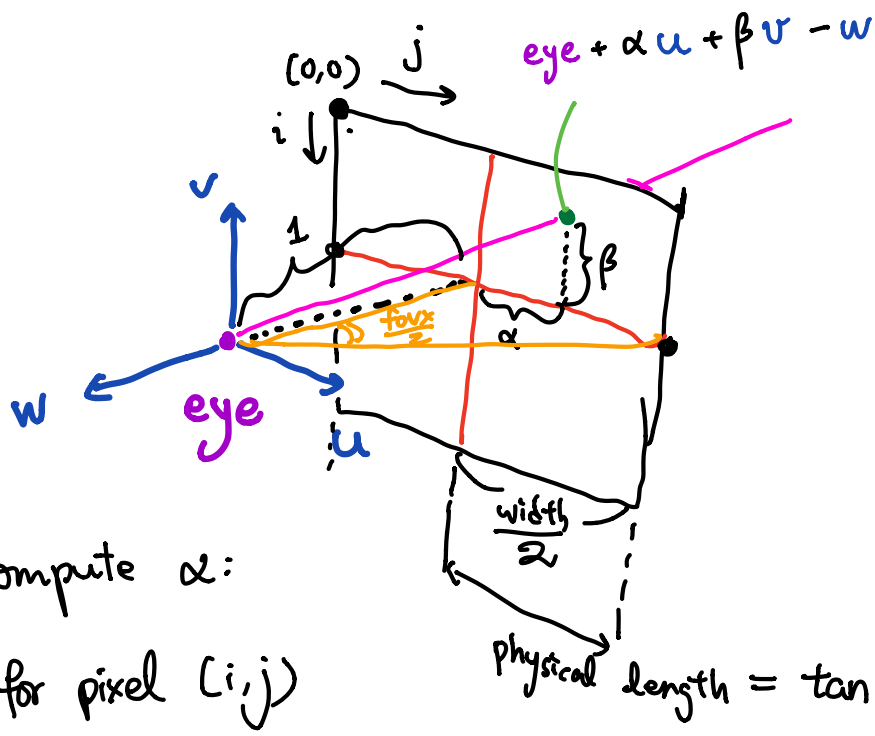


Calculate ray through pixel



Compute α :

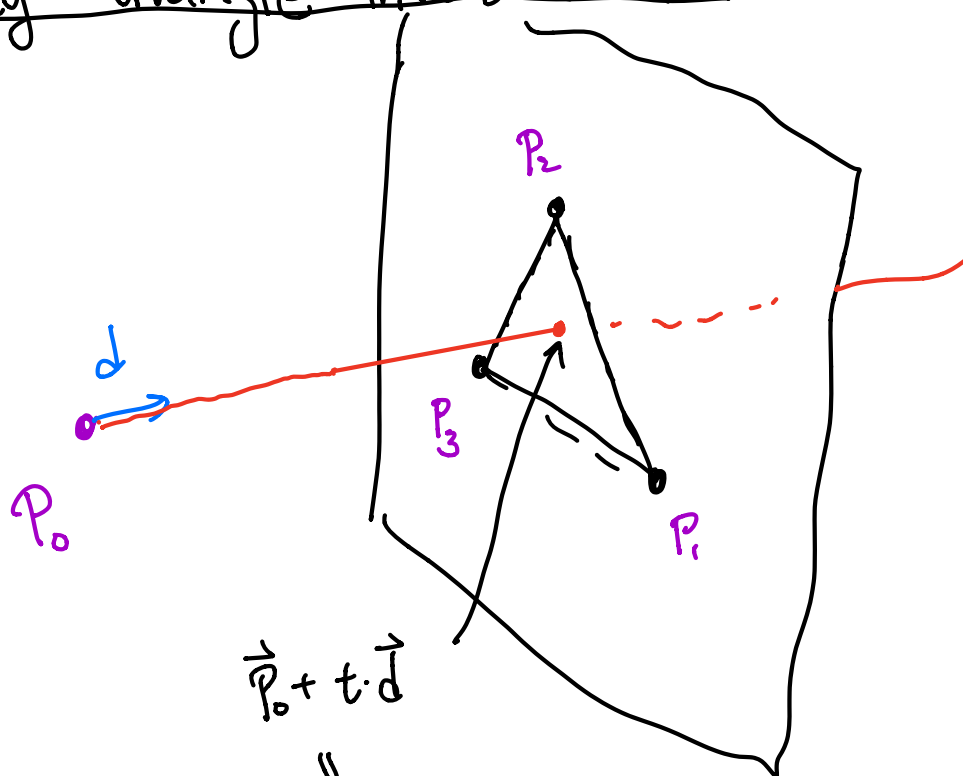
for pixel (i, j)

$$\alpha = \left(\frac{(j+0.5) - \frac{\text{width}}{2}}{\frac{\text{width}}{2}} \right) \cdot \tan\left(\frac{\text{fovx}}{2}\right)$$

$$\beta = \left(\frac{(i+0.5) - \frac{\text{height}}{2}}{\frac{\text{height}}{2}} \right) \tan\left(\frac{\text{fovy}}{2}\right).$$

$$\Rightarrow \text{ray} = \begin{cases} P_0 = \text{eye} \\ d = \frac{\alpha u + \beta v - w}{|\alpha u + \beta v - w|} \end{cases}$$

Ray-triangle intersection



$$\vec{P}_0 + t \cdot \vec{d}$$

"

$$\lambda_1 \vec{P}_1 + \lambda_2 \vec{P}_2 + \lambda_3 \vec{P}_3$$

$$\lambda_1 + \lambda_2 + \lambda_3 = 1$$

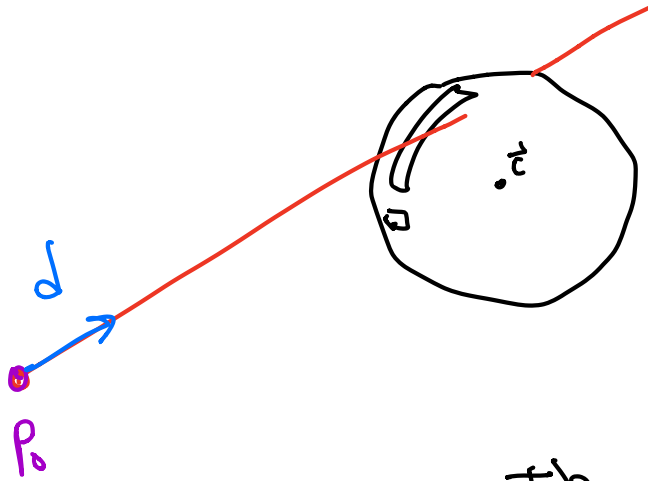
$$\left[\begin{array}{ccc|c} \vdots & \vdots & \vdots & \\ \vec{P}_1 & \vec{P}_2 & \vec{P}_3 & -\vec{d} \\ \vdots & \vdots & \vdots & \\ \hline 1 & 1 & 1 & 0 \end{array} \right] \begin{bmatrix} \lambda_1 \\ \lambda_2 \\ \lambda_3 \\ t \end{bmatrix} = \begin{bmatrix} \vec{P}_0 \\ \hline 1 \end{bmatrix}$$

We have an intersection if

$$t > 0, \underbrace{\lambda_1 > 0, \lambda_2 > 0, \lambda_3 > 0.}_{\text{barycentric coord,}}$$

can be used to interpolate vertex attributes.

Ray-sphere intersection



sphere
centered at $\vec{C}$
with radius $r > 0$.

sphere equation

$$\left\{ \vec{p} \in \mathbb{R}^3 \mid (\vec{p} - \vec{C}) \cdot (\vec{p} - \vec{C}) = r^2 \right\}$$

$$\text{ray: } \vec{p}(t) = \vec{p}_0 + t \vec{d}$$

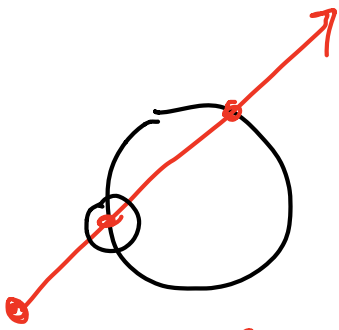
Substitute:

$$(\vec{p}_0 + t \vec{d} - \vec{C}) \cdot (\vec{p}_0 + t \vec{d} - \vec{C}) = r^2$$

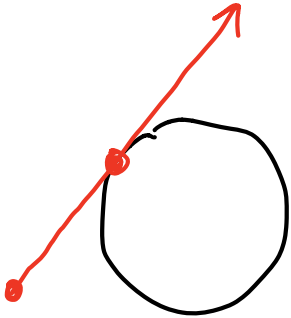
$$|\vec{d}|^2 t^2 - 2 \vec{d} \cdot (\vec{p}_0 - \vec{C}) t + |\vec{p}_0 - \vec{C}|^2 - r^2 = 0$$

$$t = \frac{2 \vec{d} \cdot (\vec{p}_0 - \vec{C}) \pm \sqrt{4 (\vec{d} \cdot (\vec{p}_0 - \vec{C}))^2 - 4 |\vec{d}|^2 (|\vec{p}_0 - \vec{C}|^2 - r^2)}}{2 |\vec{d}|^2}$$

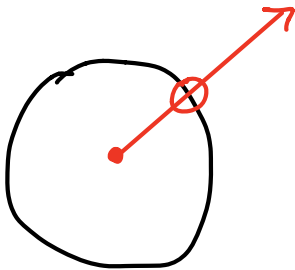
discriminate



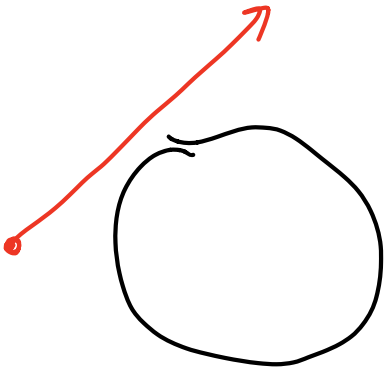
2 real positive roots.
Take the smaller one.



2 roots coincide



1 real positive
1 real negative,
take the positive one.



2 complex roots
(just check whether
the discriminant < 0)

Intersection pt: $\vec{p} = \vec{p}_0 + \vec{d} \cdot t$.

Find normal: $\frac{\vec{p} - \vec{c}}{|\vec{p} - \vec{c}|}$.