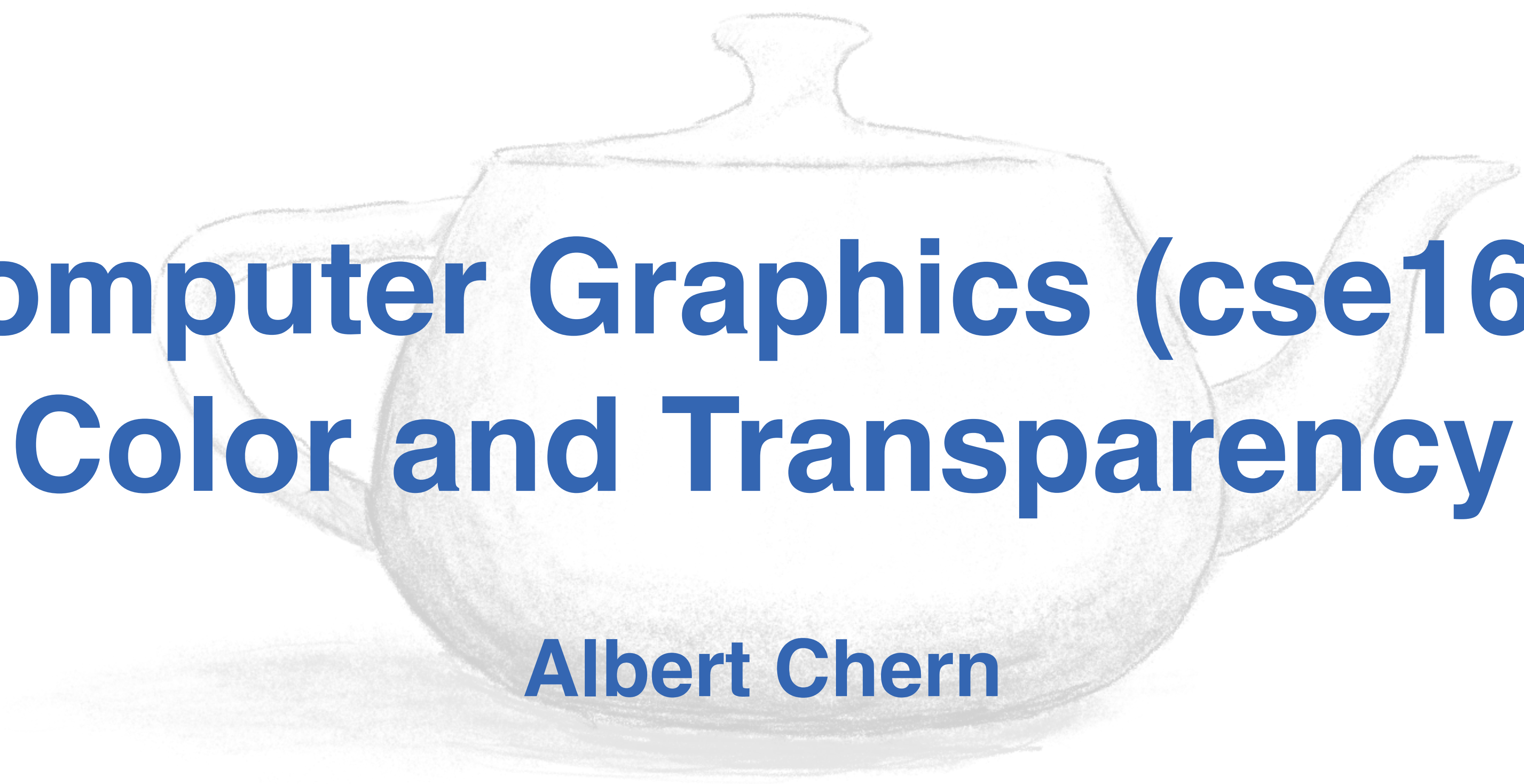




Computer Graphics (cse167)

Color and Transparency

Albert Chern

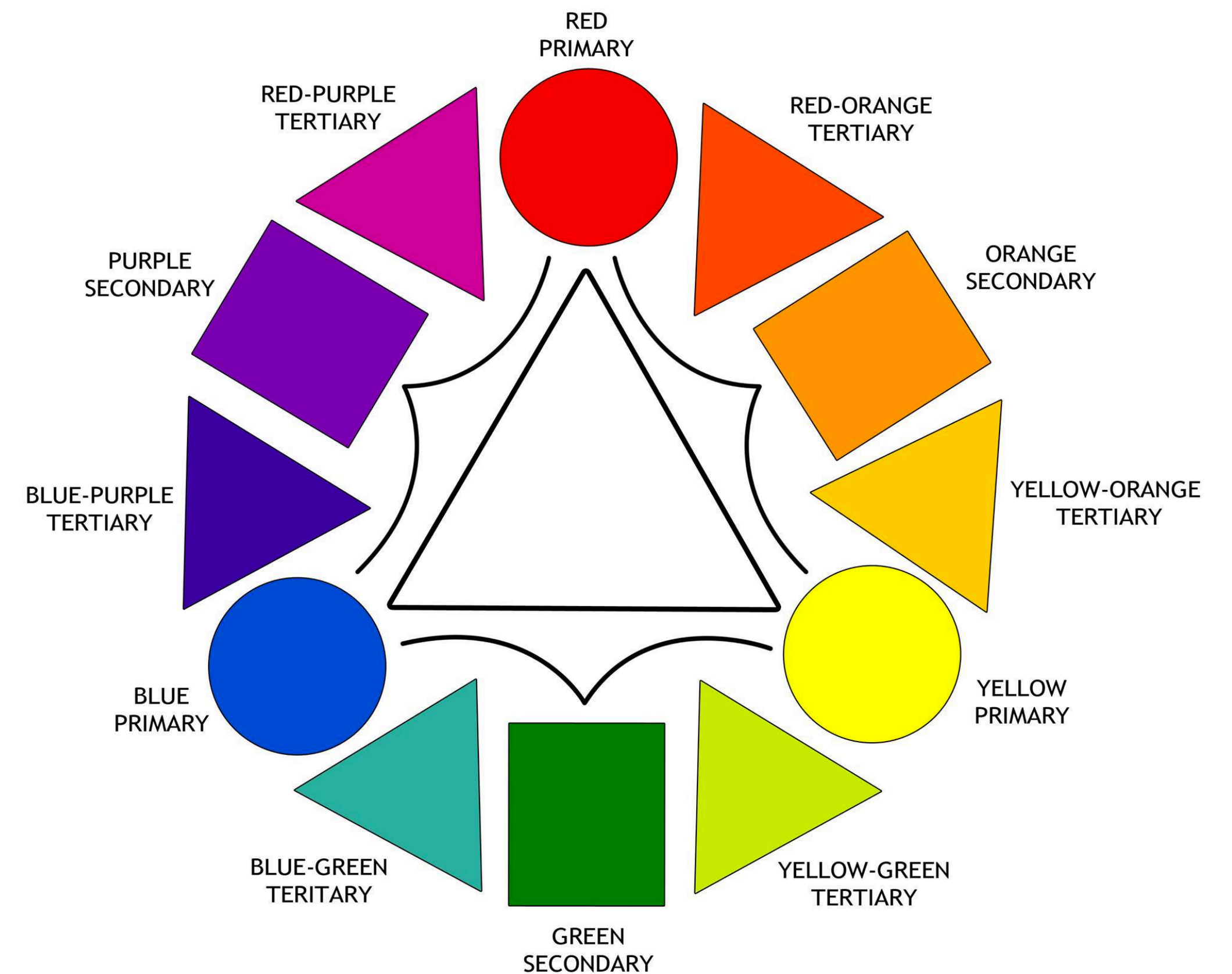
Color Science

Color

- Phenomenological description
- What causes color?
- How can we measure color?
- How can we describe color?
- How can we control color?

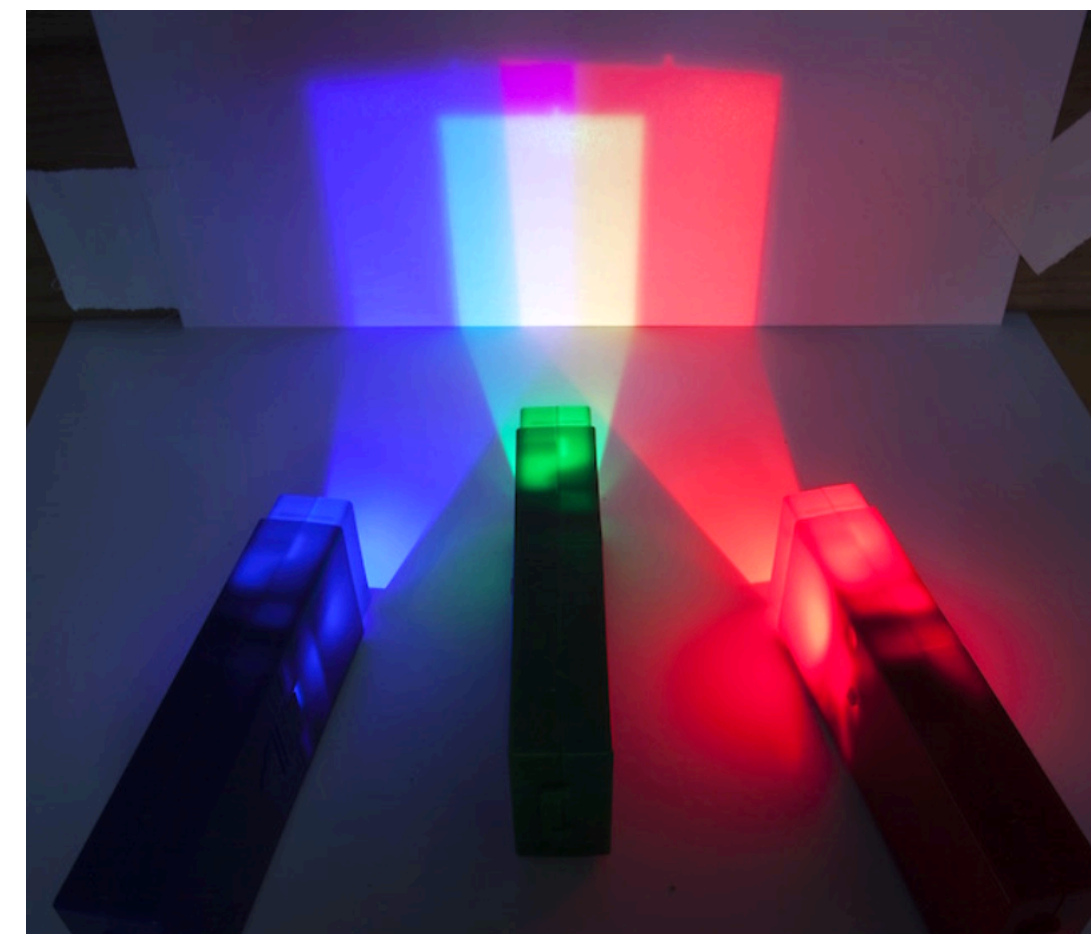
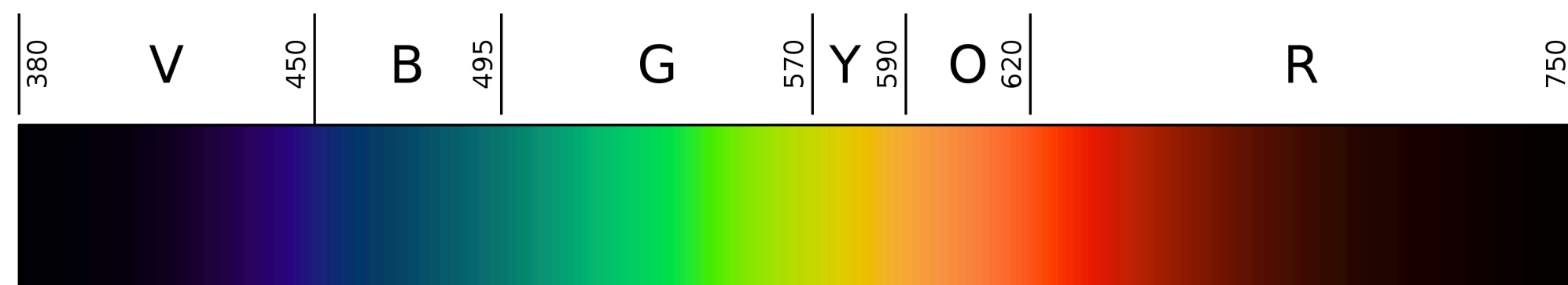
Discovering color (pigment)

- Artists studied color for millennia
- They knew that mixing 3 primary color creates a full range of hues



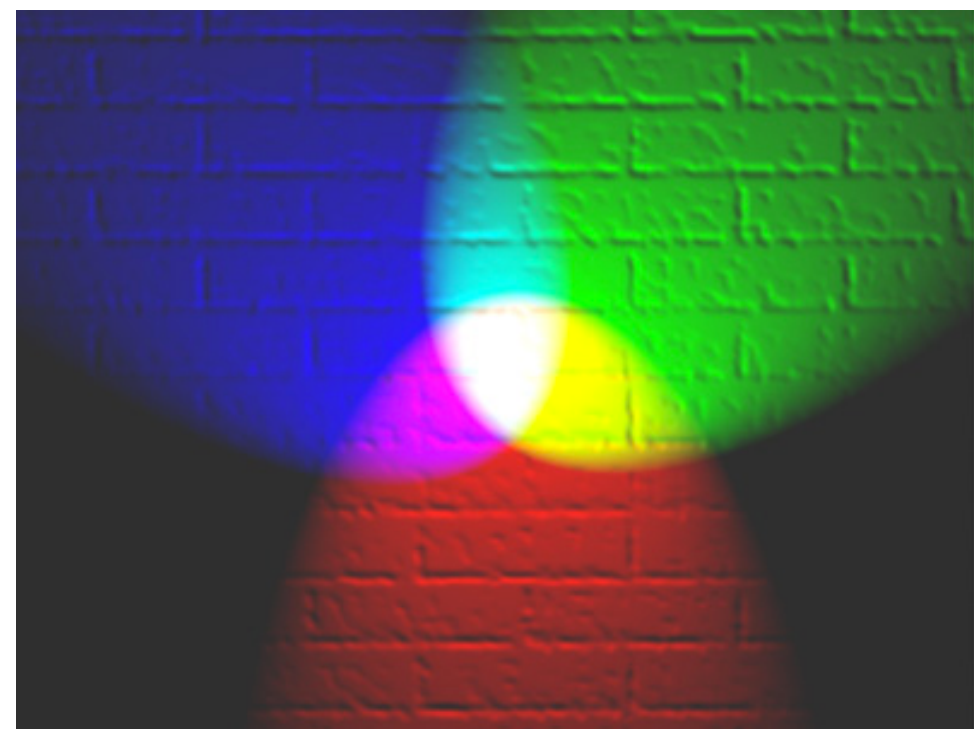
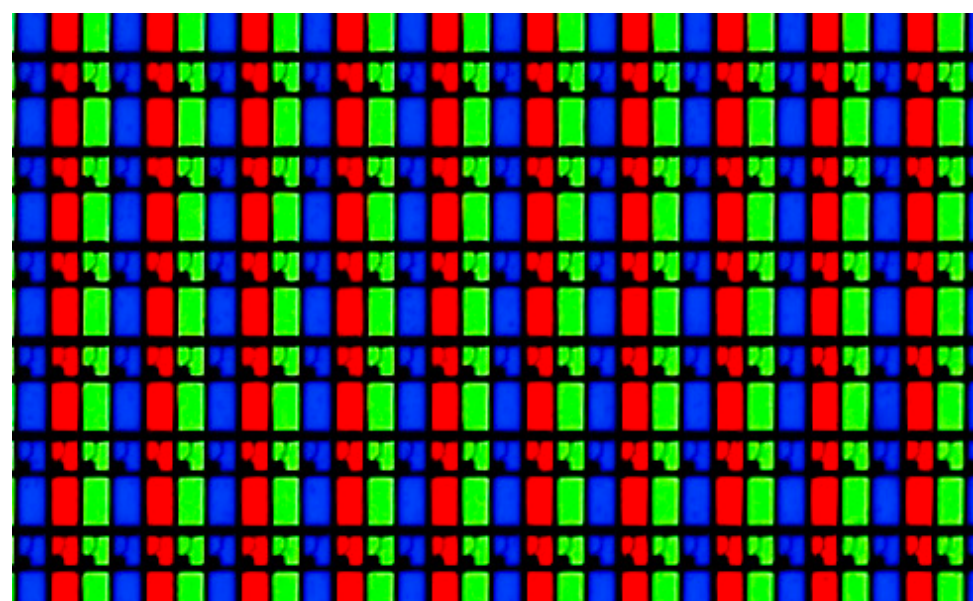
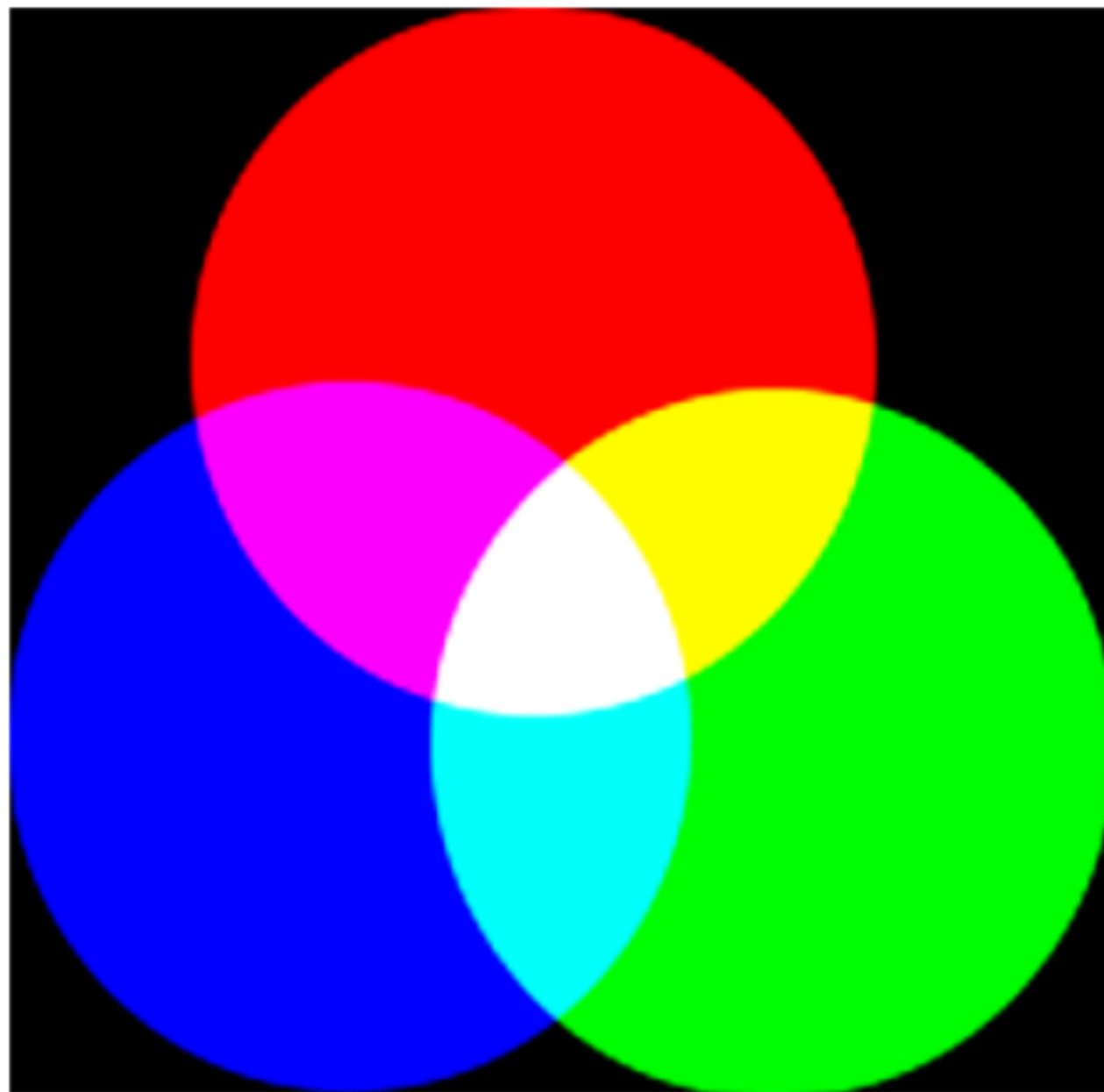
Discovering color (light)

- Isaac Newton experimented with prism
- White light contains all the “rainbow color”
- Colored light combines into white
- C. Huygens (1678), T. Young (1800): light are waves with wavelength around 380–760nm
- Young, Helmholtz, Maxwell: 3 primary color of light

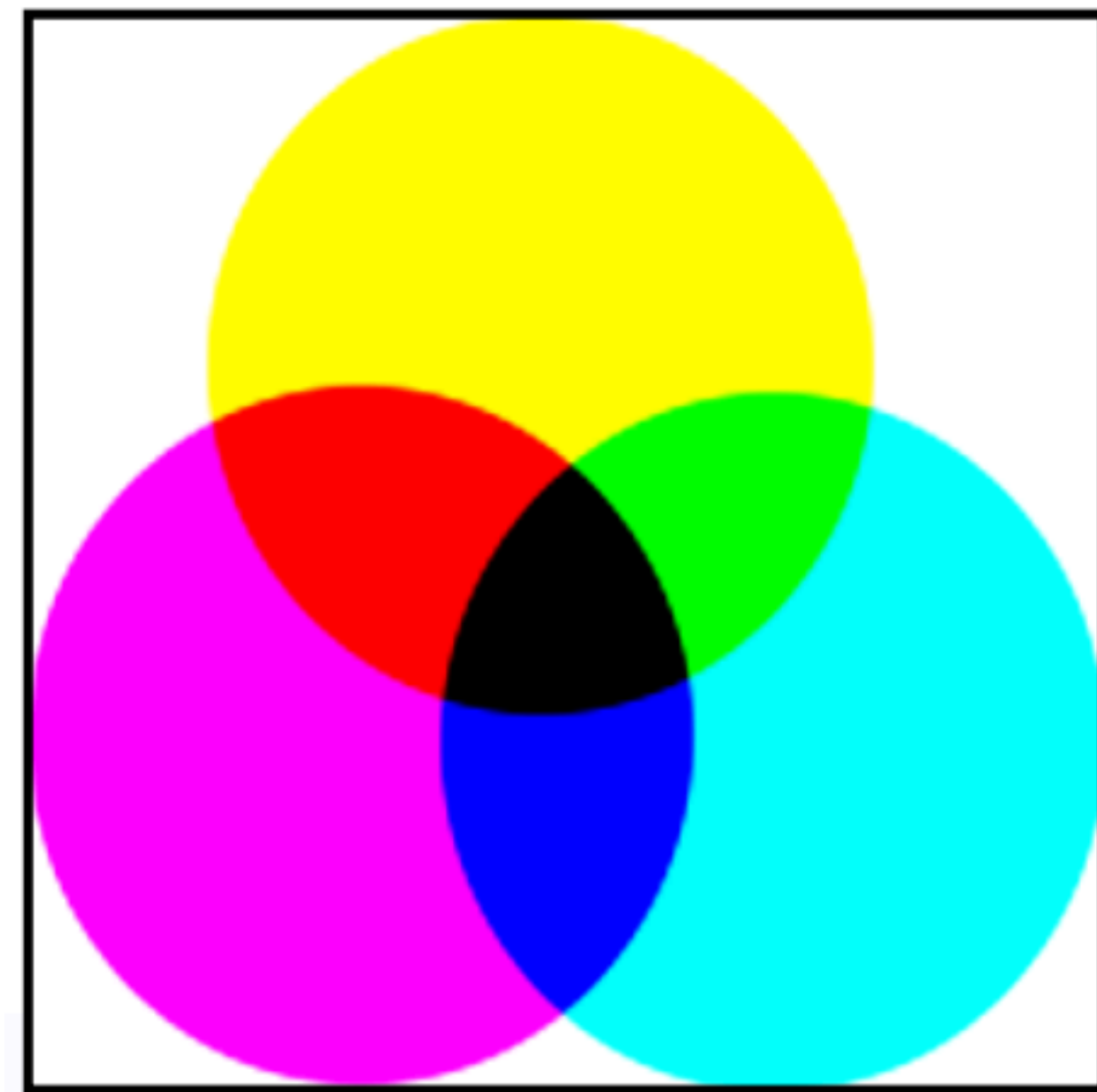


Additive and subtractive color

Additive



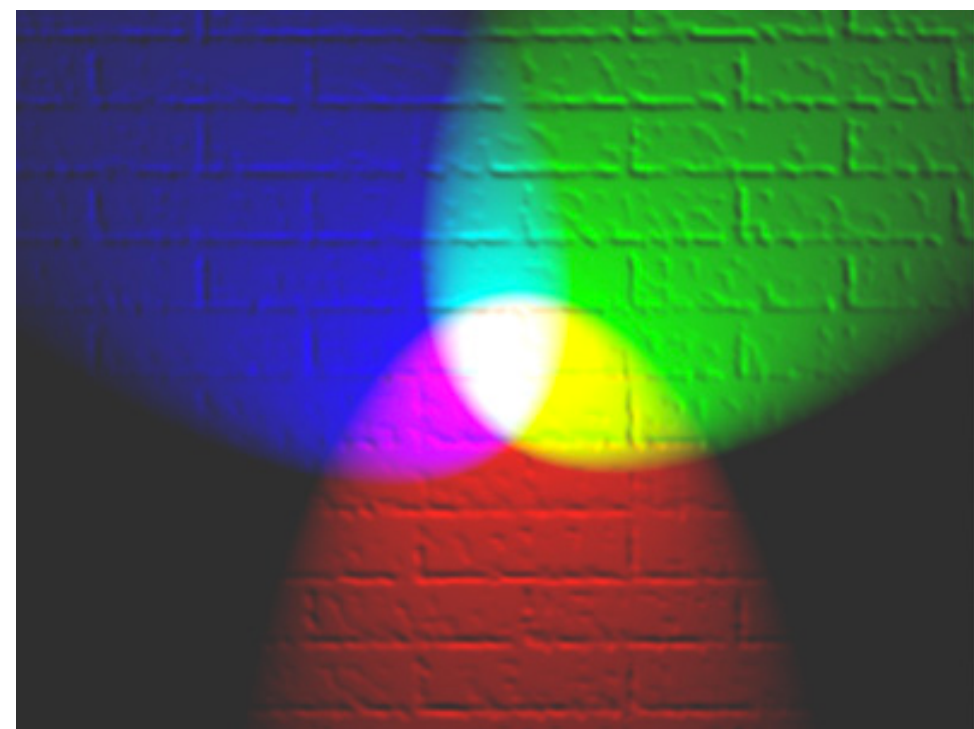
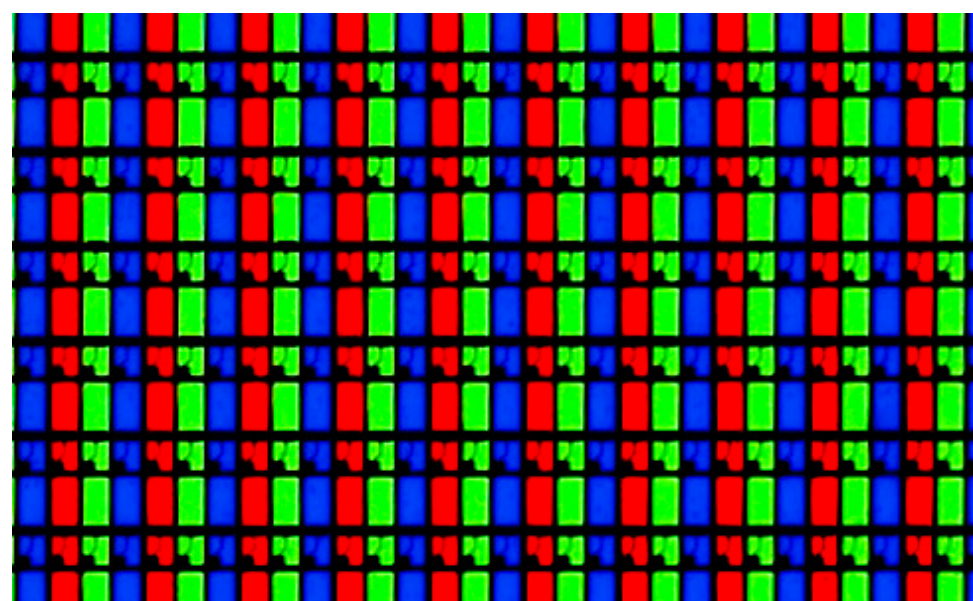
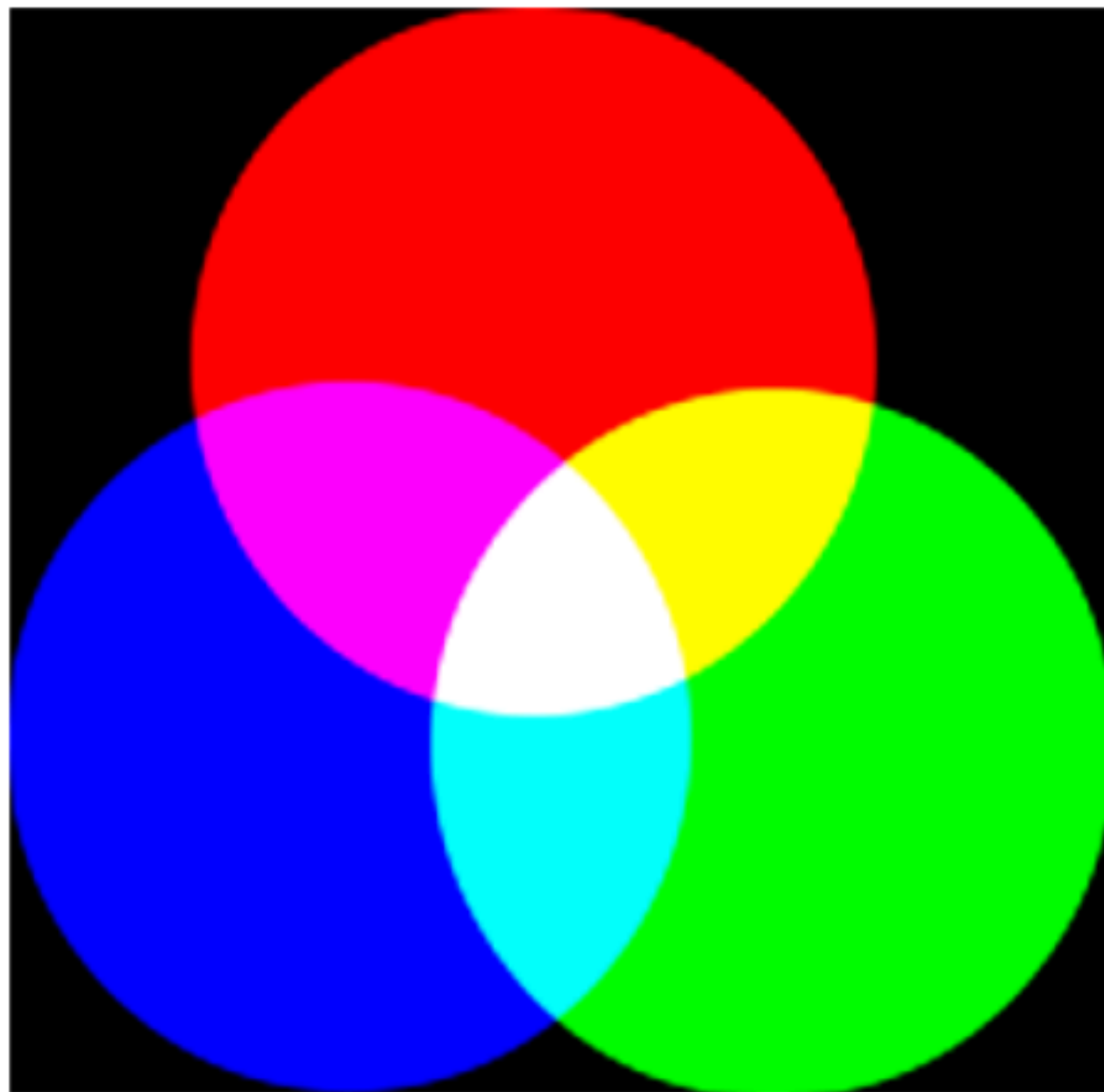
Subtractive



Descriptions of color

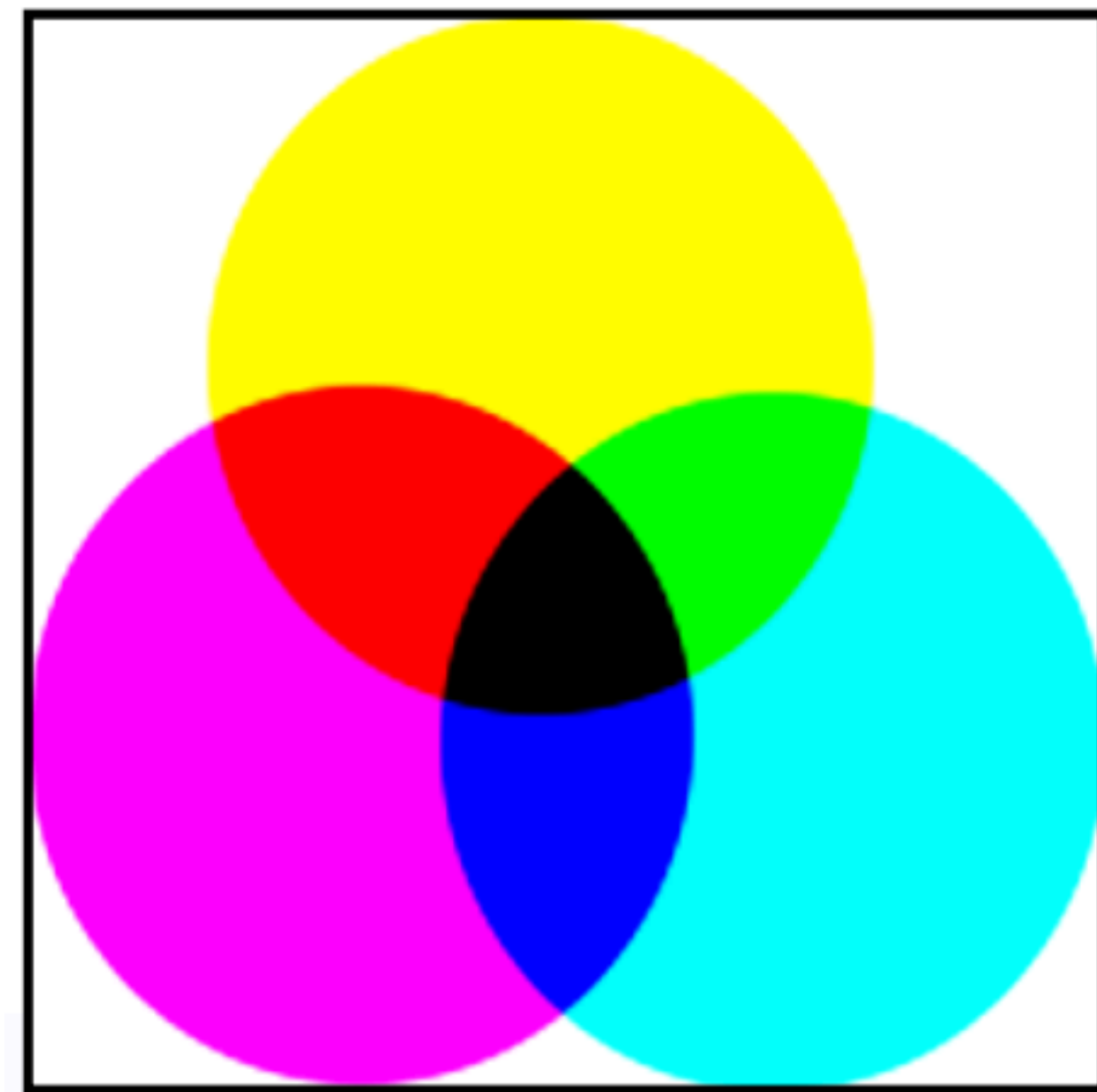
RGB

- Controls the light bulbs in each pixel



CMYK

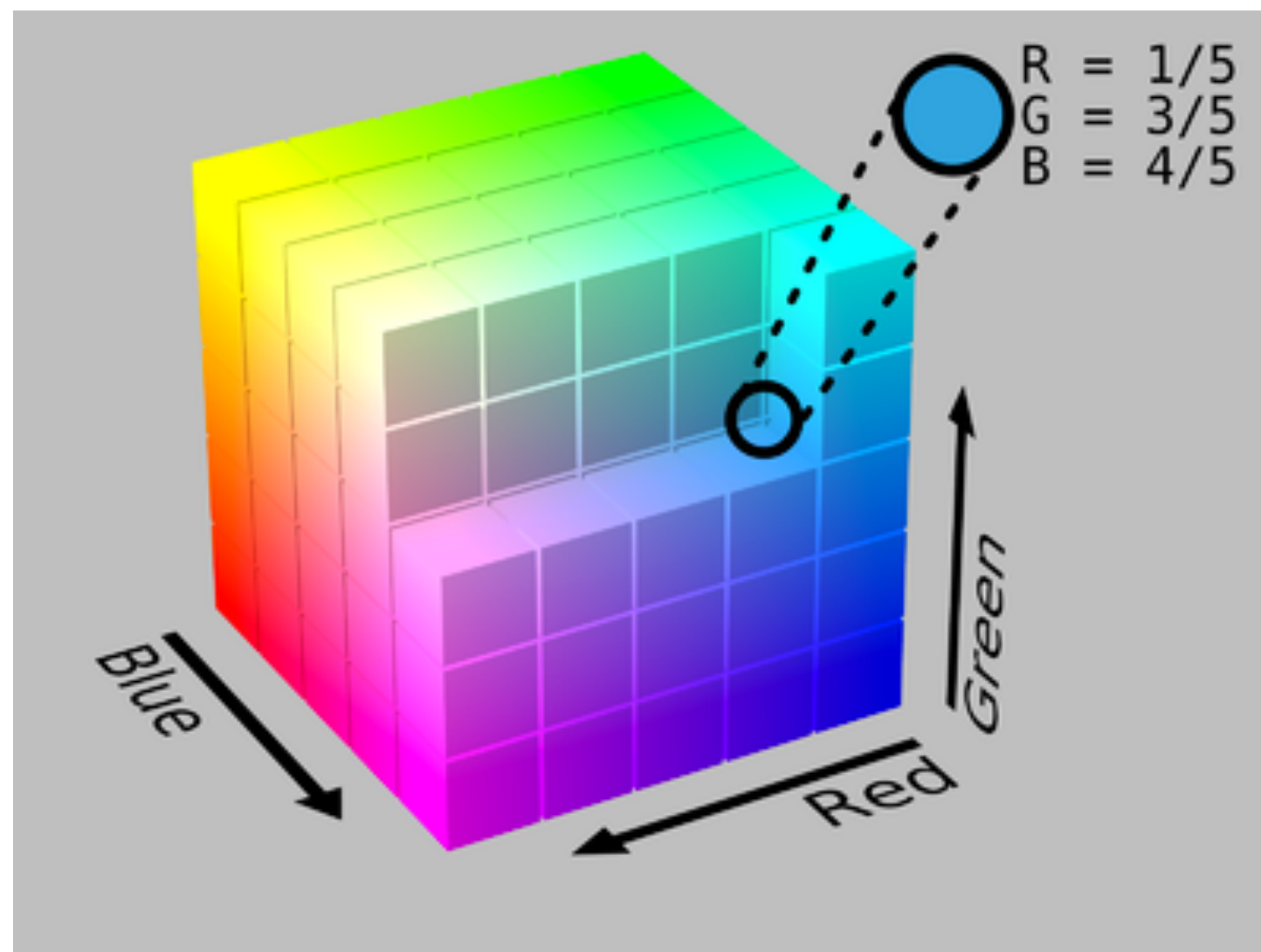
- Controls the ink in a printer



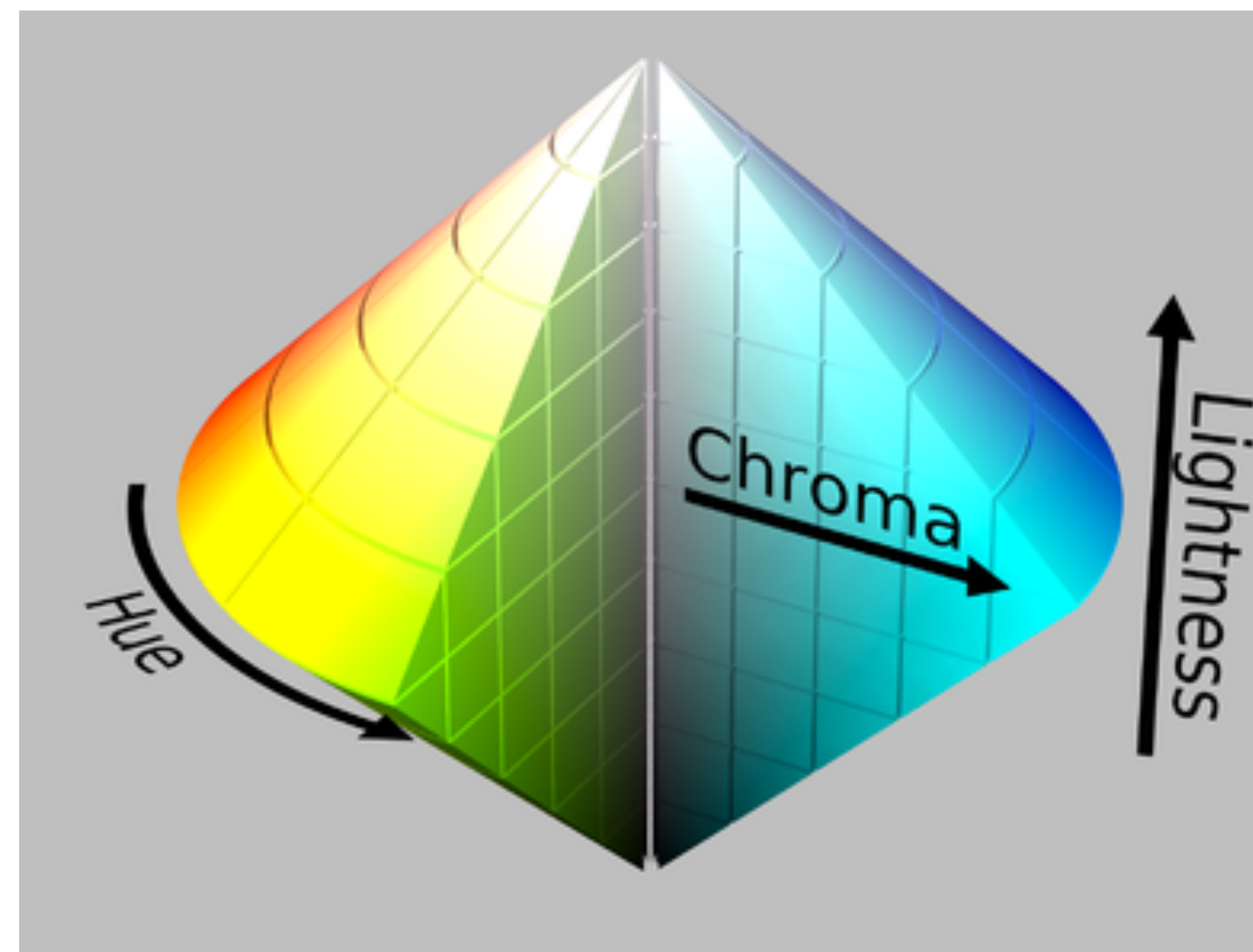
Descriptions of color

Hue, Saturation, Lightness/Brightness/Value (HSL, HSB, HSV)

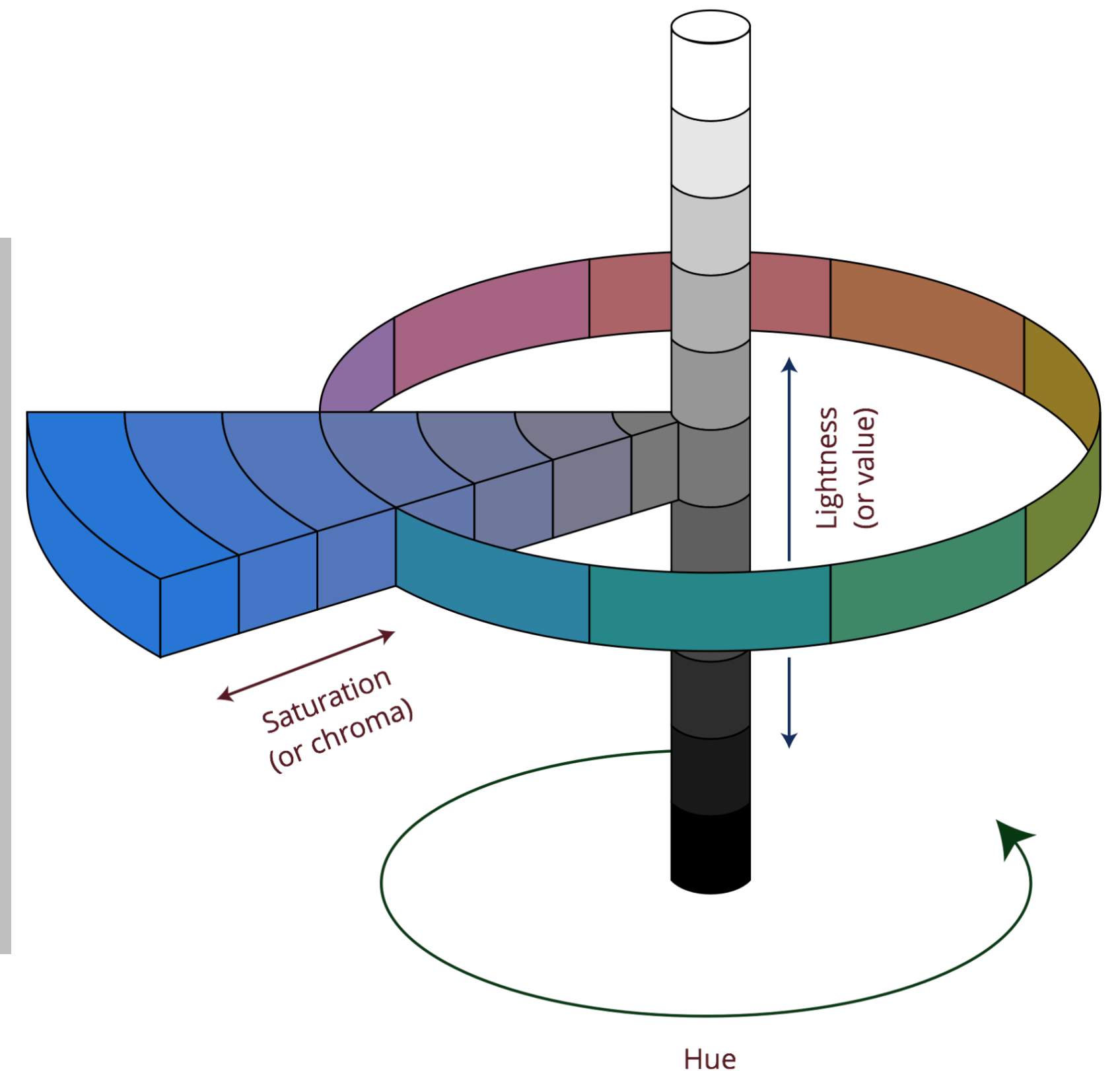
- ▶ More intuitive
- ▶ Useful in design



RGB



HSL



Real Story

- The above description of color is based on what we perceive.
- What is the objective and mathematical description of color?
- We need to distinguish **physical color** & **perceived color**.

Crash course on the L^2 pairing

Definition. The L^2 pairing between two functions

$$f : [a, b] \rightarrow \mathbb{R}$$

$$g : [a, b] \rightarrow \mathbb{R}$$

on an interval $[a, b]$ is the integral

$$\int_a^b f(x)g(x) dx$$

- You can think of it as the inner product between functions.
- Here, we will view f, g playing different roles.

Crash course on the L^2 pairing

$$\int_a^b f(x)g(x)dx$$

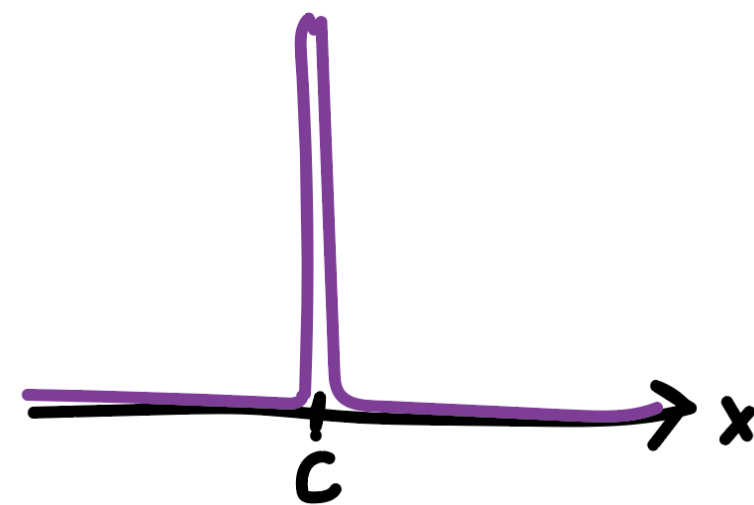
- Here, we will view f, g playing different roles.
 - ▶ $f(x)$ is assumed to be a **continuous function**.
 - ▶ $g(x)$ can be a **distribution** (derivative of a possibly discontinuous function).

Crash course on the L^2 pairing

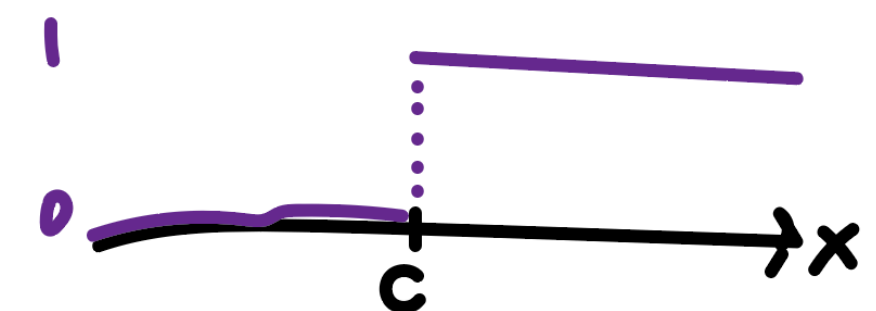
- ▶ $g(x)$ can be a **distribution** (derivative of a possibly discontinuous function).
- ▶ Continuous functions are distribution
- ▶ Integrable functions are distribution
- ▶ Generalized functions such as the δ -functions are distribution

Definition. The δ -function $\delta_c(x)$ concentrated at $c \in [a, b]$ is the derivative of the step function jumping at c :

$$\delta_c(x) = \frac{d}{dx} H_c(x)$$



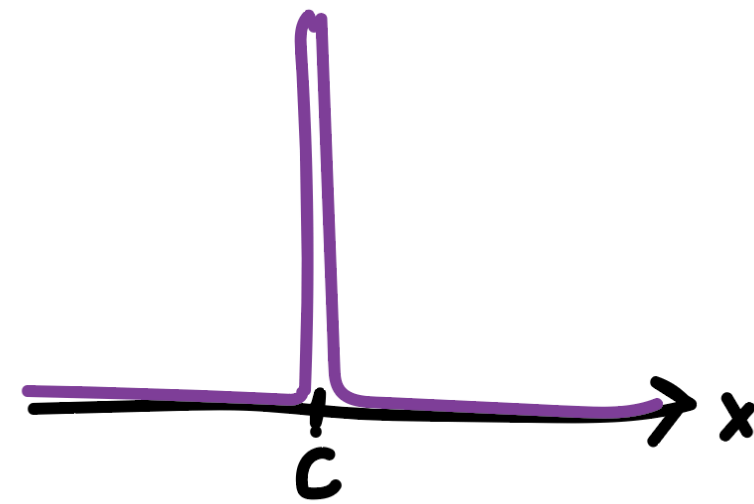
$$H_c(x) = \begin{cases} 0, & x < c \\ 1, & x \geq c \end{cases}$$



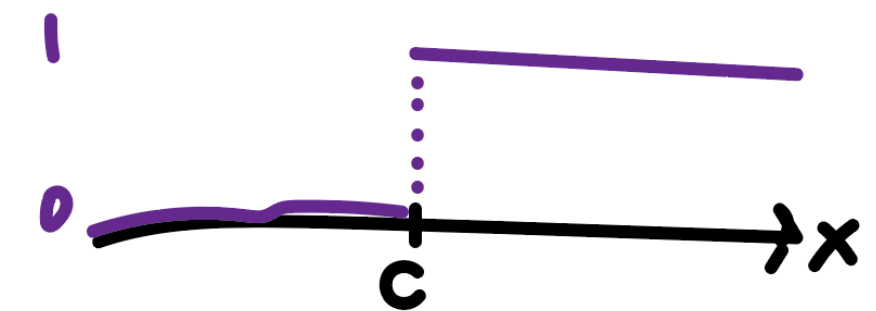
Crash course on the L^2 pairing

Definition. The δ -function $\delta_c(x)$ concentrated at $c \in [a, b]$ is the derivative of the step function jumping at c :

$$\delta_c(x) = \frac{d}{dx} H_c(x)$$



$$H_c(x) = \begin{cases} 0, & x < c \\ 1, & x \geq c \end{cases}$$



► Infinite impulse: $\delta_c(x) = \begin{cases} \infty, & x = c \\ 0, & x \neq c \end{cases}$

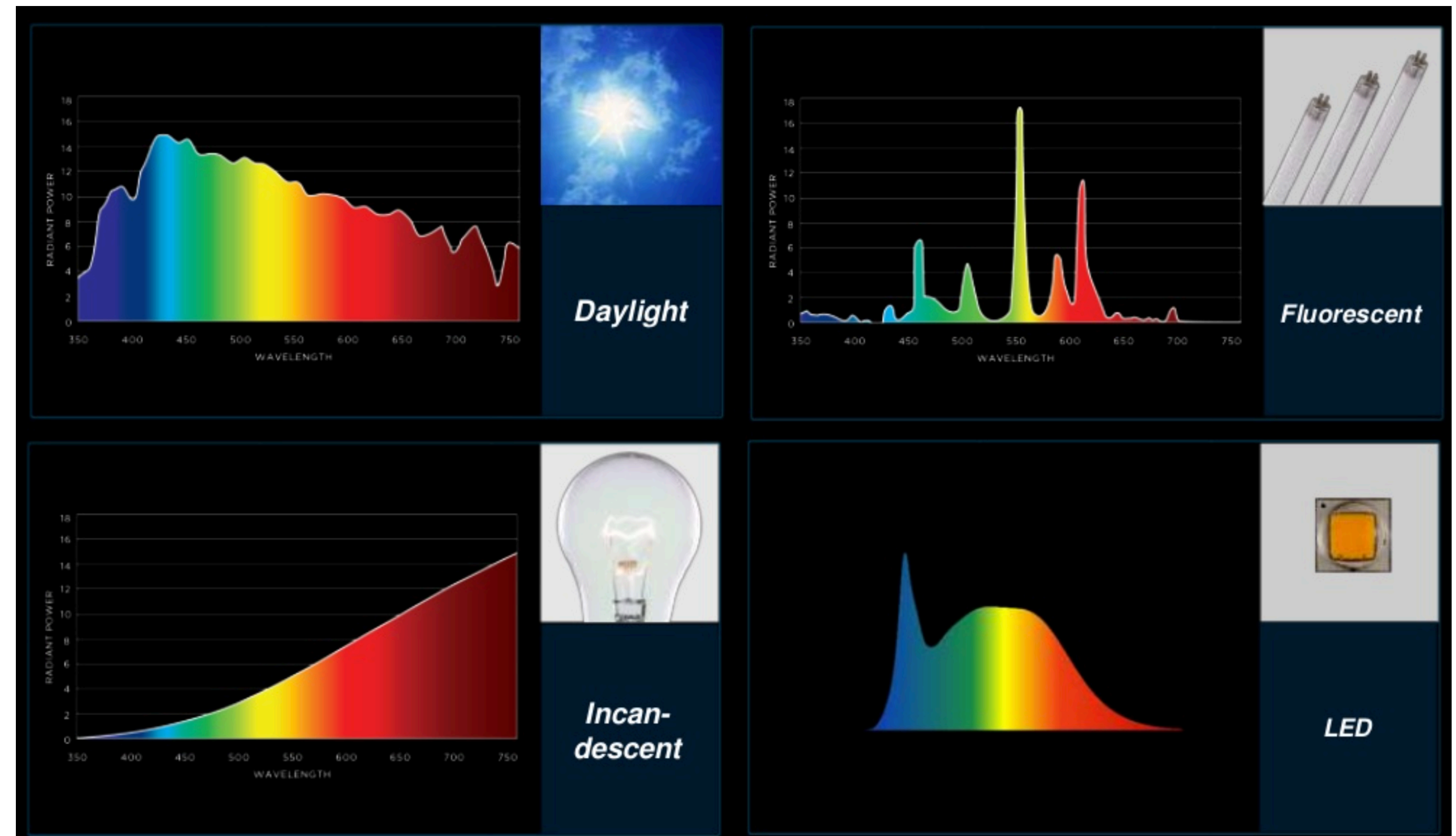
► Translation $\delta_c(x) = \delta_0(x - c)$

► L^2 -pairing $\int_a^b f(x) \delta_c(x) dx = f(c)$

Physical colored light

Definition. A **colored light** is a nonnegative distribution over the spectral interval [380nm, 750nm] representing the radiance power for each wavelength.

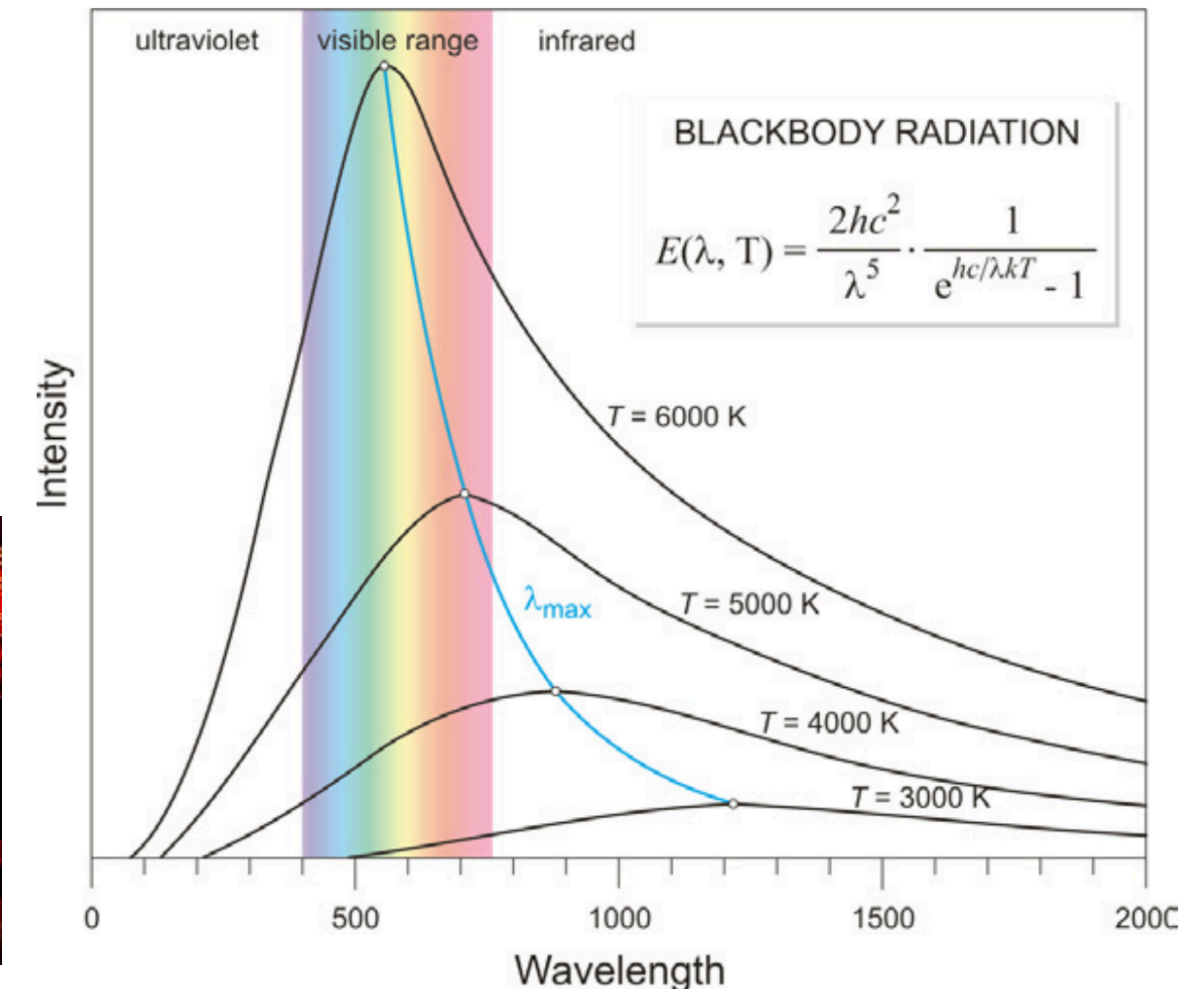
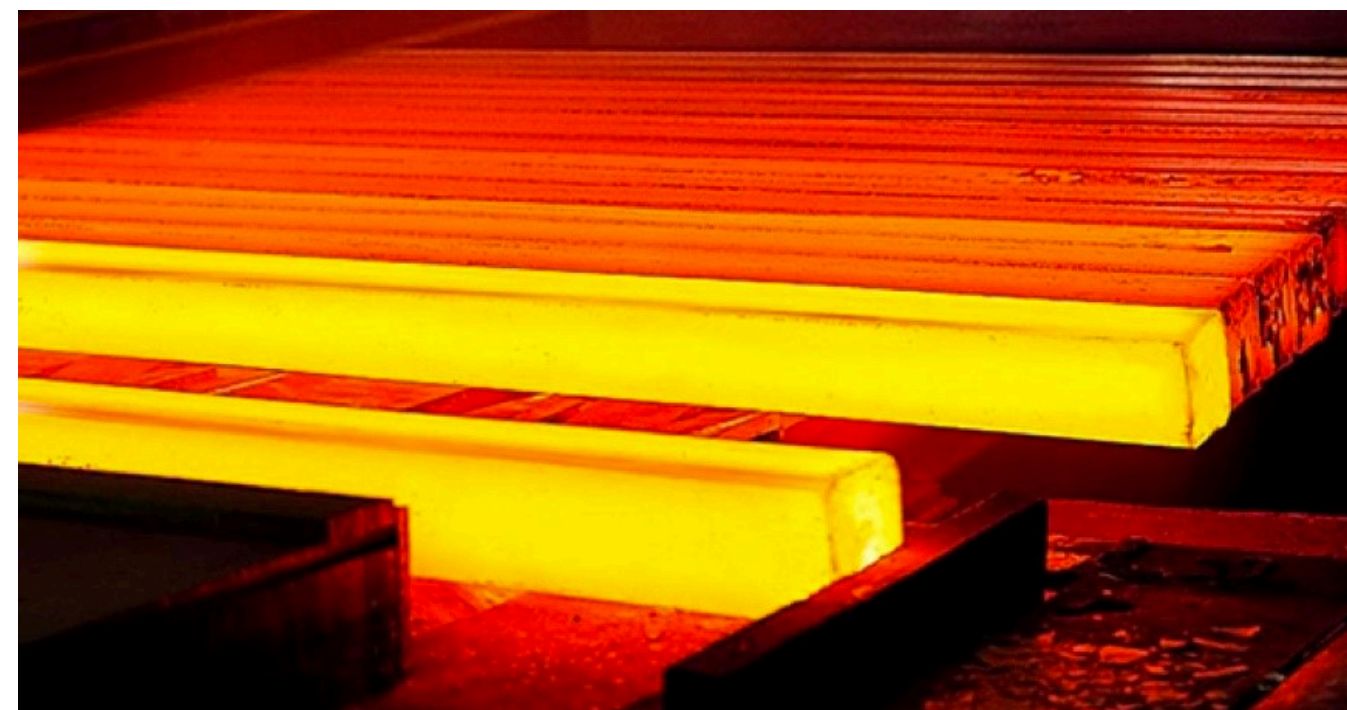
- This is called the **spectral power distribution**.
- Single-spectral light is a δ -function.



Physical colored light

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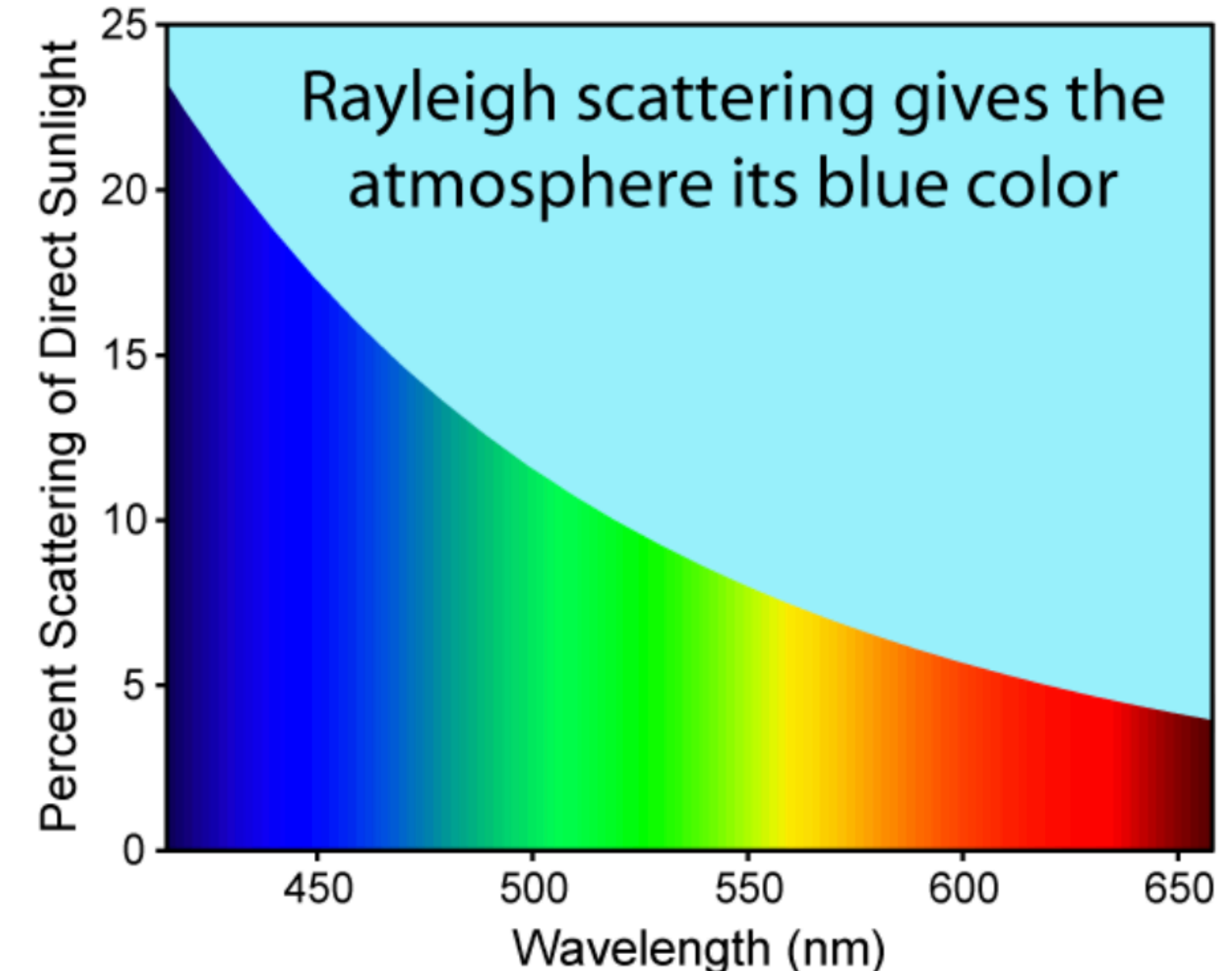
- This is called the **spectral power distribution**.
- Single-spectral light is a δ -function.
- Blackbody radiation



Physical colored light

Definition. A **colored light** is a nonnegative distribution over the spectral interval [380nm, 750nm] representing the radiance power for each wavelength.

- This is called the **spectral power distribution**.
- Single-spectral light is a δ -function.
- Blackbody radiation
- Rayleigh scattering



Physical colored light

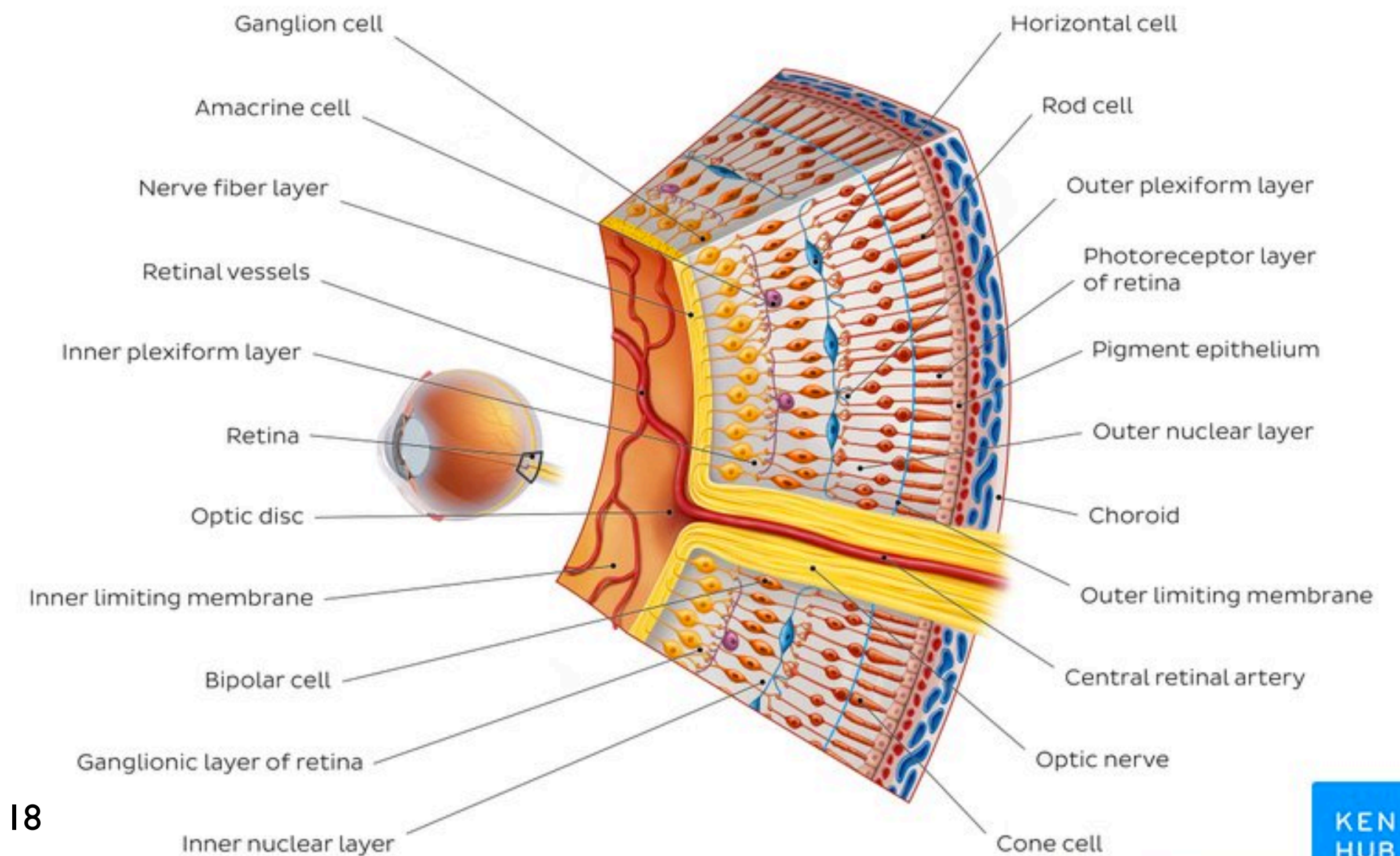
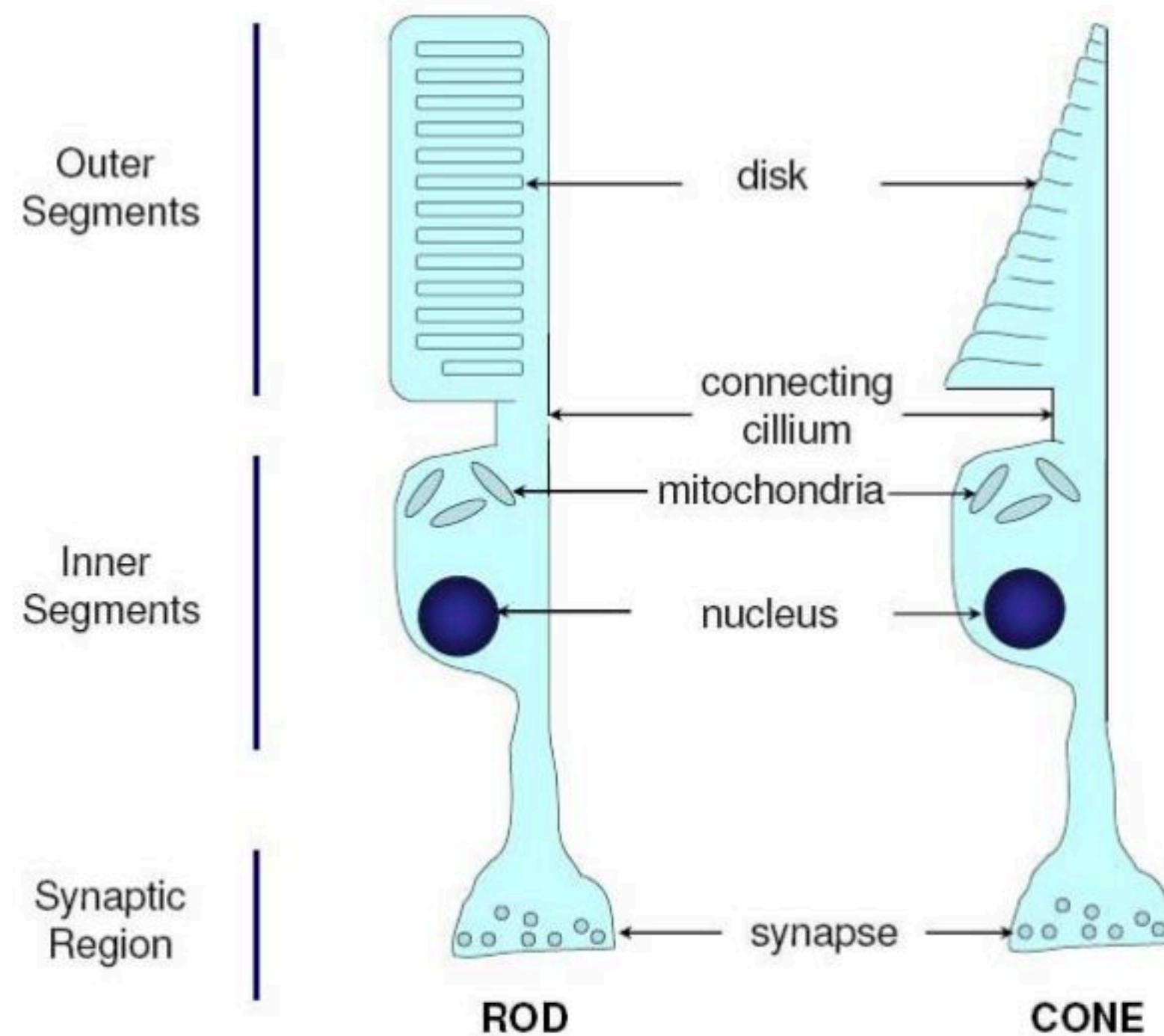
Definition. Two colored lights $P_1(\lambda)$, $P_2(\lambda)$ have the same chromatic color if they differ only by a scale

$$P_1(\lambda) = \alpha P_2(\lambda)$$

- The **chromaticity** lives in a projective space (infinite dimensional).

Vision

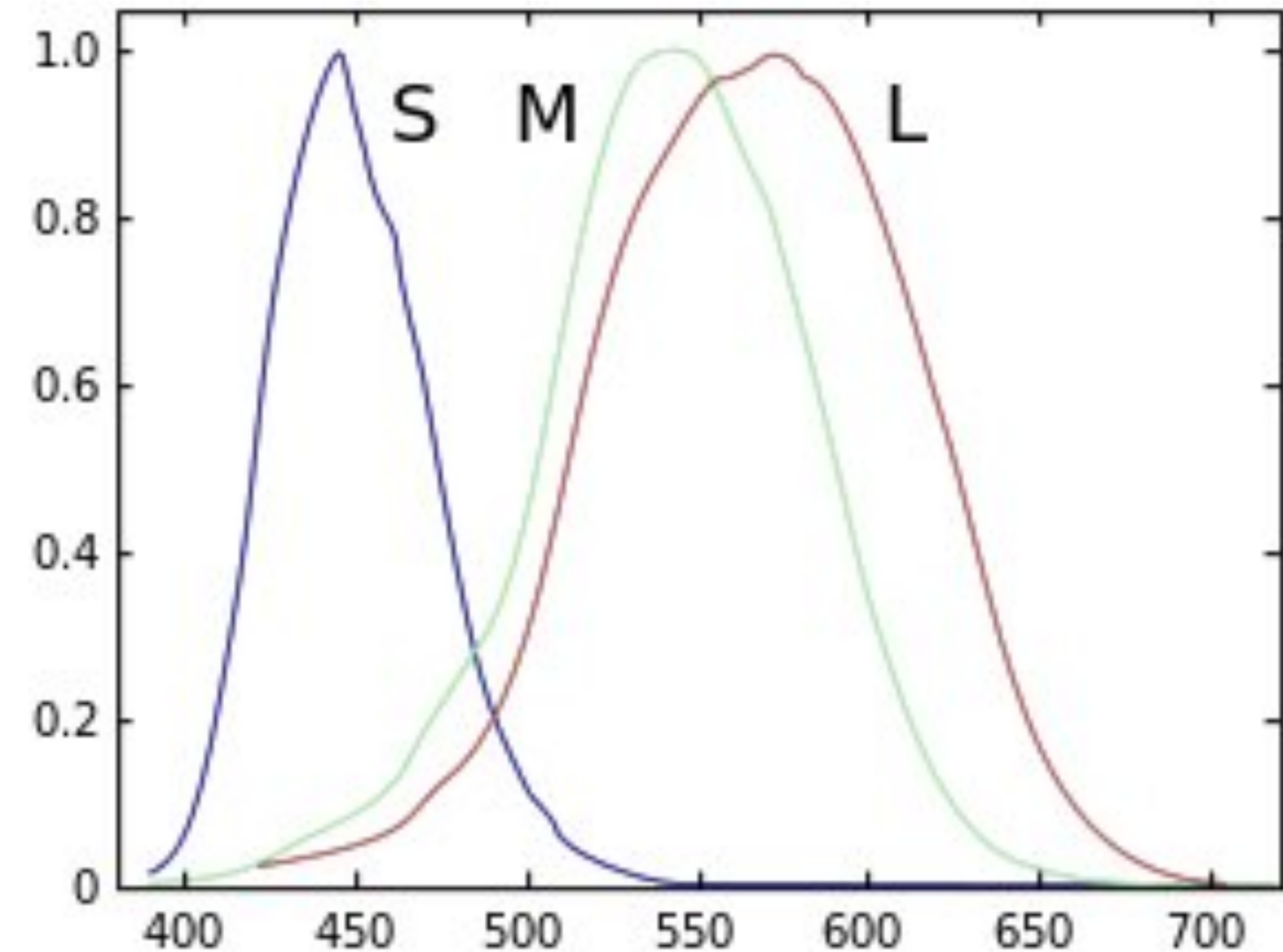
- Cone and rod photoreceptors in the retina turn light into neural signals.
- Rod is 100 times more sensitive to the cones, but slow response time. Rod is responsible for night vision (scotopic vision).
- Under sufficient illumination we use cone cells (photopic vision).
- It can take 10–30 minutes to switch between scotopic/photopic vision.



Color vision

- Under sufficient illumination we use cone cells (photopic vision).
- 90% of you have 3 types of cone cells with different sensitivity depending on the spectrum (long, middle, short).
- The sensitivity are continuous functions of the wavelength:

$$\bar{\ell}(\lambda) \quad \bar{m}(\lambda) \quad \bar{s}(\lambda)$$



Color vision

- The sensitivity are continuous functions of the wavelength:

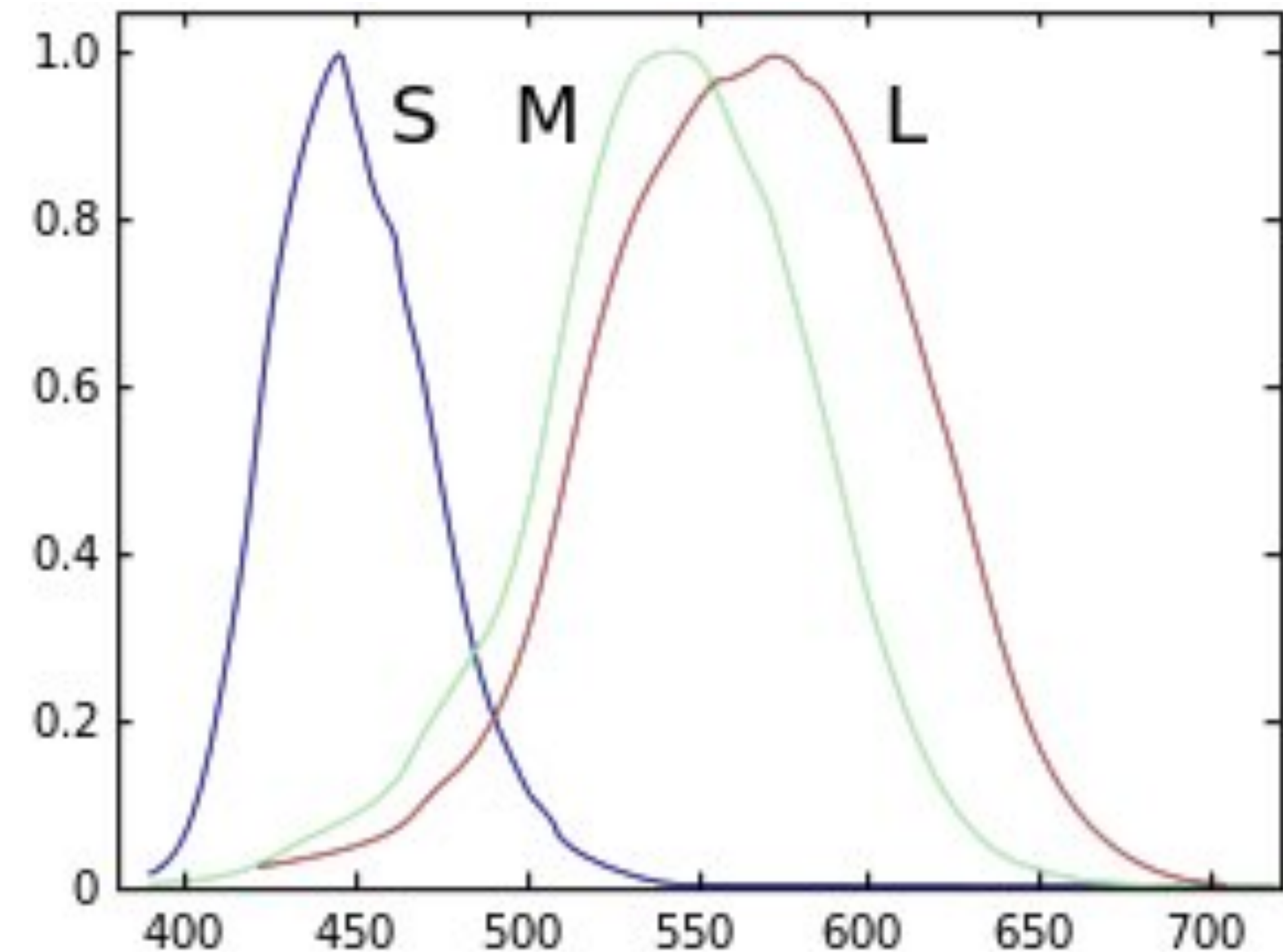
$$\bar{\ell}(\lambda) \quad \bar{m}(\lambda) \quad \bar{s}(\lambda)$$

- When you watch a colored light $P(\lambda)$, the strength of the neural signals from the 3 cones are the L^2 -pairings:

$$L = \int_{380}^{750} \bar{\ell}(\lambda) P(\lambda) d\lambda$$

$$M = \int_{380}^{750} \bar{m}(\lambda) P(\lambda) d\lambda$$

$$S = \int_{380}^{750} \bar{s}(\lambda) P(\lambda) d\lambda$$

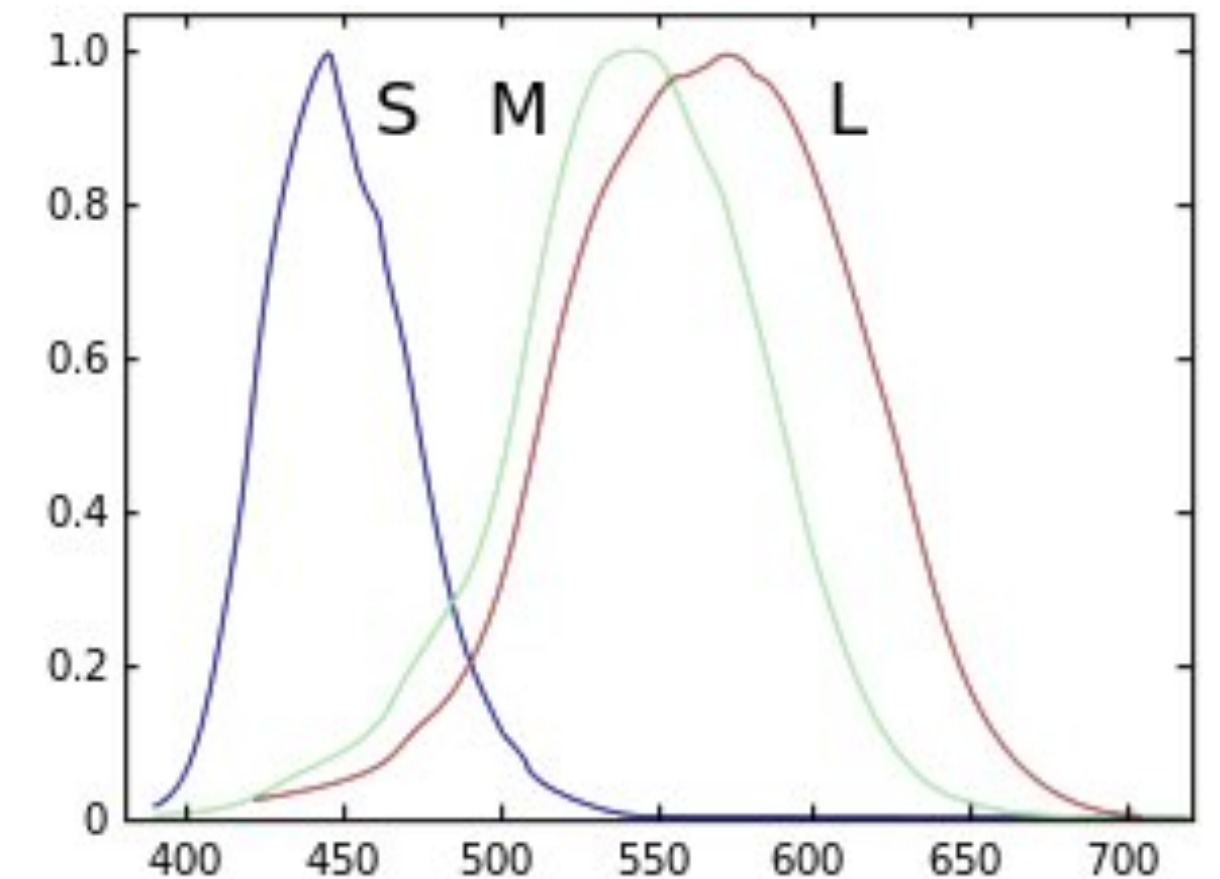


Color vision

$$L = \int_{380}^{750} \bar{\ell}(\lambda) P(\lambda) d\lambda \quad M = \int_{380}^{750} \bar{m}(\lambda) P(\lambda) d\lambda \quad S = \int_{380}^{750} \bar{s}(\lambda) P(\lambda) d\lambda$$

Definition. Two colored lights $P_1(\lambda)$, $P_2(\lambda)$ *appear the same* if the LMS values match

$$(L_1, M_1, S_1) = (L_2, M_2, S_2)$$

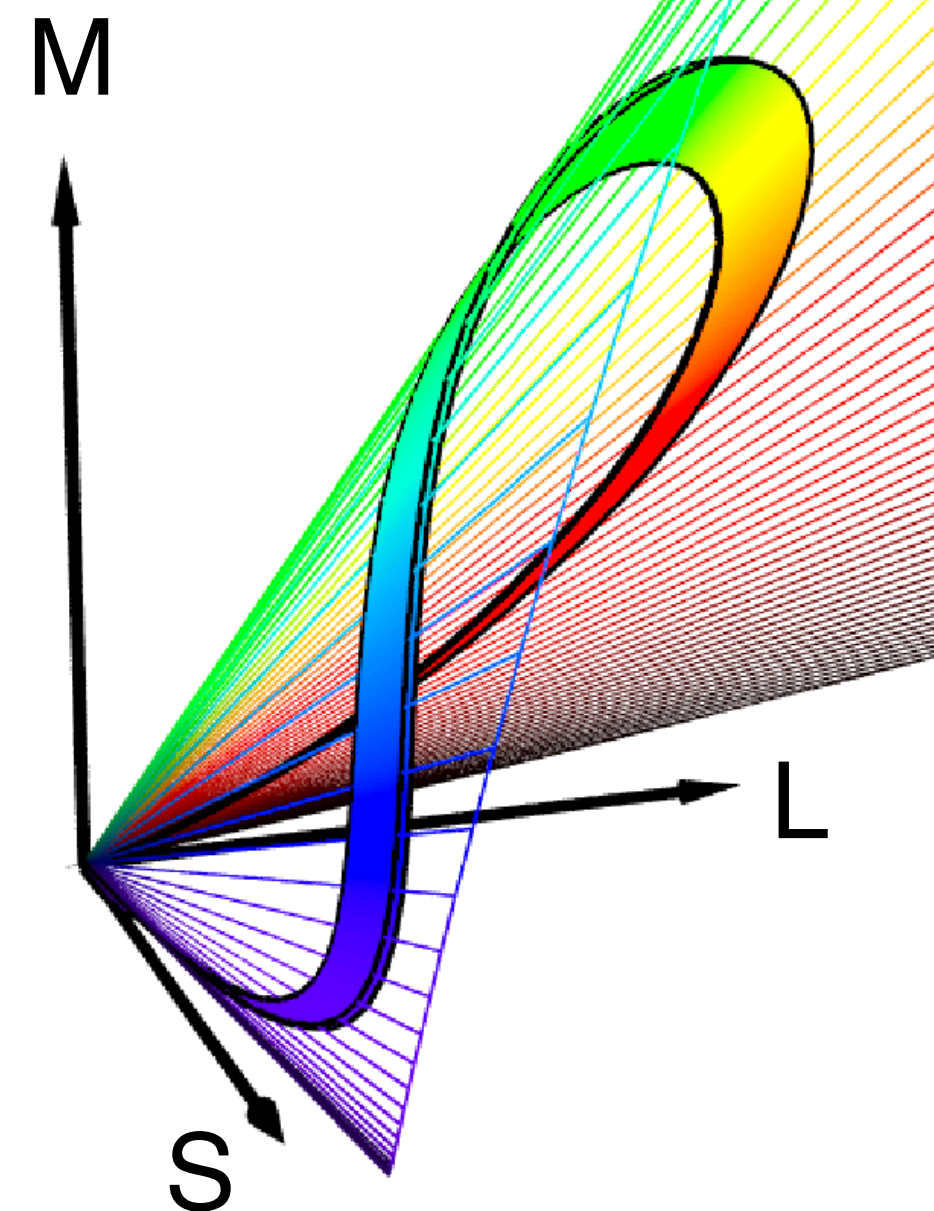
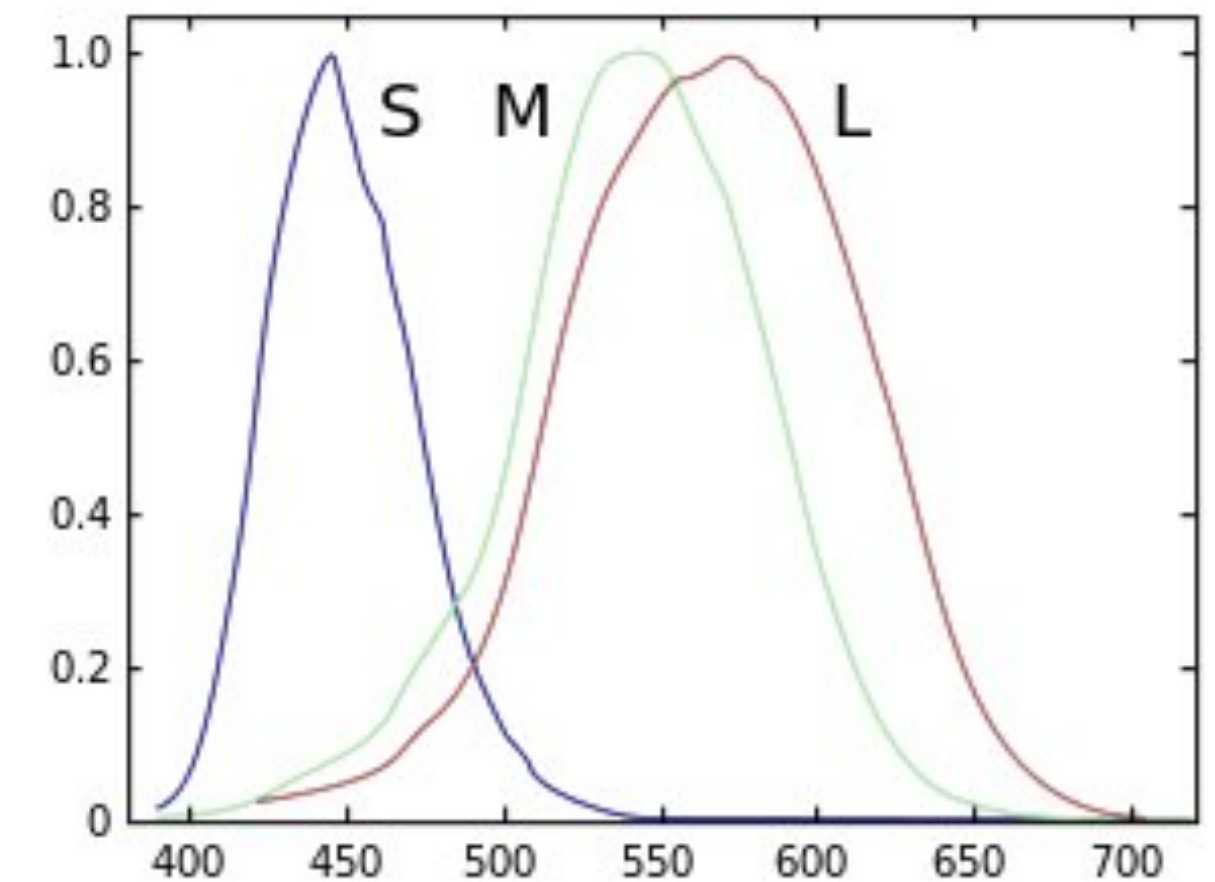


Definition. Two perceived colors (L_1, M_1, S_1) , (L_2, M_2, S_2) have the same chromatic color if $L_1 : M_1 : S_1 = L_2 : M_2 : S_2$

- The perceived chromaticities live on a 2-dimensional projective space.
- The colored light may be a combination of 3 monotone lights (RGB screen), but still appear the same as a natural colored light.

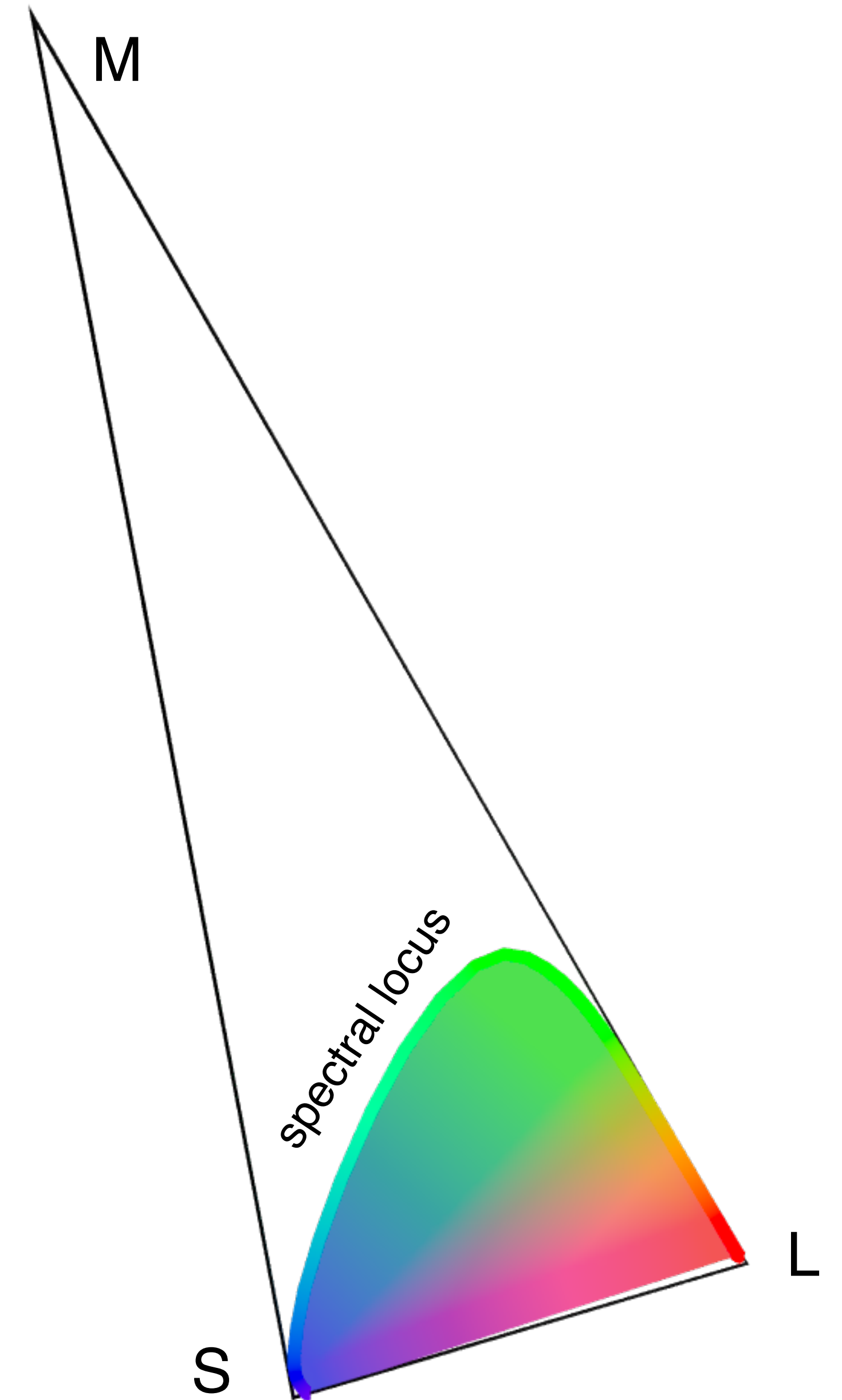
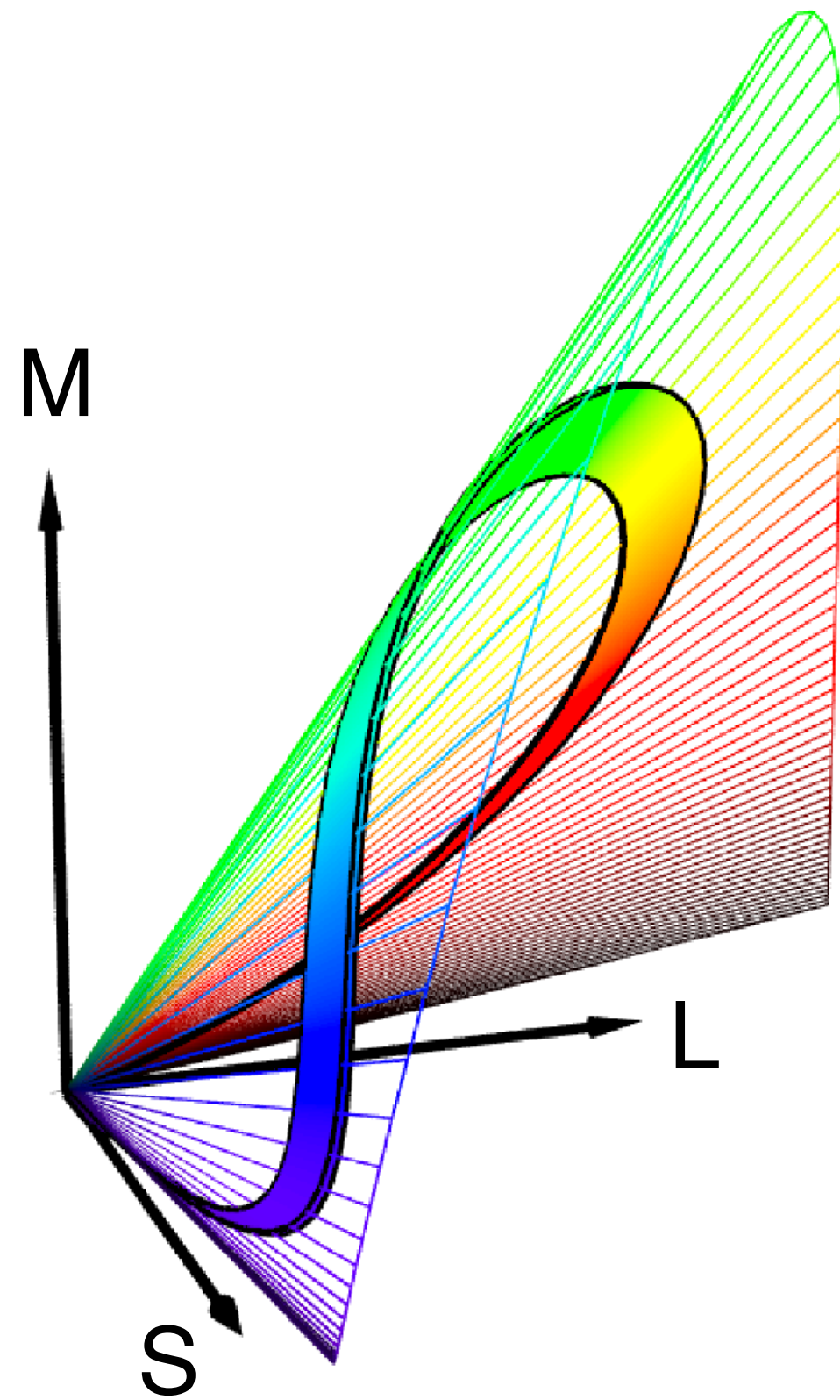
LMS color space

- Not all points in the 3D LMS space correspond to a color.
 - ▶ L,M,S are always nonnegative.
 - ▶ The source light is always a nonnegative distribution.
 - ▶ There is no light that would stimulate only the M cone cell but not the L cone cell.
 - ▶ All colored lights are mapped into the convex hull of the cone spanned by the spectral locus.



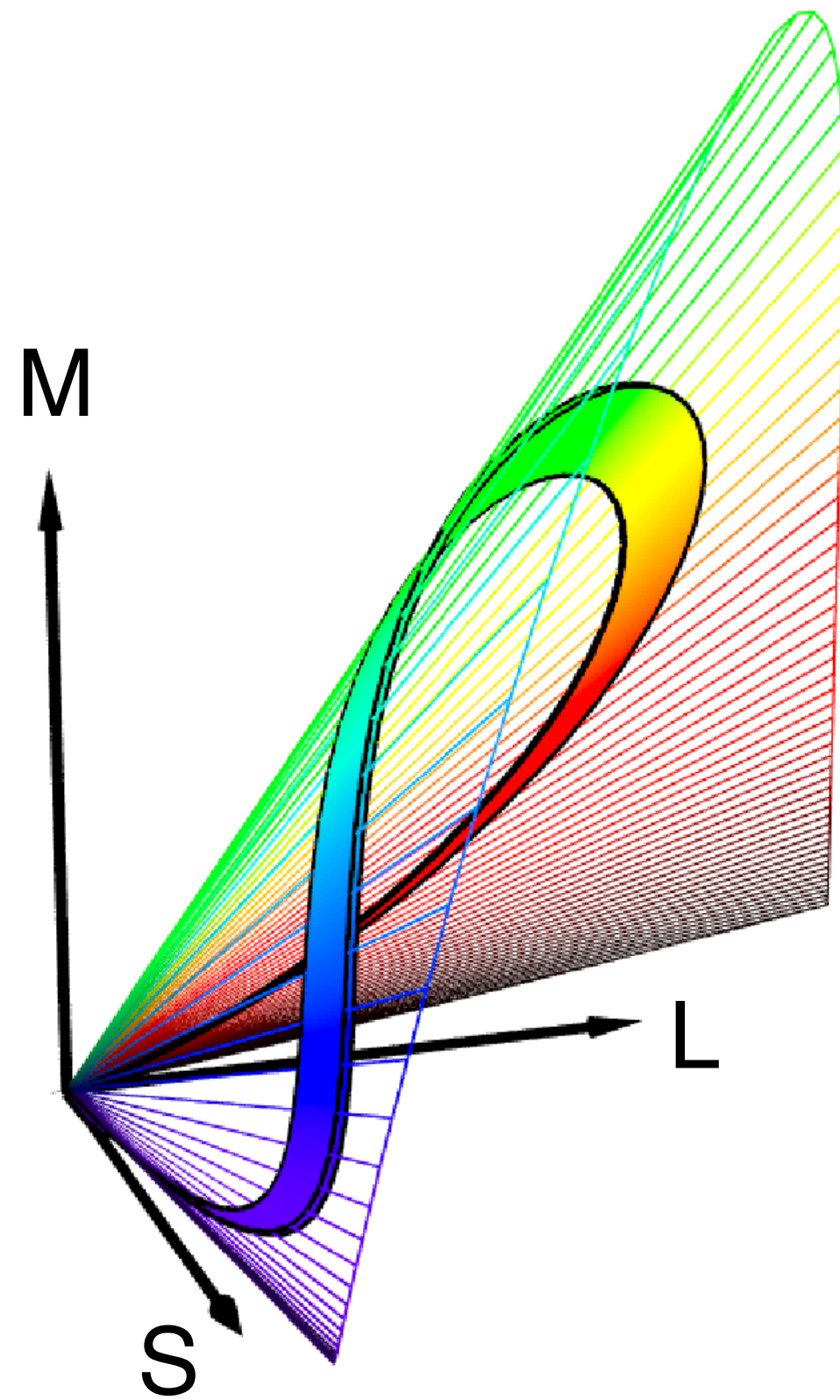
LMS color space

- Visualize the chromatic color on a 2D slice

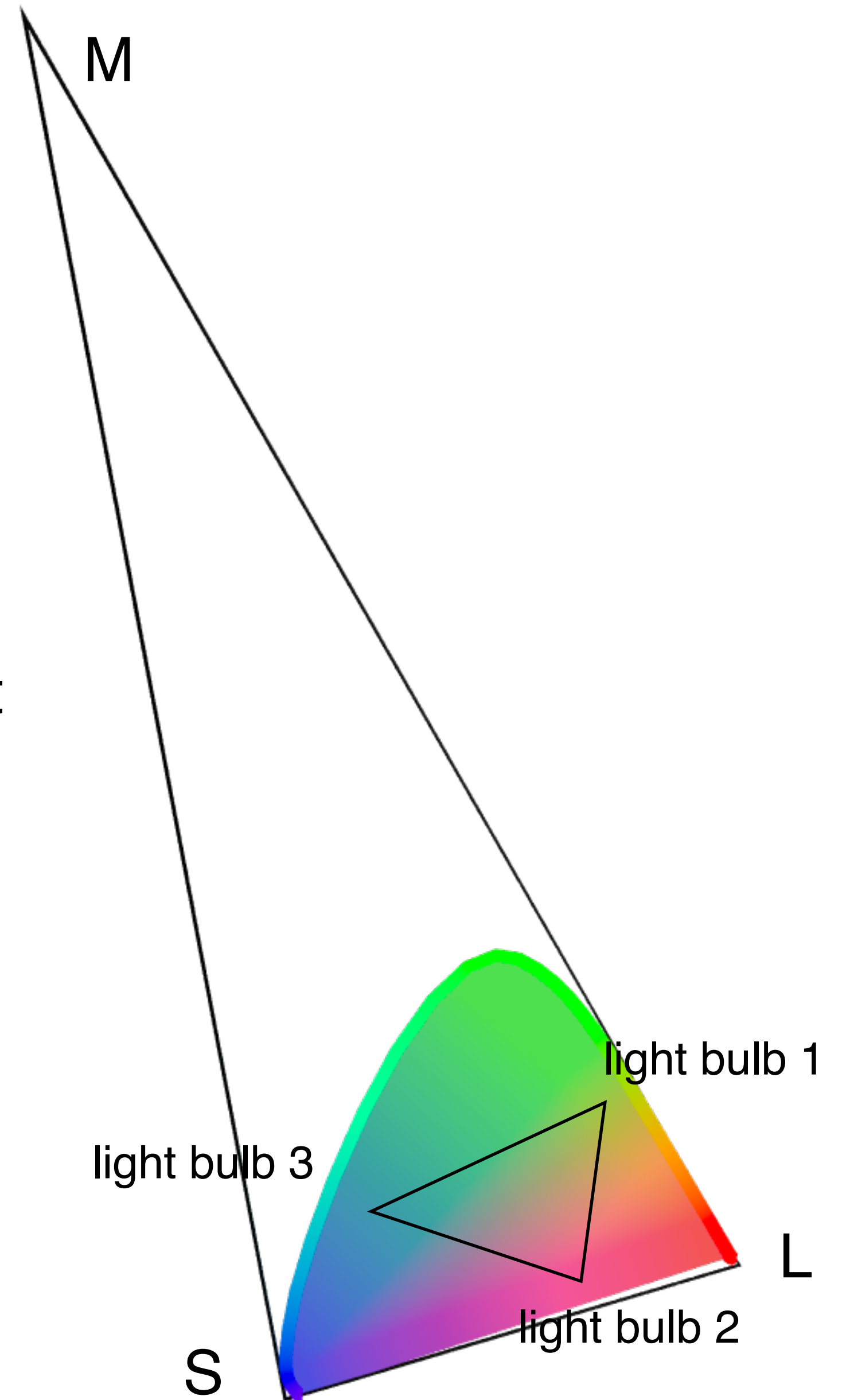


LMS color space

- Visualize the chromatic color on a 2D slice

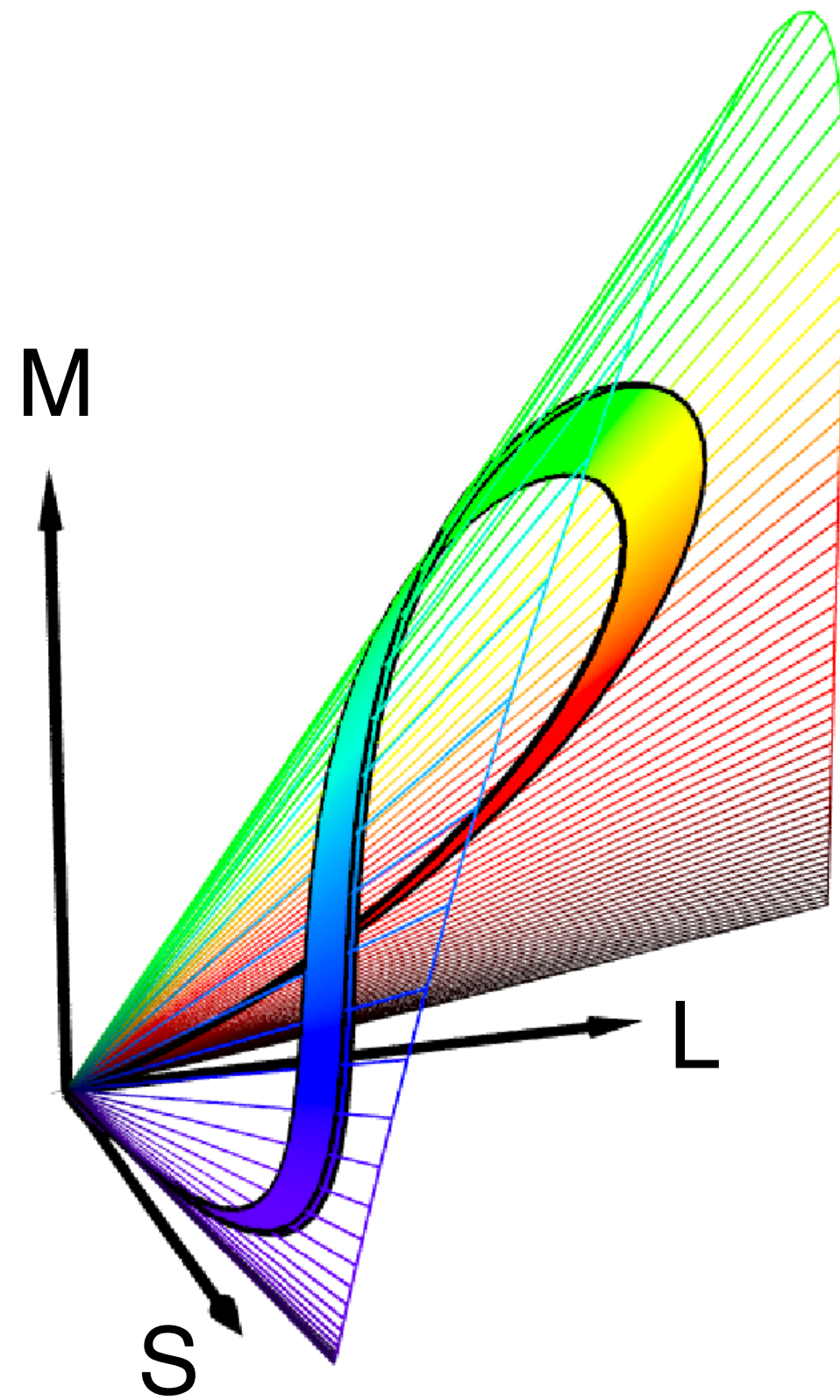


- ▶ Every artificial light that combines 3 lights must lie in the interior of a triangle.
- ▶ It is impossible to reproduce all colors by using 3 light bulbs.

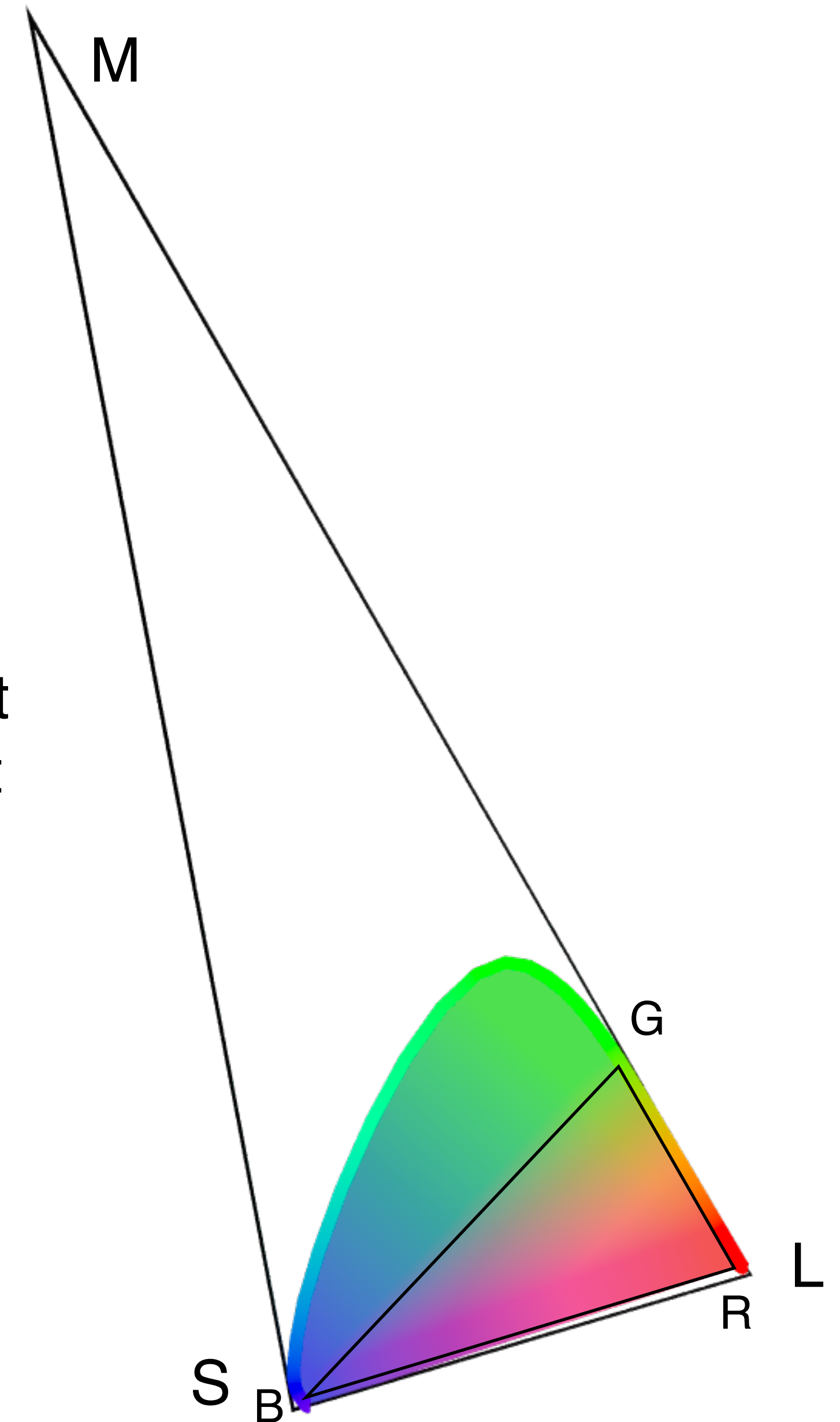


LMS color space

- Visualize the chromatic color on a 2D slice



- ▶ Every artificial light that combines 3 lights must lie in the interior of a triangle.
- ▶ It is impossible to reproduce all colors by using 3 light bulbs.
- ▶ The standard RGB



Grassmann Law

- **Grassmann Law**

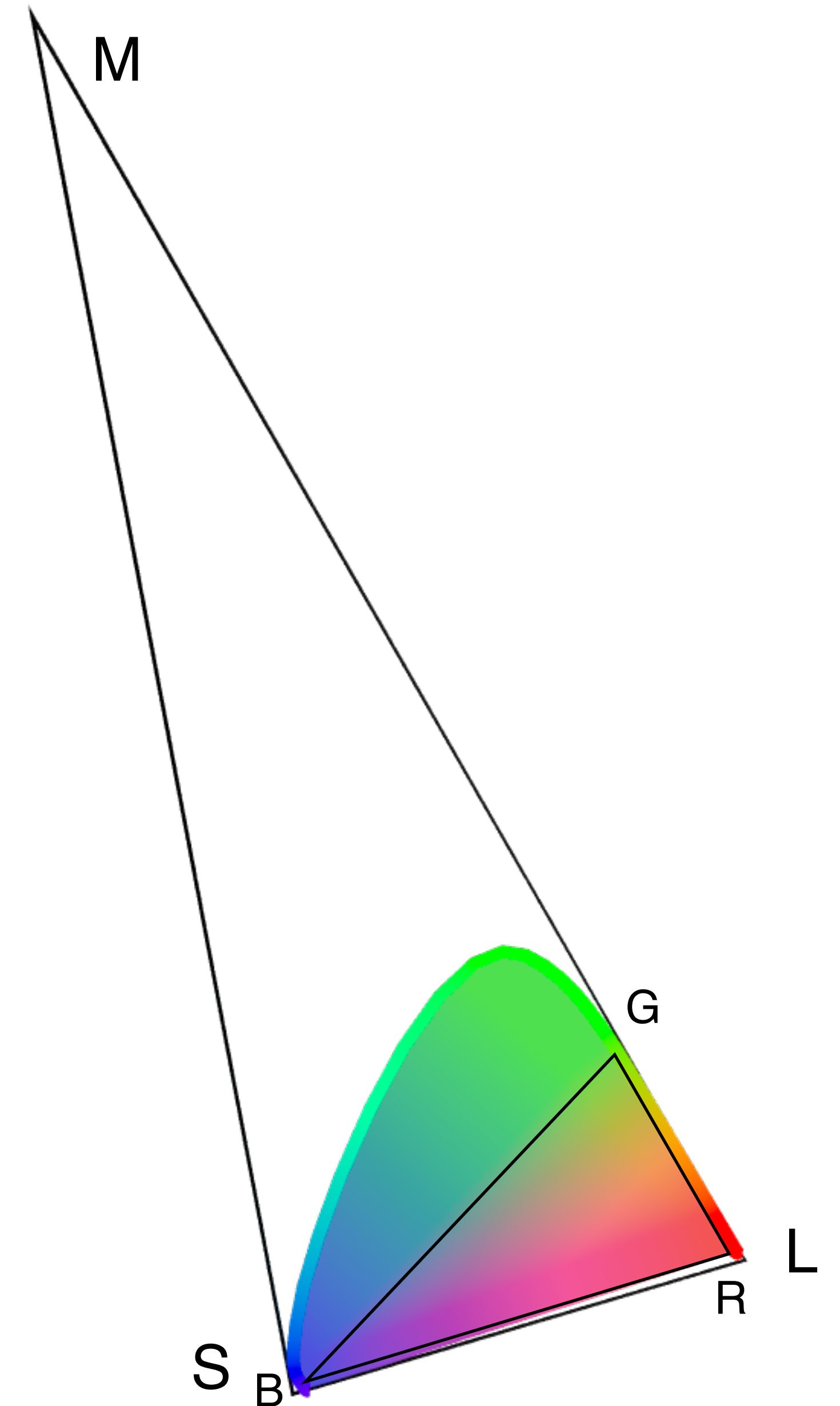
Given any triangle as the basis, say RGB, there exists functions $\bar{r}(\lambda)$, $\bar{g}(\lambda)$, $\bar{b}(\lambda)$ such that any physical light $P(\lambda)$ appear to have the same color as the point

$$rR + gG + bB$$

with the coefficients computed by the pairing

$$r = \int \bar{r}(\lambda)P(\lambda)d\lambda \quad g = \int \bar{g}(\lambda)P(\lambda)d\lambda$$

$$b = \int \bar{b}(\lambda)P(\lambda)d\lambda$$



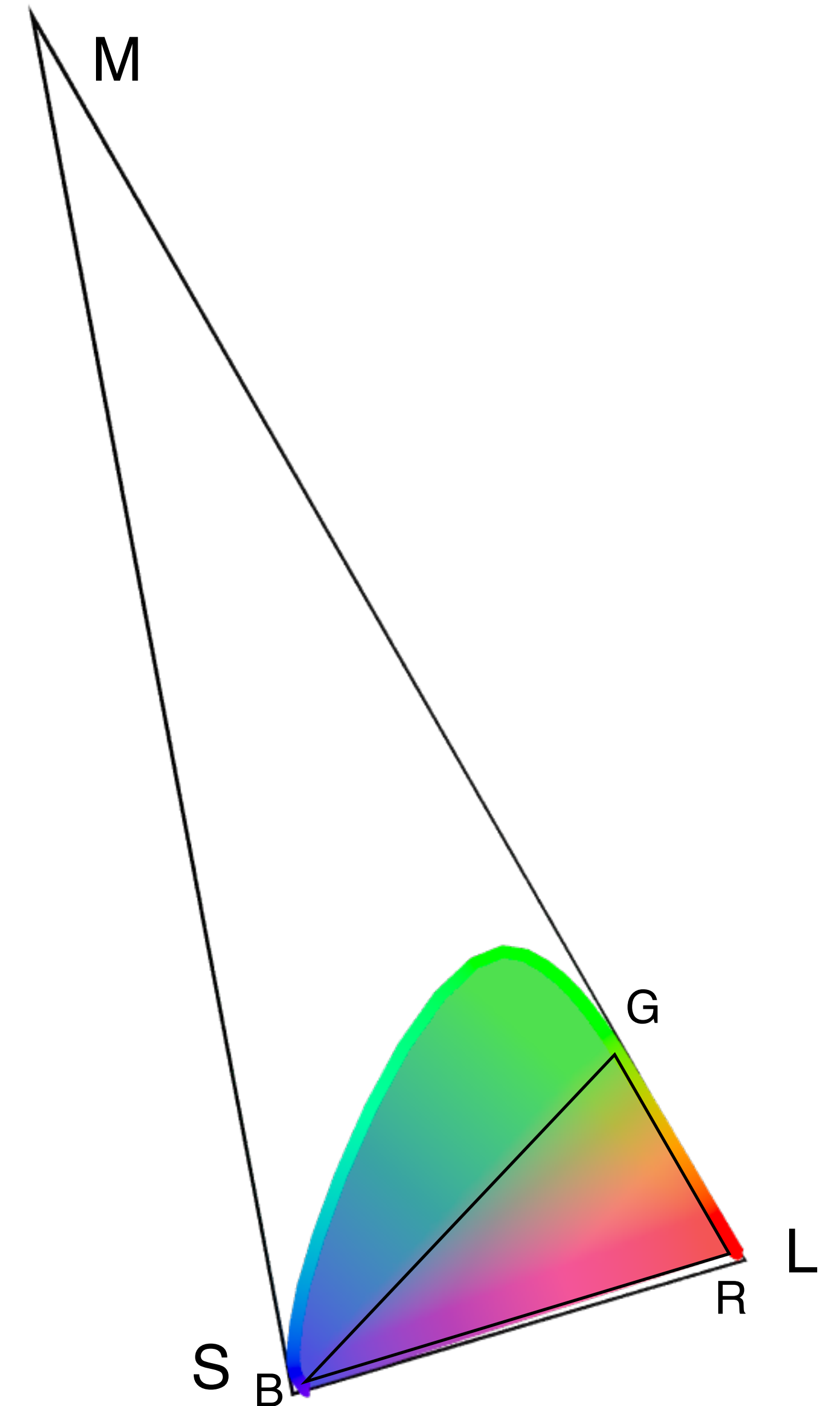
Grassmann Law

$$rR + gG + bB$$

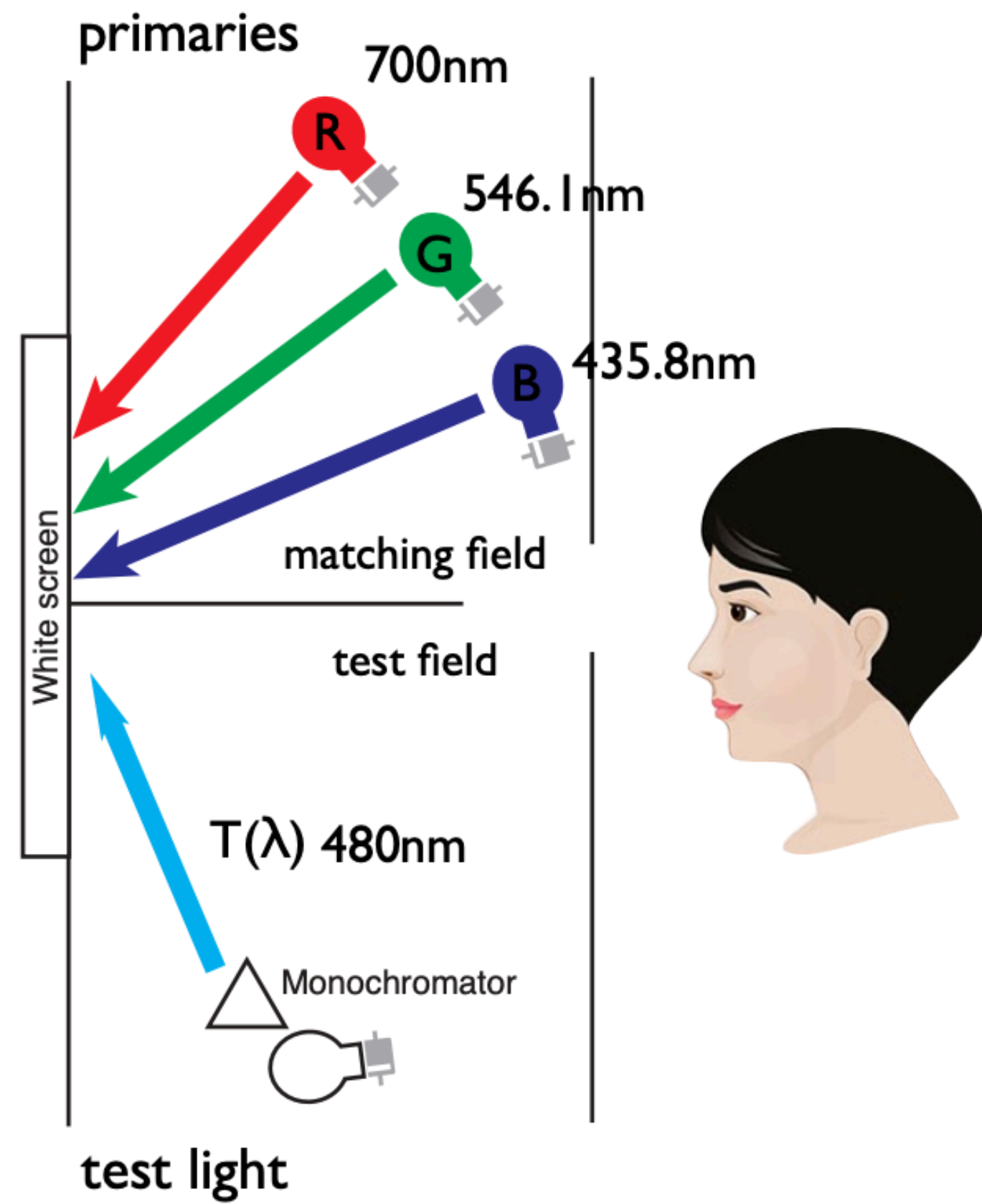
$$r = \int \bar{r}(\lambda)P(\lambda)d\lambda \quad g = \int \bar{g}(\lambda)P(\lambda)d\lambda$$

$$b = \int \bar{b}(\lambda)P(\lambda)d\lambda$$

- ▶ To determine $\bar{r}, \bar{g}, \bar{b}$, take RGB as 3 known light bulbs, and take $P(\lambda) = \delta_{\lambda_0}(\lambda)$
- ▶ Tune the coefficients r, g, b in $rR + gG + bB$ so that the synthetic light looks the same as $P(\lambda) = \delta_{\lambda_0}(\lambda)$
- ▶ Then we obtain $\bar{r}(\lambda_0) = r, \bar{g}(\lambda_0) = g, \bar{b}(\lambda_0) = b$.
- ▶ We call $\bar{r}, \bar{g}, \bar{b}$ the matching function.

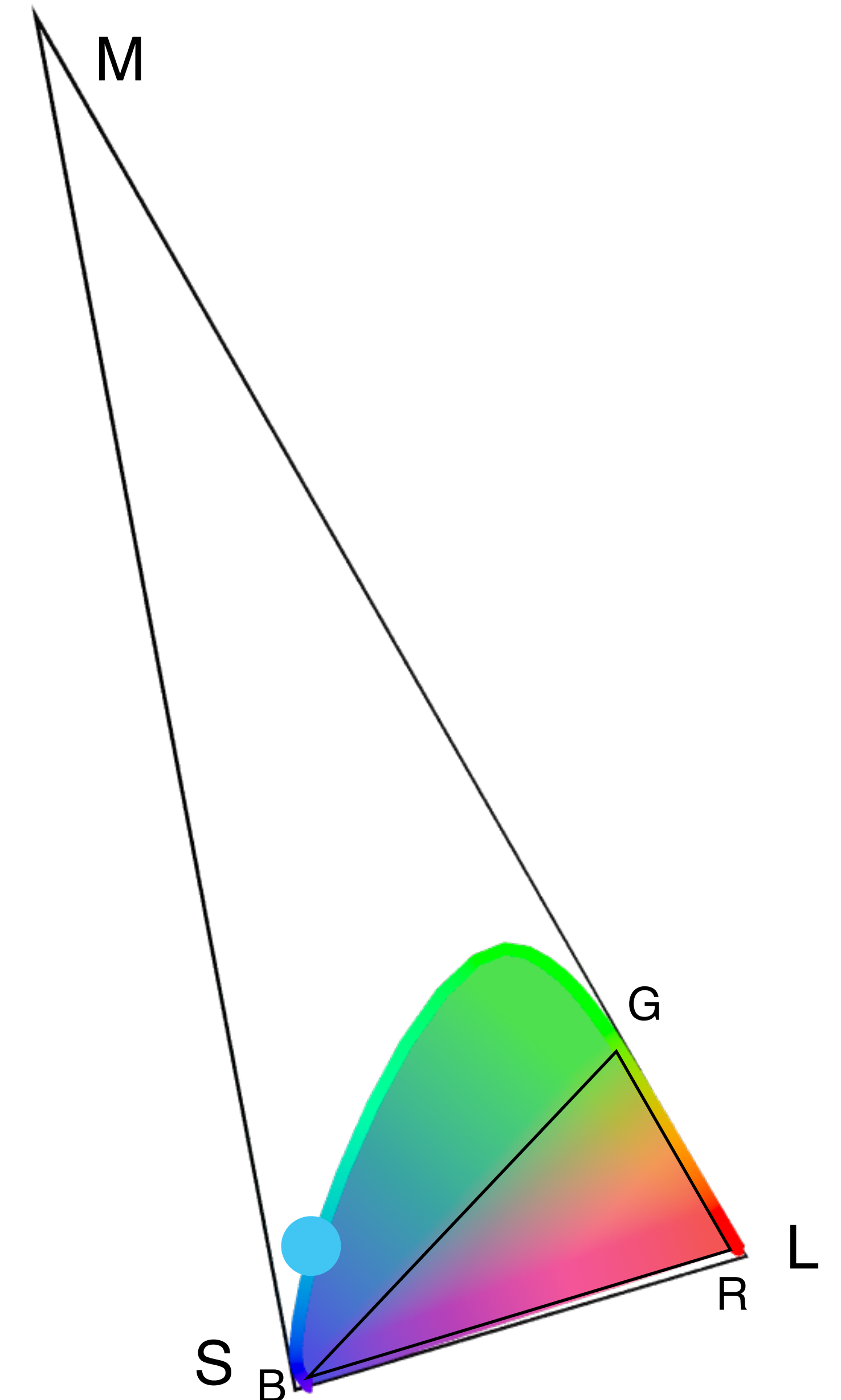
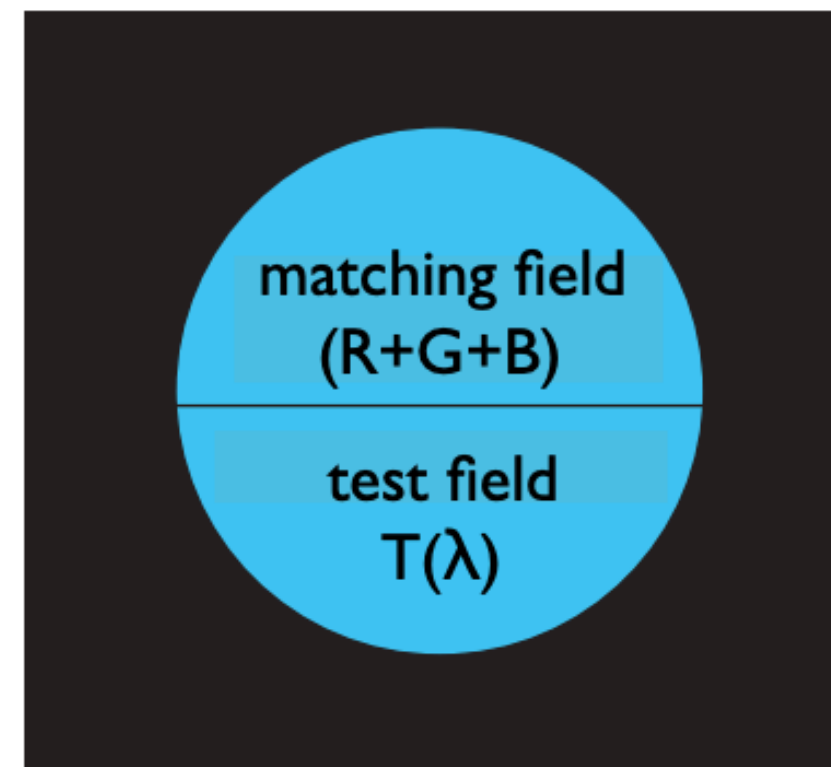


Color matching experiment

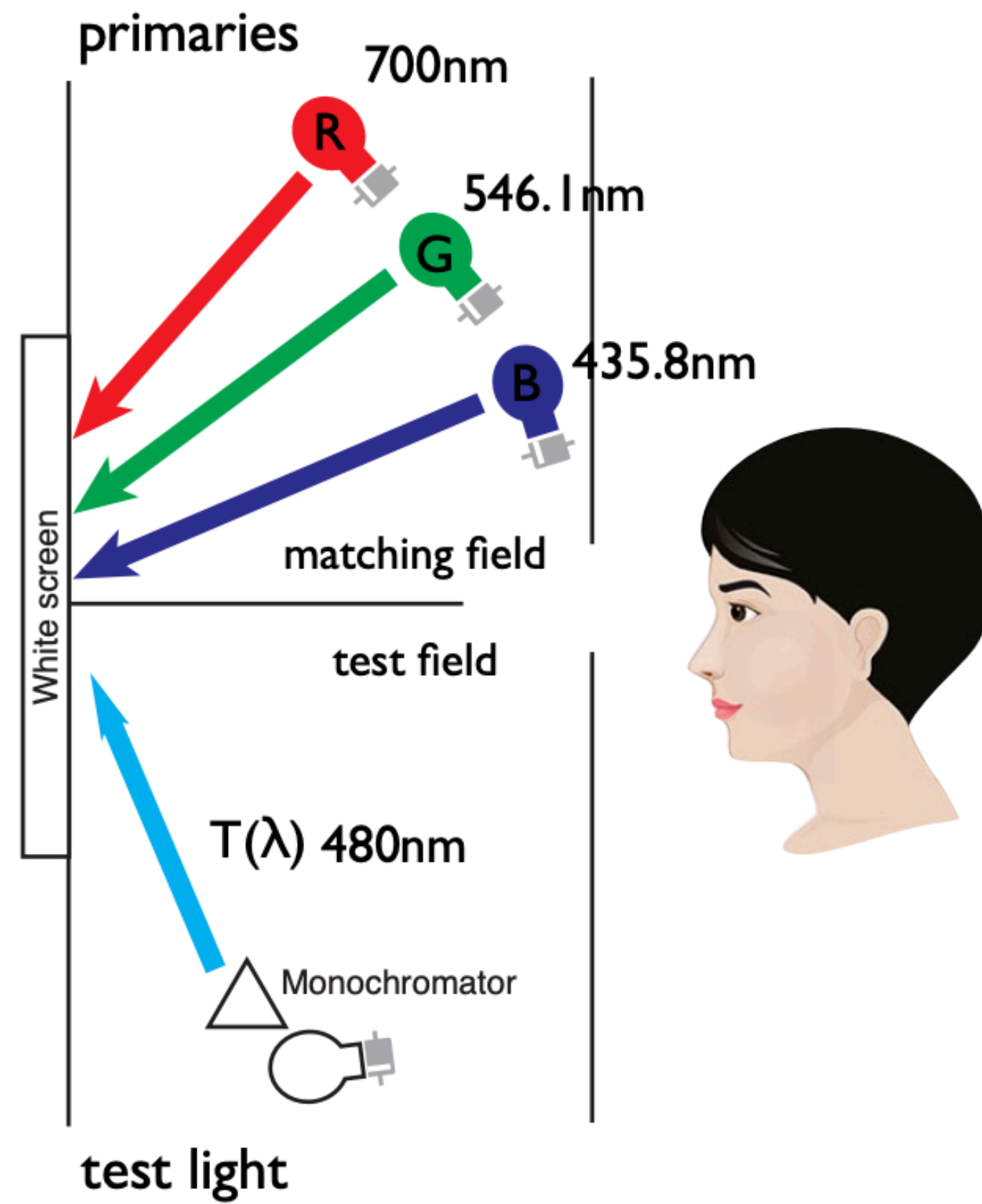


$$T(\lambda) = rR + gG + bB$$

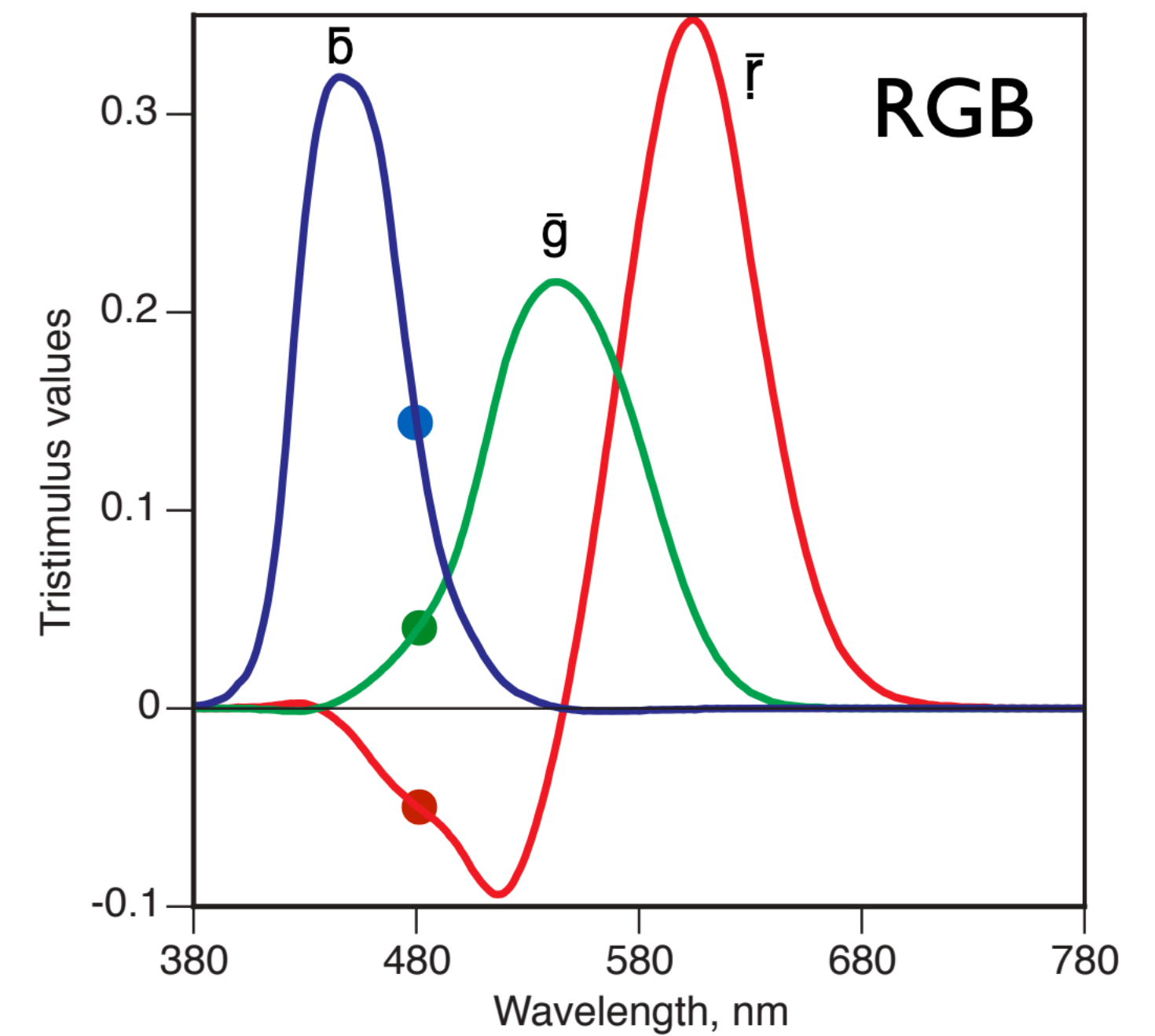
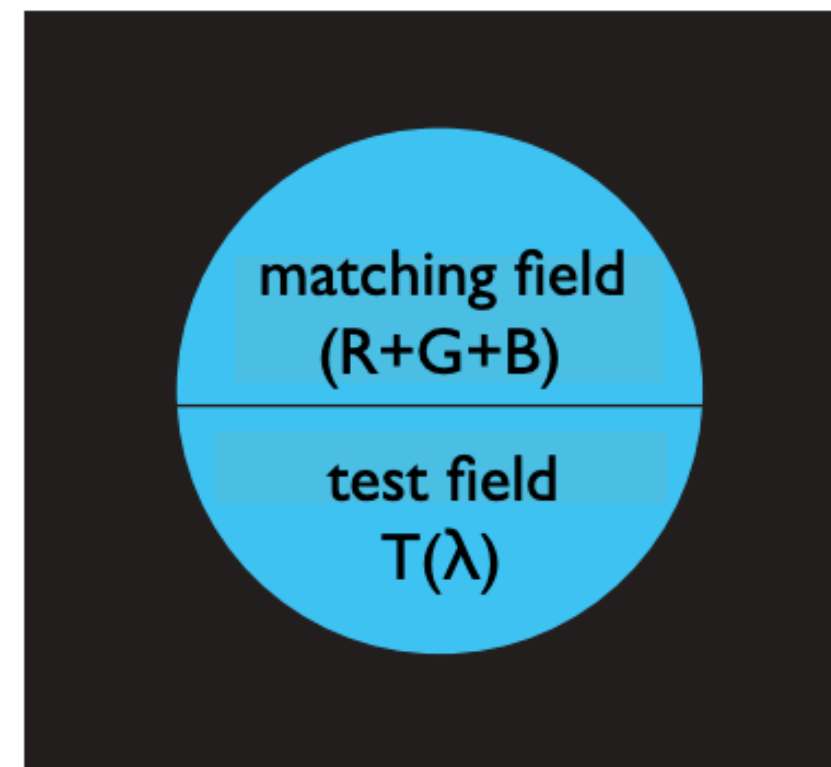
$$T(480) = -0.04R + 0.04G + 0.14B$$



Color matching experiment



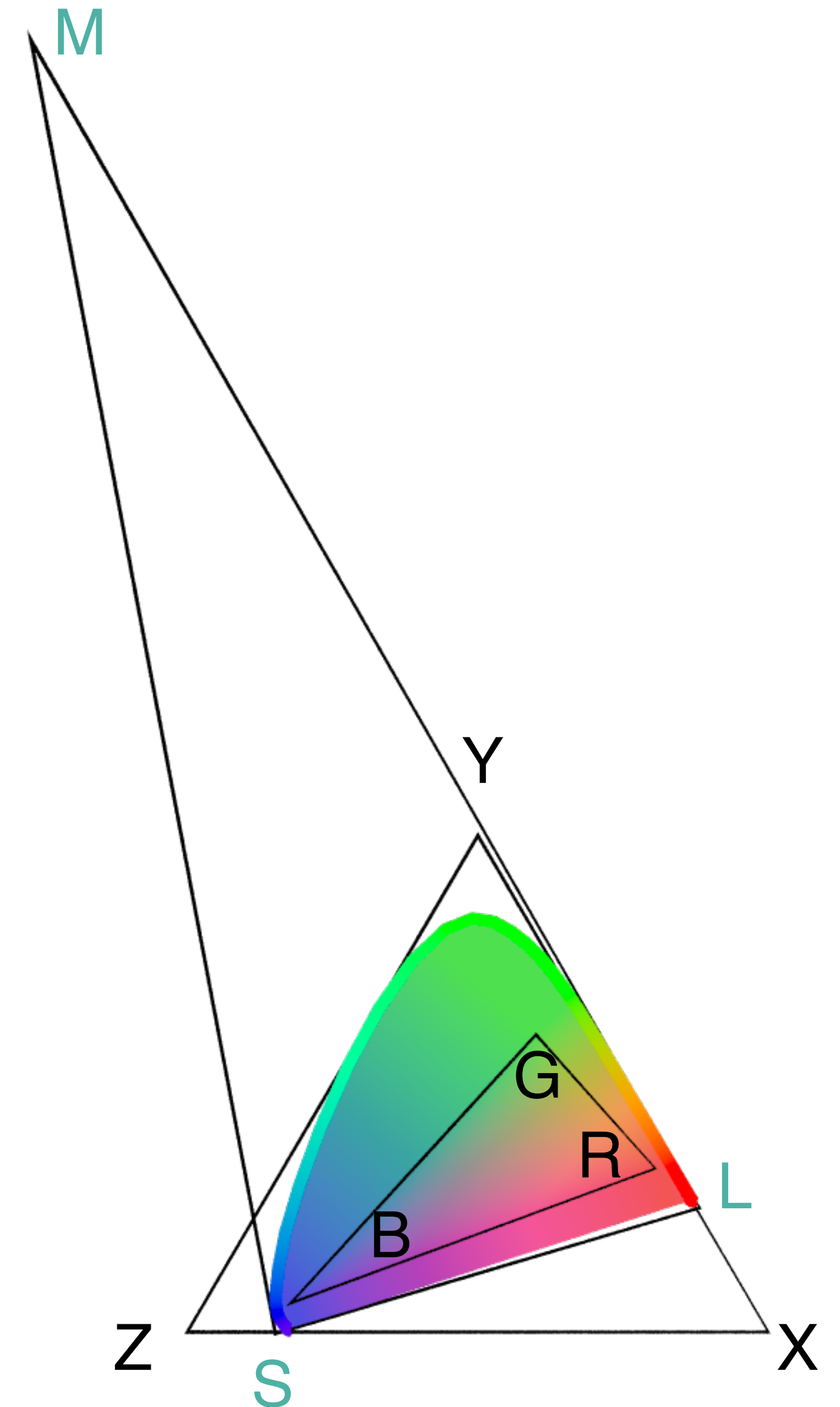
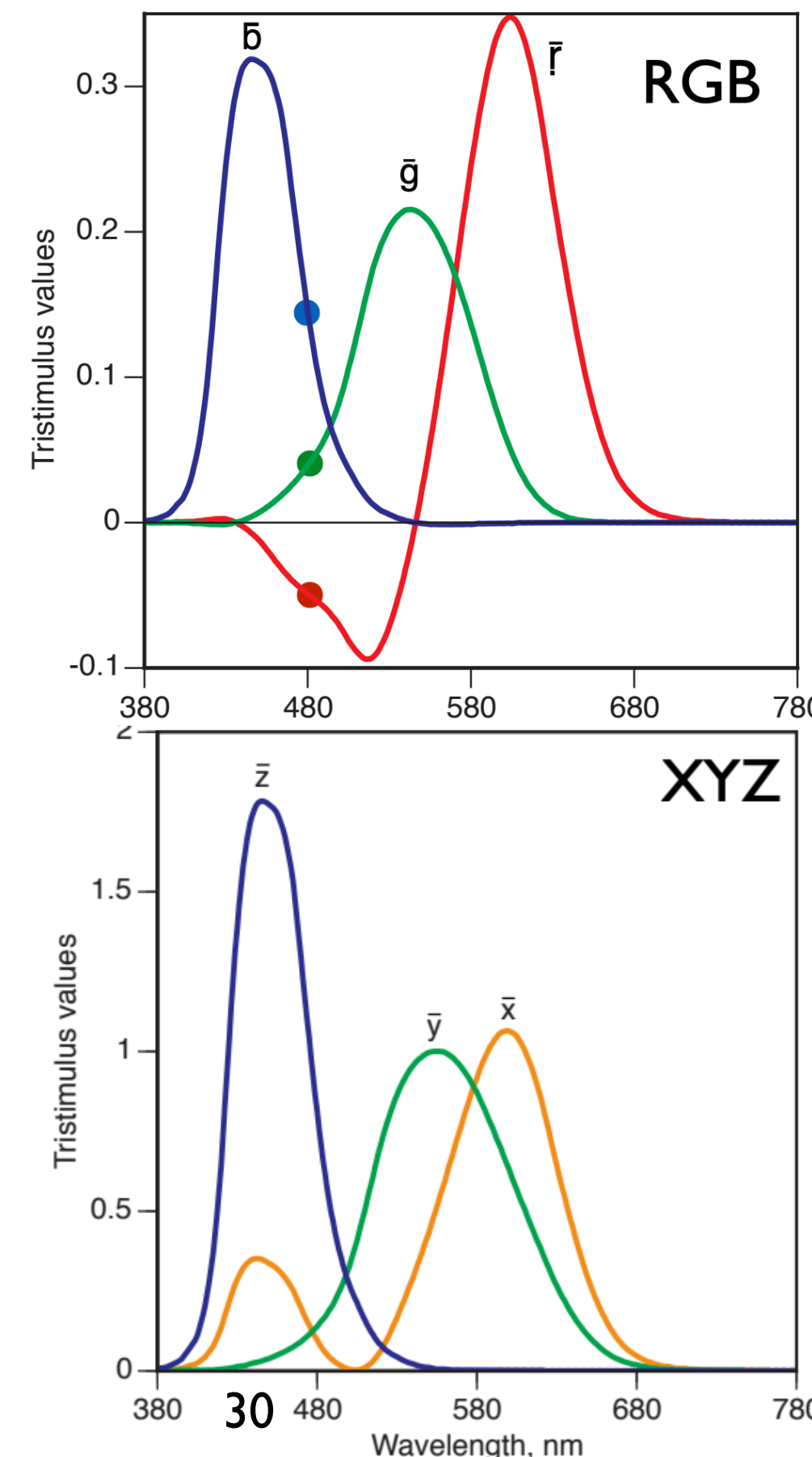
$$T(\lambda) = rR + gG + bB$$
$$T(480) = -0.04R + 0.04G + 0.14B$$



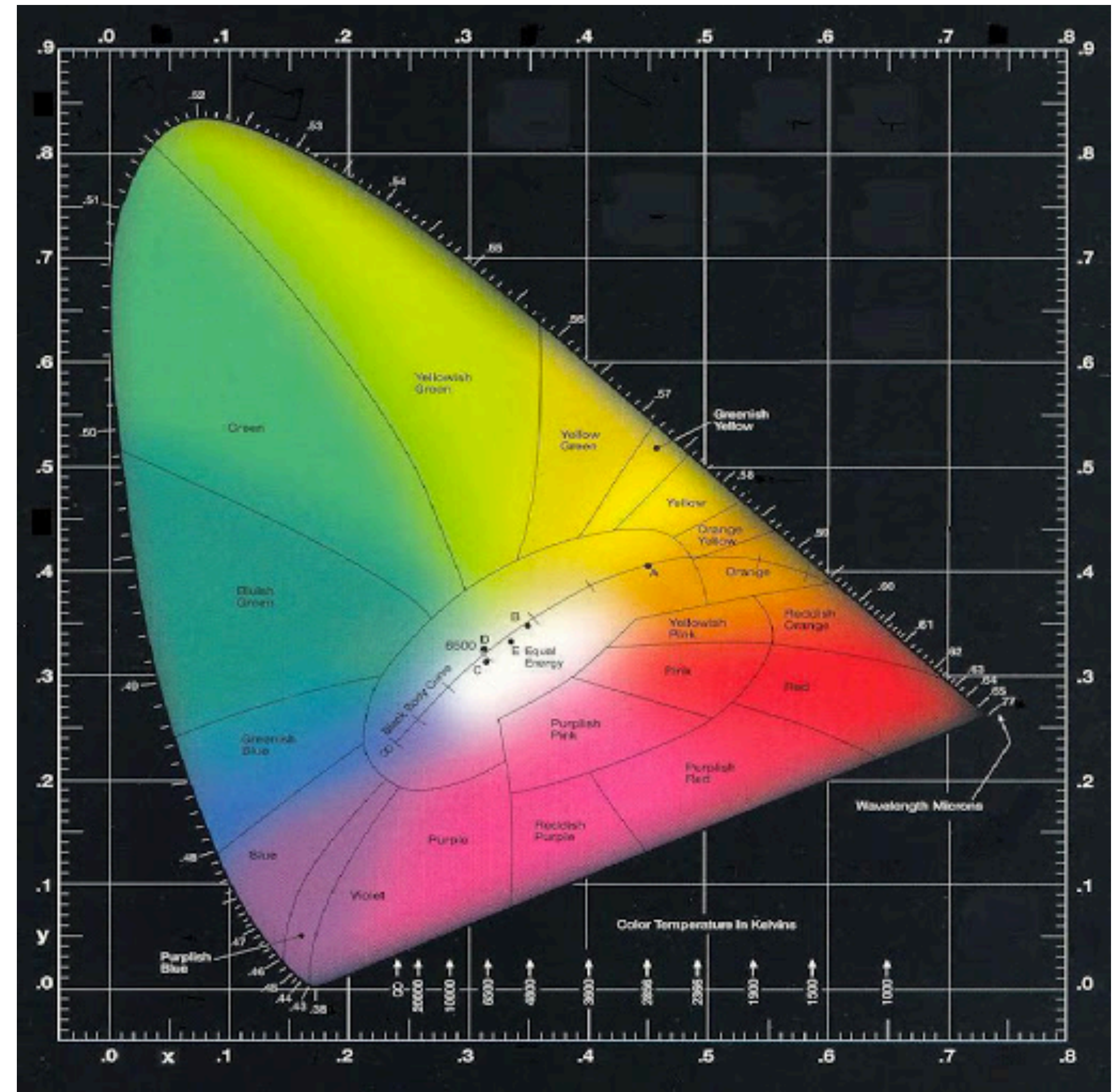
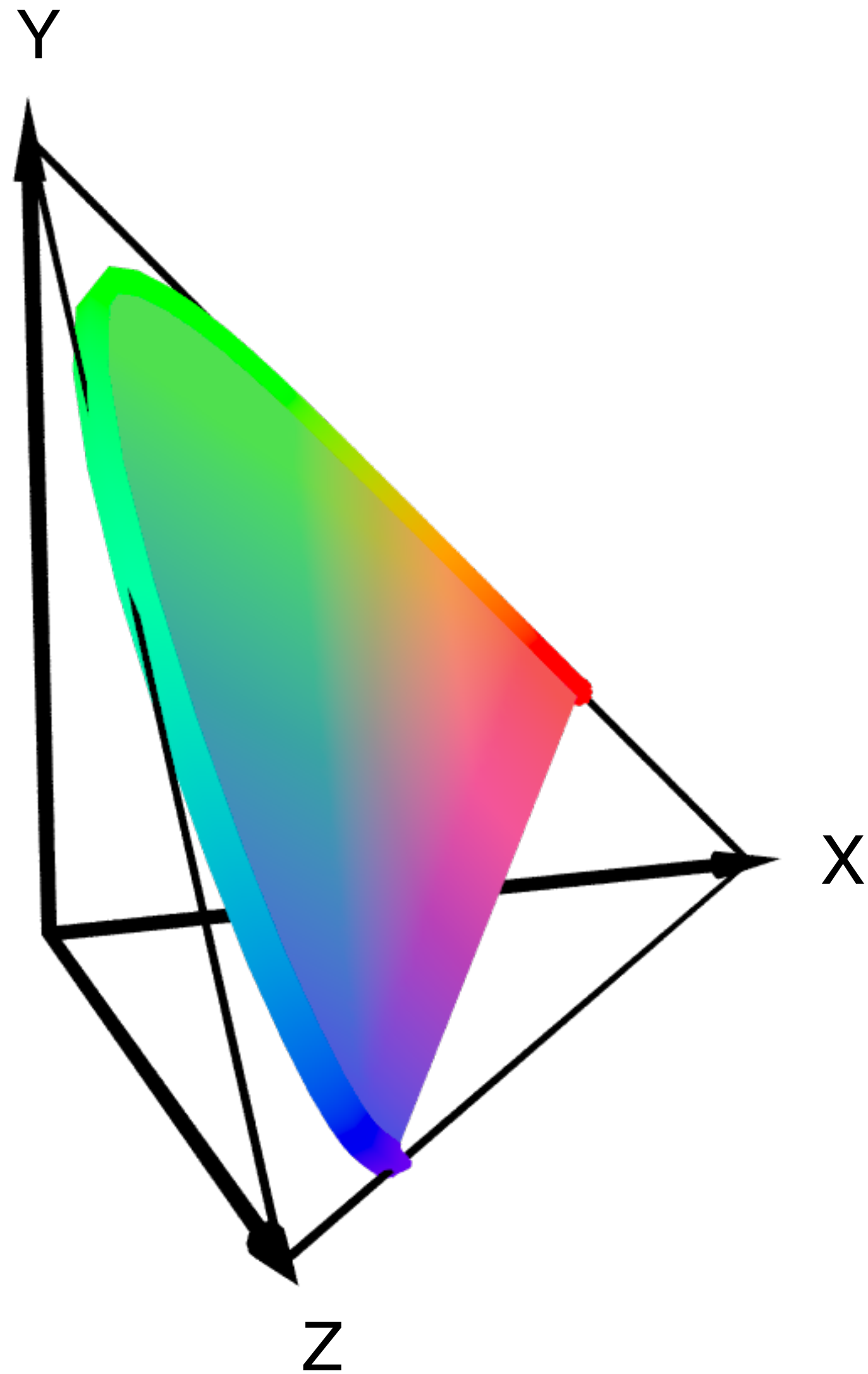
CIE 1931 XYZ color space

- In 1931, CIE (International Commission on Illumination) proposed another basis based on 3 imaginary primaries X, Y, Z.

- ▶ Mixture of XYZ gives all color
- ▶ The corresponding matching functions $\bar{x}, \bar{y}, \bar{z}$ are linear transforms of $\bar{r}, \bar{g}, \bar{b}$
- ▶ $\bar{x}, \bar{y}, \bar{z}$ are nonnegative (equivalent to the first item above)
- ▶ $\bar{x}, \bar{y}, \bar{z}$ have equal areas under their curves.
- ▶ The y-coefficient $\int \bar{y}(\lambda)P(\lambda)d\lambda$ is defined to be the **luminance**.

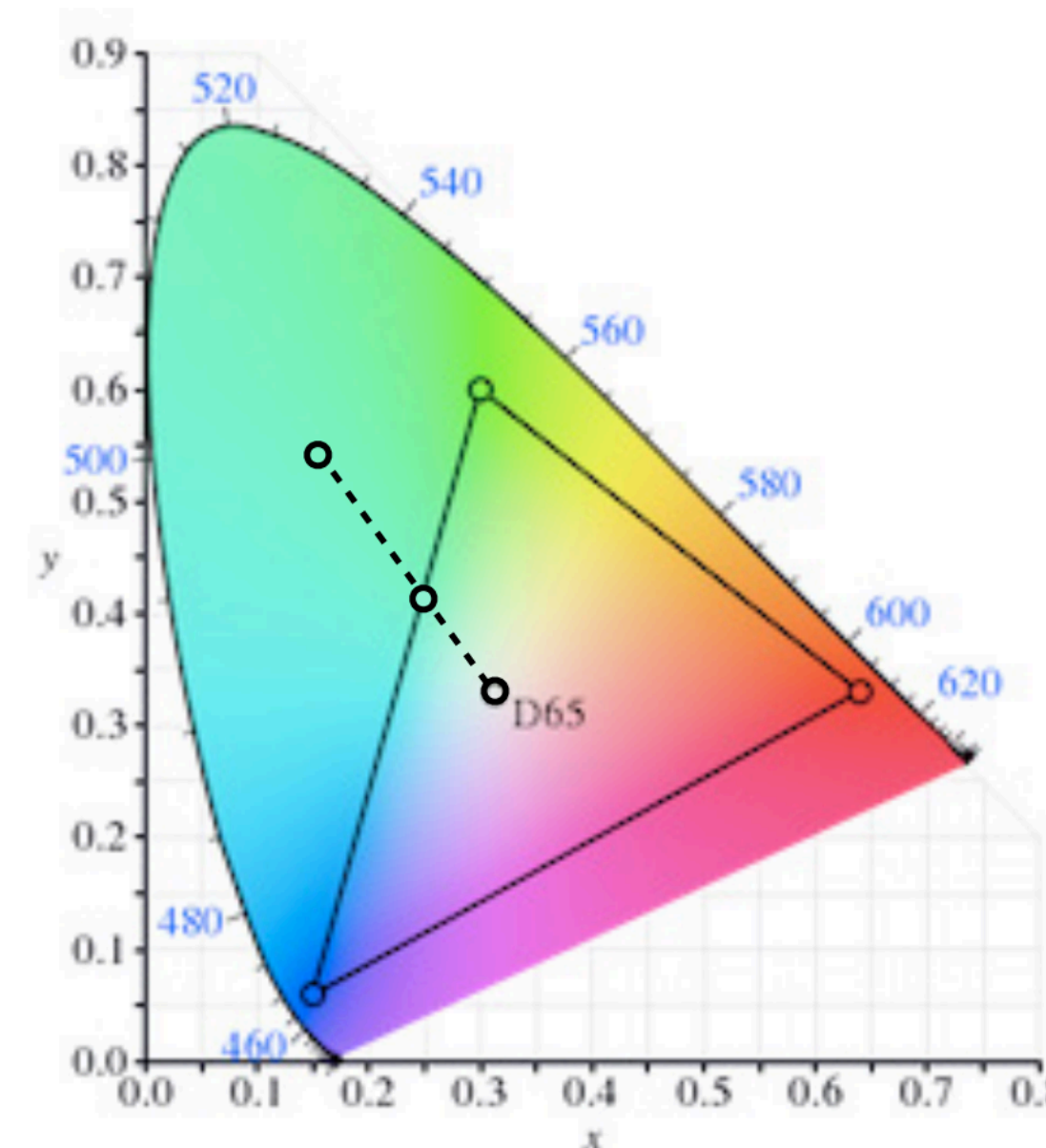


CIE 1931 XYZ color space



Color gamut

- Color gamut: The range of chromaticities that can be produced by mixing primaries.
- Real device gamut are limited, can't produce all chromaticities.
- Gamut mapping: Methods to approximate out-of-gamut chromaticities
 - ▶ Typically involves desaturating source chromaticities until they fall within the gamut boundaries.



sRGB Specification

Primary	x	y
Red	0.6400	0.3300
Green	0.3000	0.6000
Blue	0.1500	0.0600
White Point	0.3127	0.3290

Transparency

Transparency

Go to whiteboard



Next time

- Ray tracing

