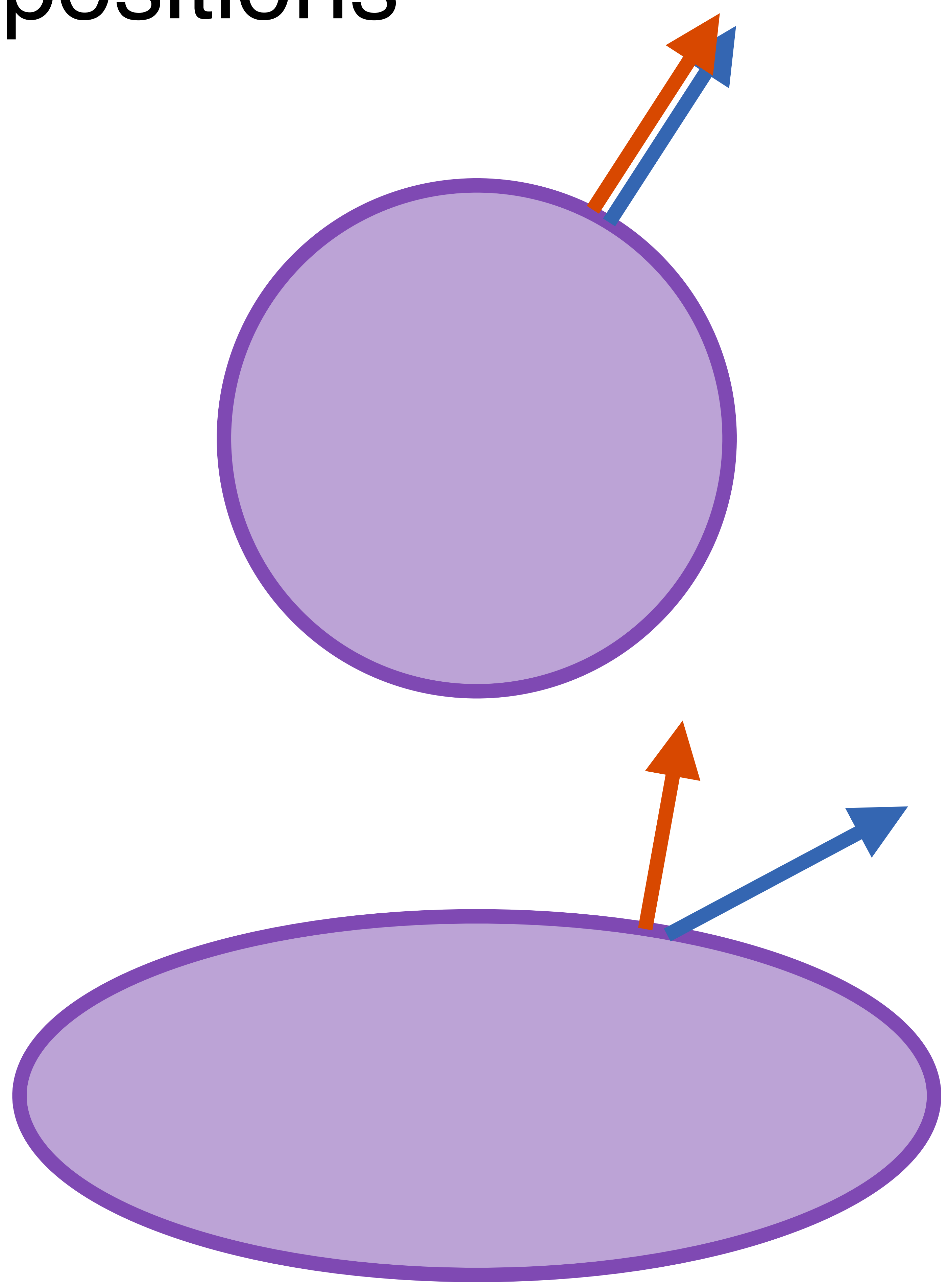
$$\begin{bmatrix} \mathbf{u} \end{bmatrix} \mapsto \begin{bmatrix} \mathbf{A} \end{bmatrix} \begin{bmatrix} \mathbf{u} \end{bmatrix}$$



Induced Transformation on Normals

- Suppose we have an affine transformation on positions

$$\begin{bmatrix} \mathbf{p} \\ 1 \end{bmatrix} \mapsto \begin{bmatrix} \mathbf{A} & \mathbf{b} \\ \mathbf{0} & 1 \end{bmatrix} \begin{bmatrix} \mathbf{p} \\ 1 \end{bmatrix}$$



- and normal vectors transform according to

$$\begin{bmatrix} \mathbf{n} \end{bmatrix} \mapsto \begin{bmatrix} \text{What is this} \\ \text{3x3 matrix?} \end{bmatrix} \begin{bmatrix} \mathbf{n} \end{bmatrix}$$

(followed by a normalization)

Induced Transformation on Normals

- Suppose we have an affine transformation on positions

$$\begin{bmatrix} \mathbf{p} \\ 1 \end{bmatrix} \mapsto \begin{bmatrix} \mathbf{A} & \mathbf{b} \\ \mathbf{0} & 1 \end{bmatrix} \begin{bmatrix} \mathbf{p} \\ 1 \end{bmatrix}$$

- and normal vectors transform according to

$$\begin{bmatrix} \mathbf{n} \end{bmatrix} \mapsto \begin{bmatrix} \mathbf{A}^{-\mathsf{T}} \end{bmatrix} \begin{bmatrix} \mathbf{n} \end{bmatrix}$$

(followed by a normalization)