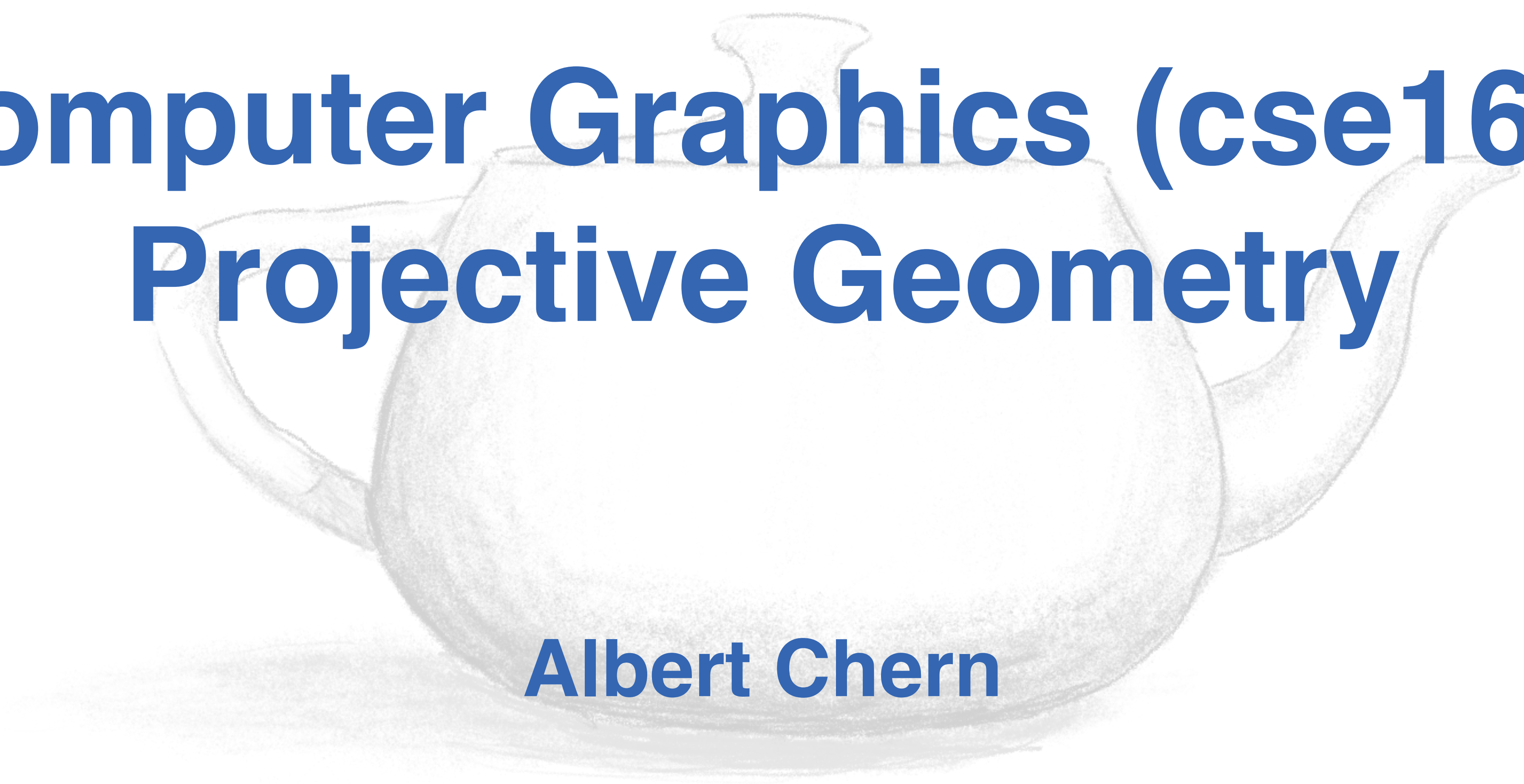




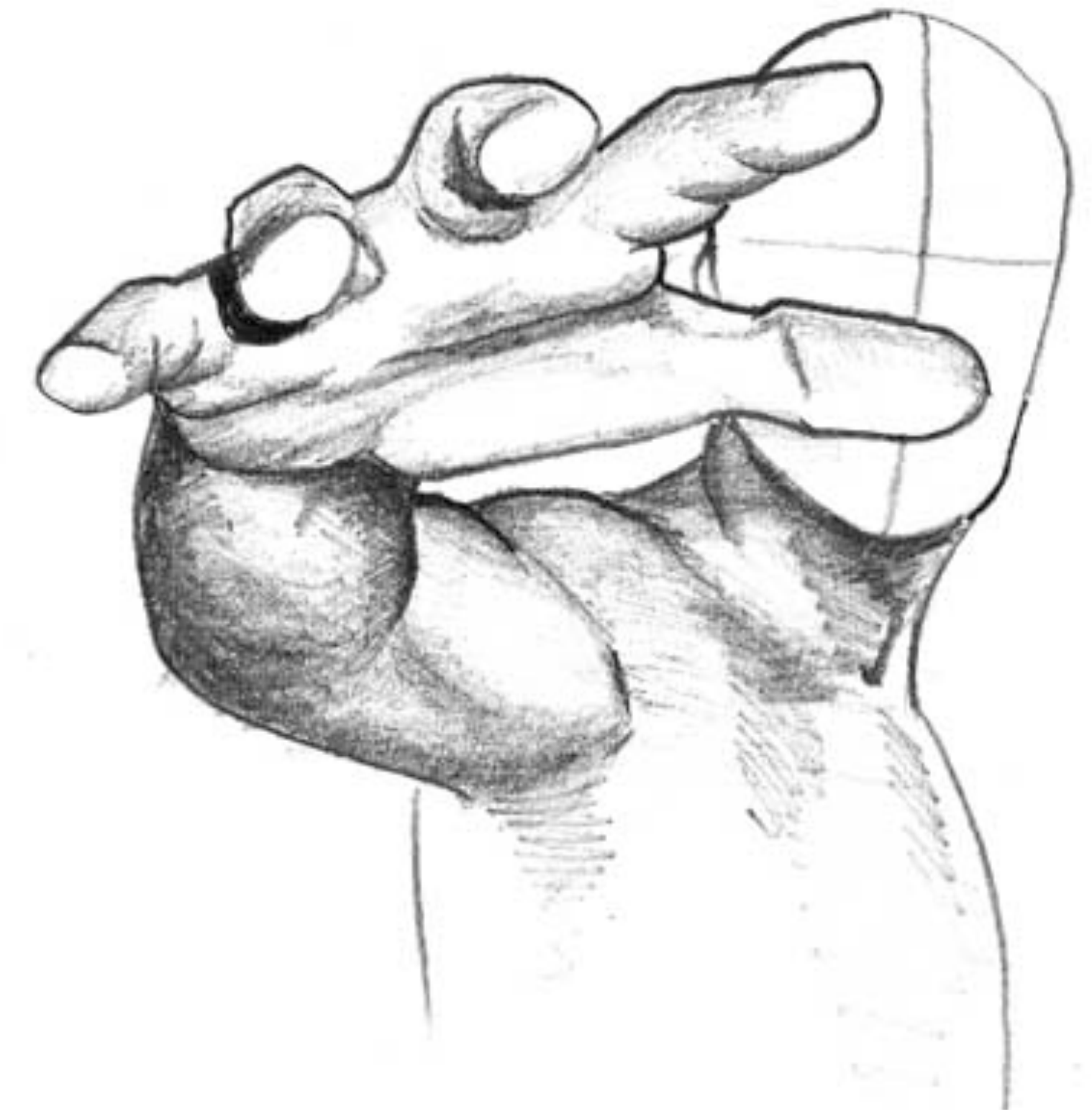
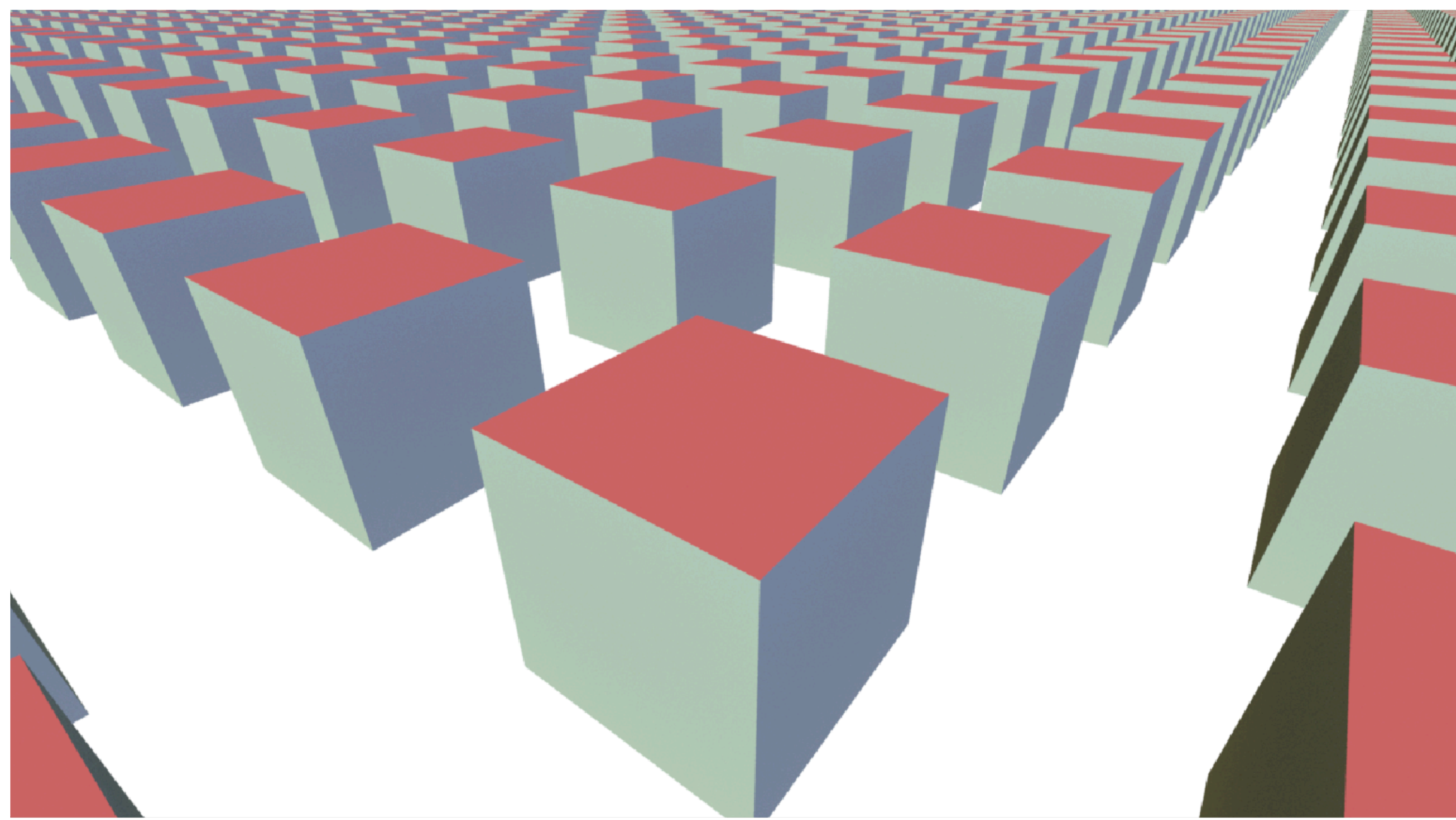
Computer Graphics (cse167) Projective Geometry

Albert Chern

Perspective

Perspective

- Foreshortening
 - ▶ Further objects look smaller
 - ▶ The scaling is non-uniform

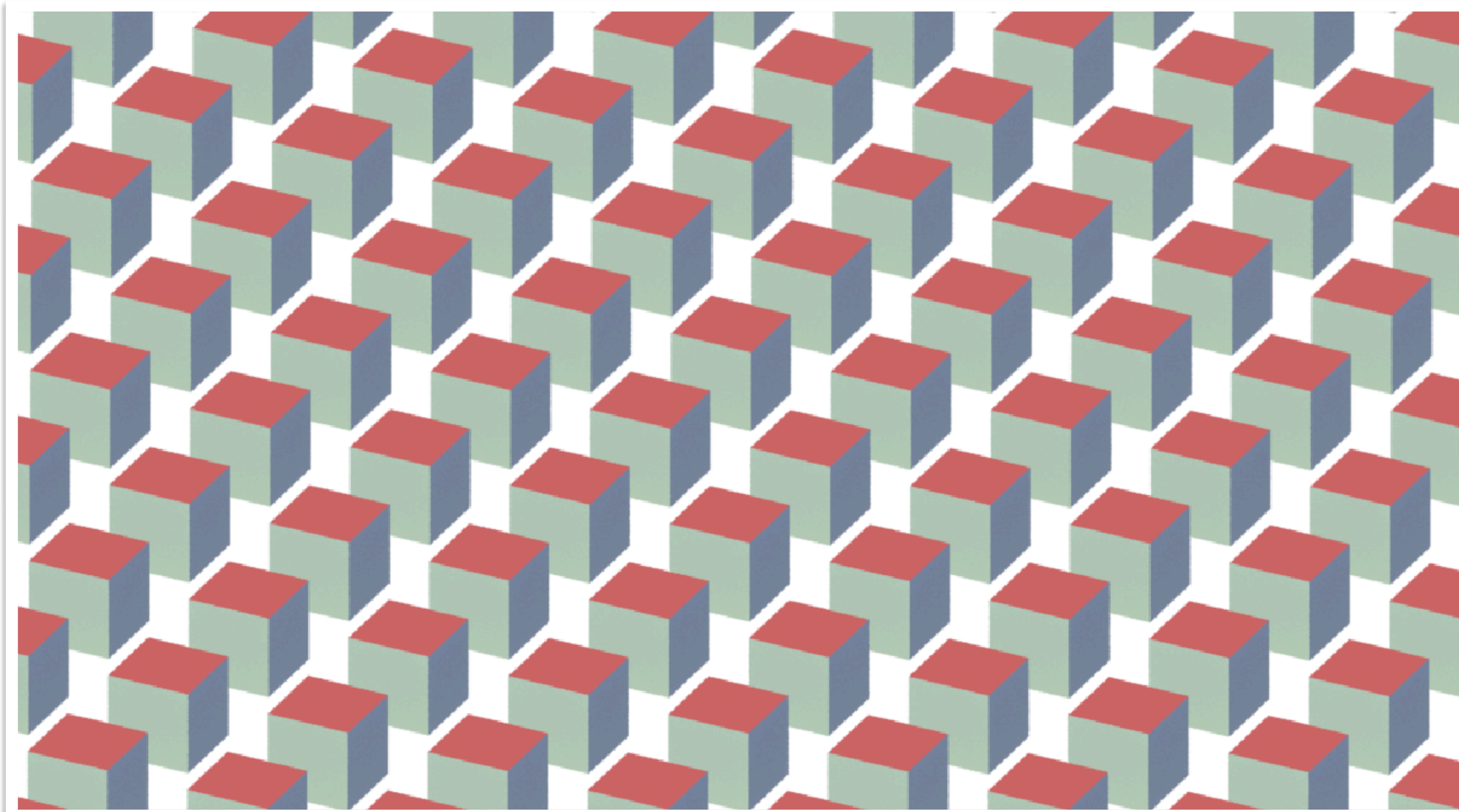


learn-to-draw.com

Perspective

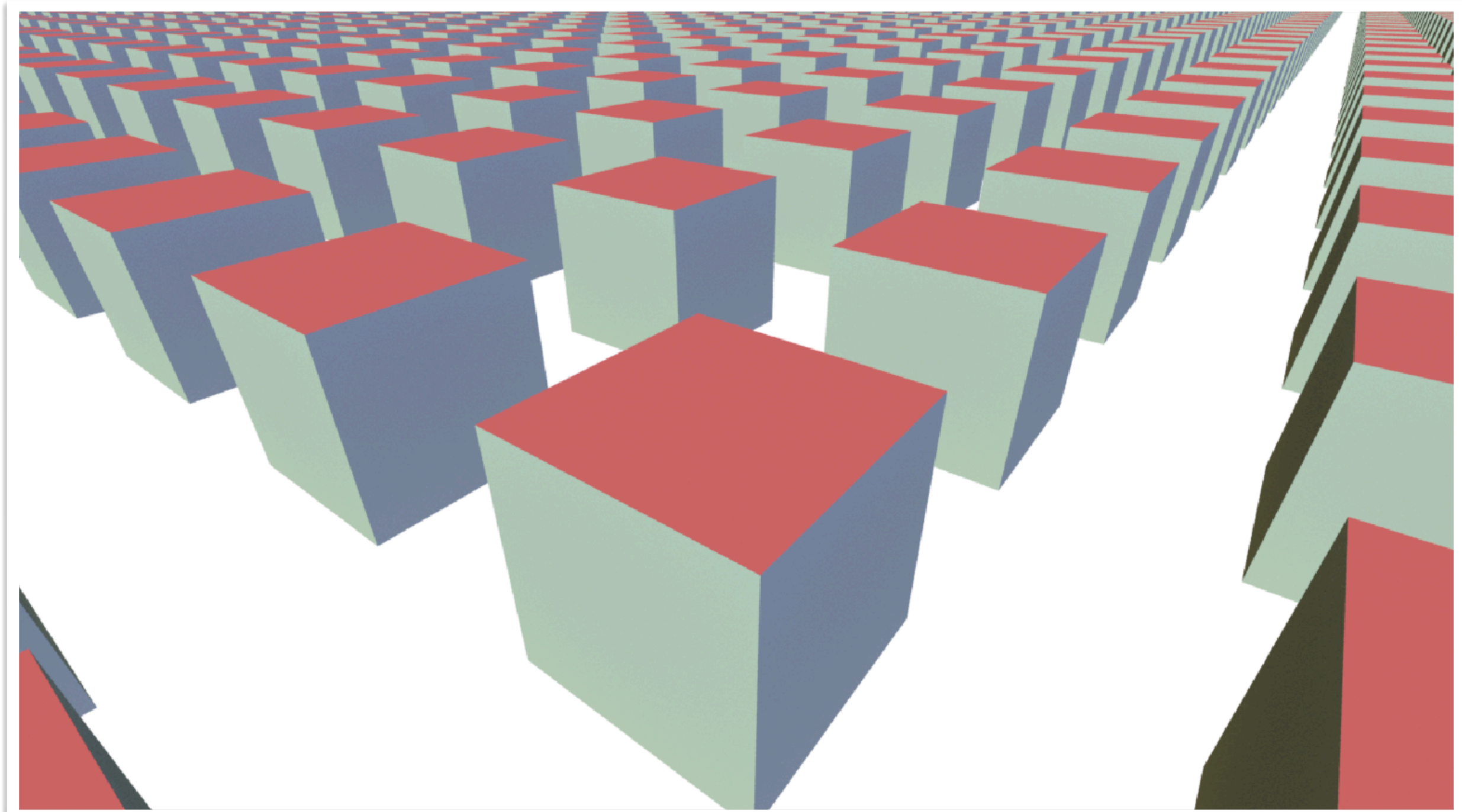
- Orthographic view

- ▶ Telescopic view standing infinitely far away.
- ▶ A view preserving parallelism, proportions
- ▶ Lack a sense of depth



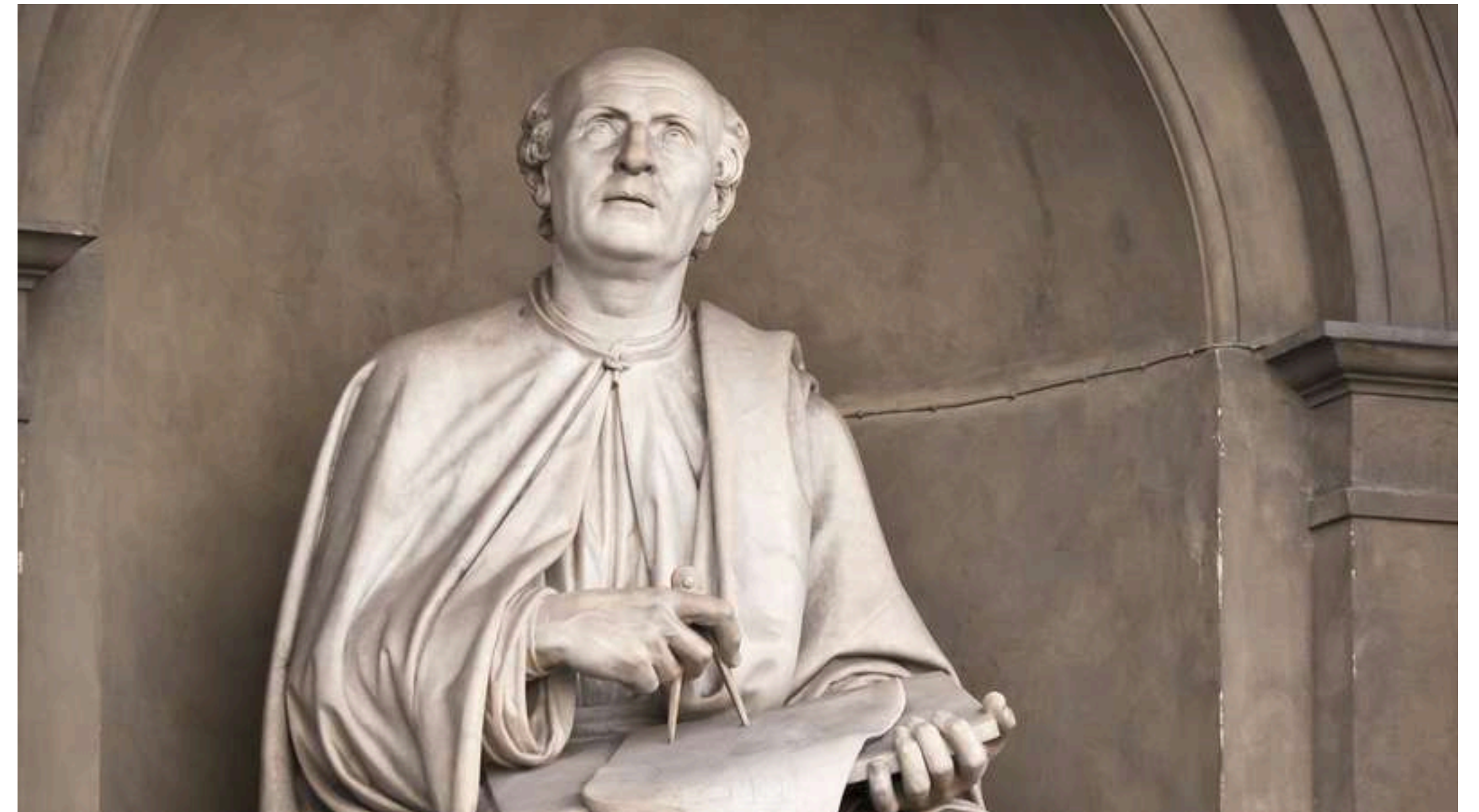
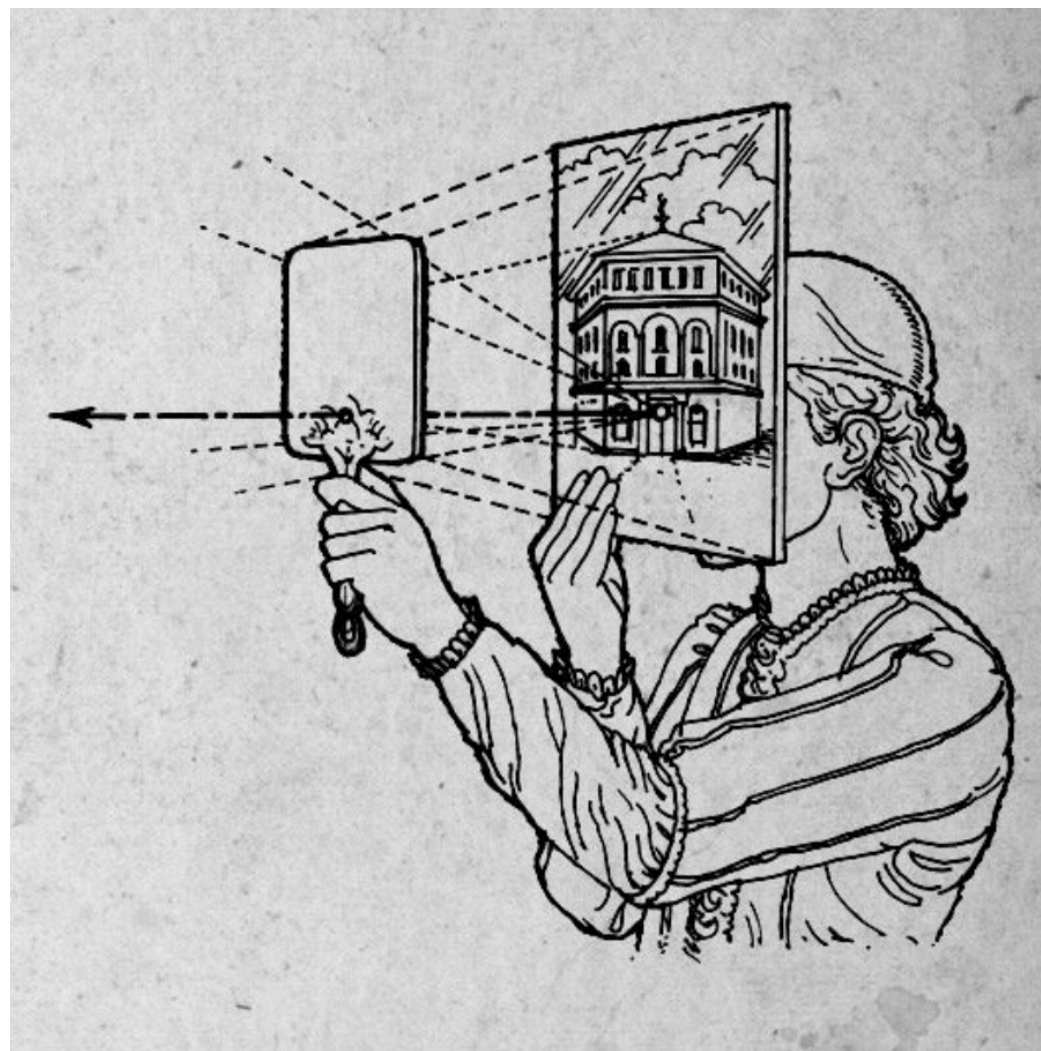
- Perspective view

- ▶ The vantage point is immersed in the scene.
- ▶ Does not preserve parallelism, proportions
- ▶ Provides a sense of depth



Renaissance Art*

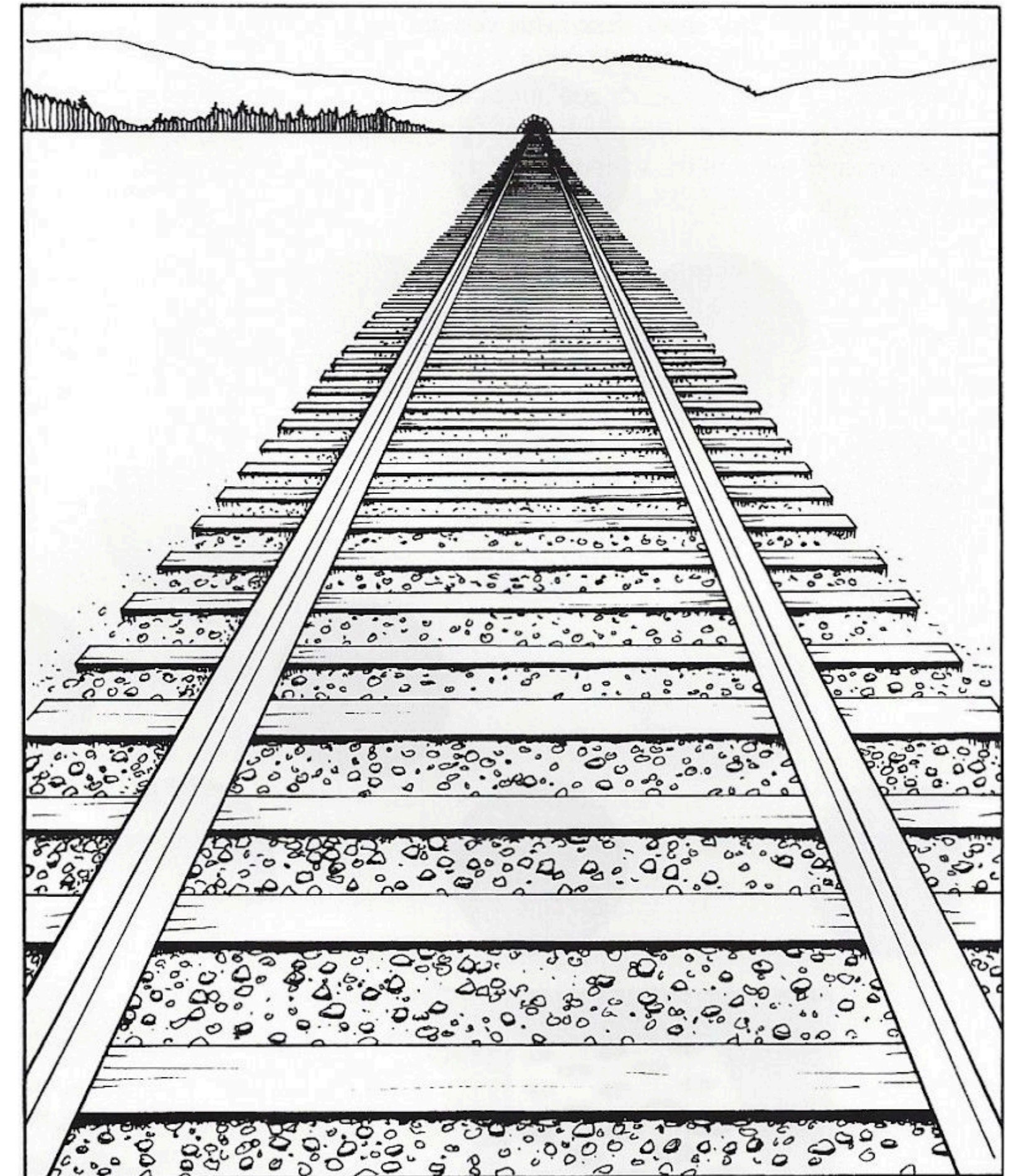
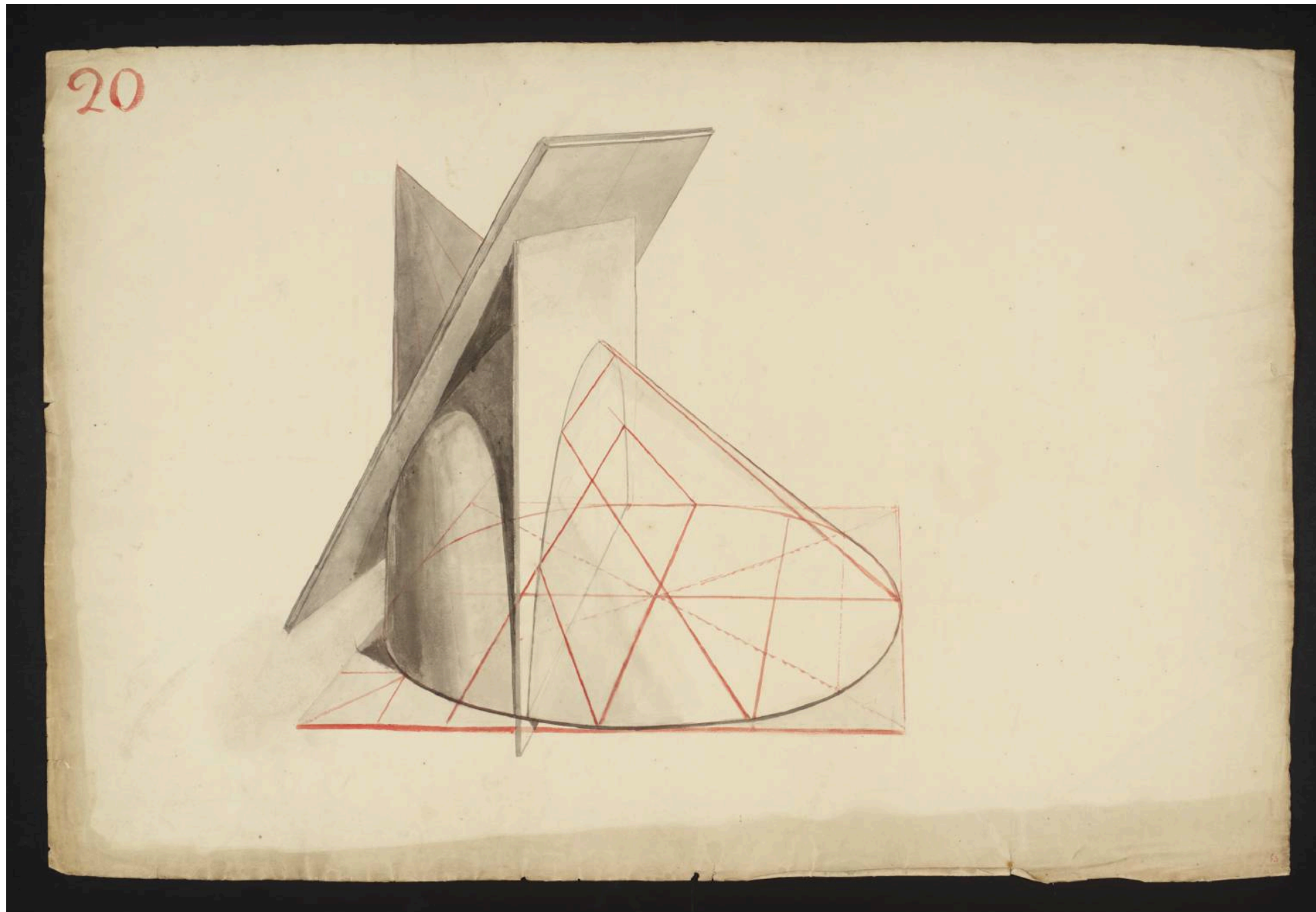
- Filippo Brunelleschi 1415: Linear perspective
 - ▶ 2D representation of a 3D object?
 - ▶ Found method accurate perspective drawing.
 - ▶ Soon after Brunelleschi, most of the artists in Florence adopted the method.



Filippo Brunelleschi 1337–1446

Renaissance Art*

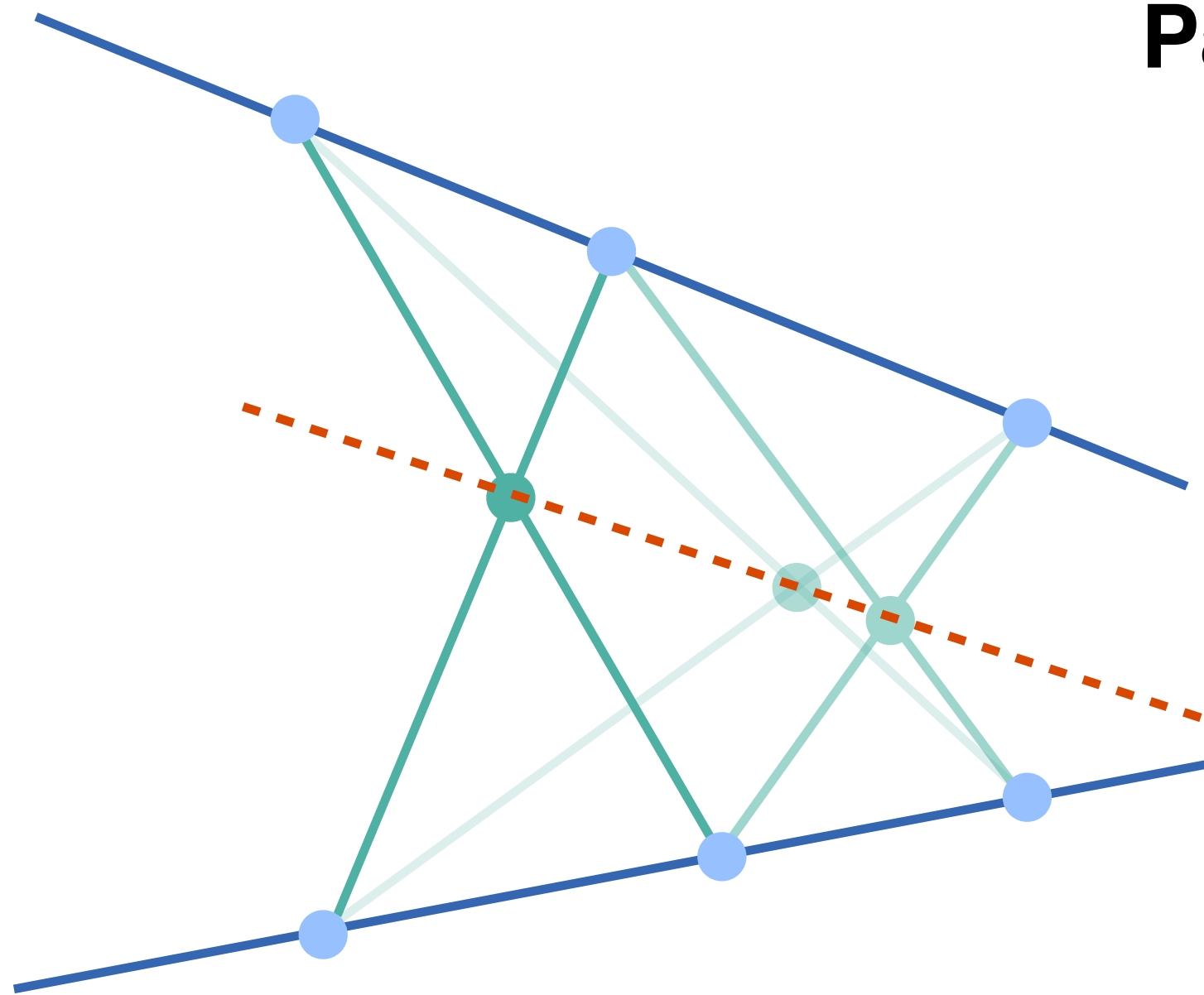
- The rules for representing 3D picture on a plane
 - ▶ Straight lines must be represented by straight lines.
 - ▶ Parallel lines meet at the horizon.
 - ▶ Image of a conic is a conic.



Inclusion of points at infinity*

- Girard Desargues 1600s

- ▶ Projective Geometry: Geometry of straight edges.
- ▶ There has been geometric theorems that involve only straight lines. (e.g. Pappus Theorem 300 AD)
- ▶ Insist that every two lines in the plane intersect at a point.



Pappus Theorem

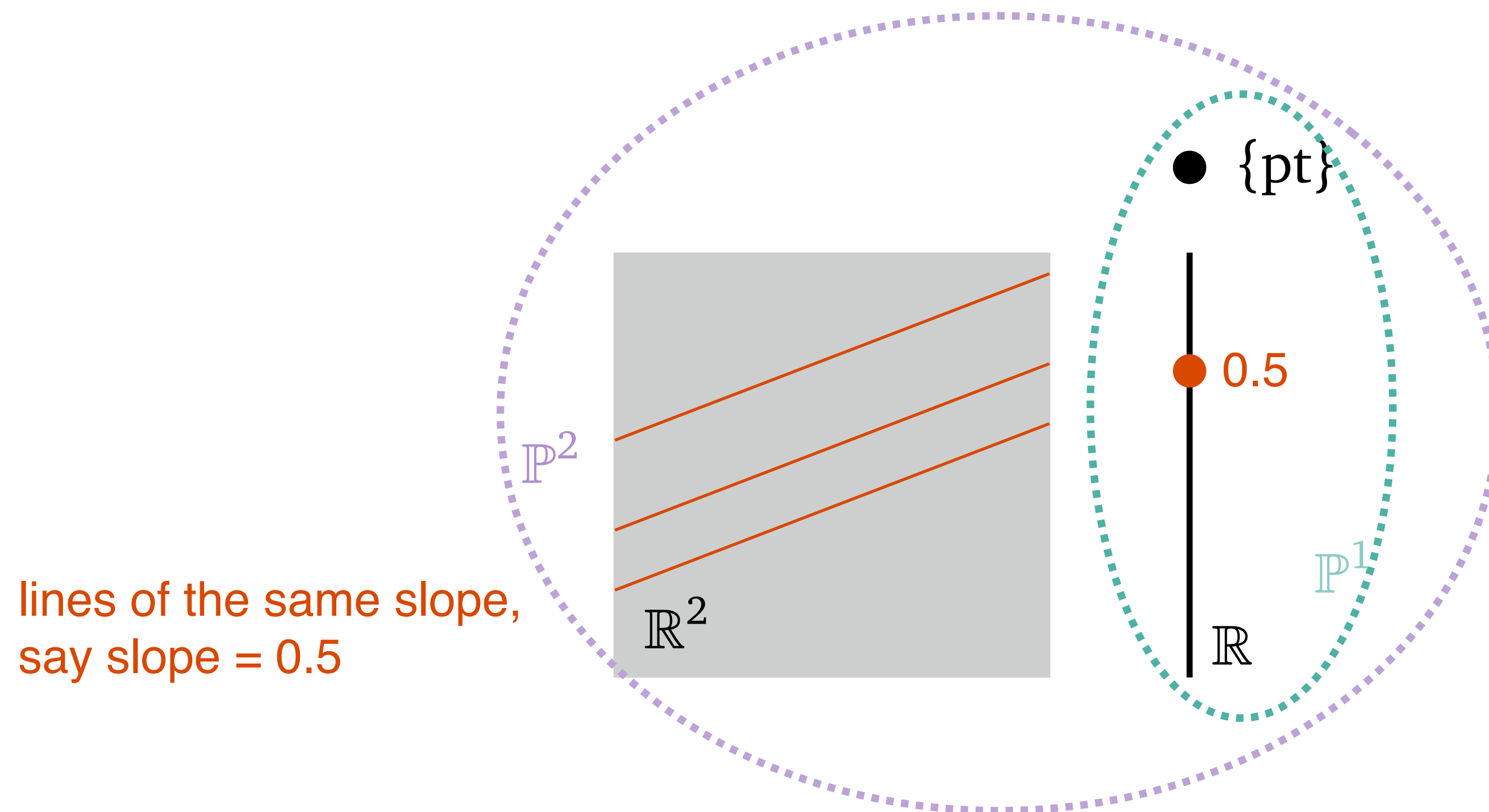
- ▶ Given two arbitrary lines, take 3 arbitrary points on each line.
- ▶ Find the intersections of the lines joining the crossing pairs.
- ▶ Then these intersections will always be collinear.



Girard Desargues 1591–1661

Inclusion of points at infinity*

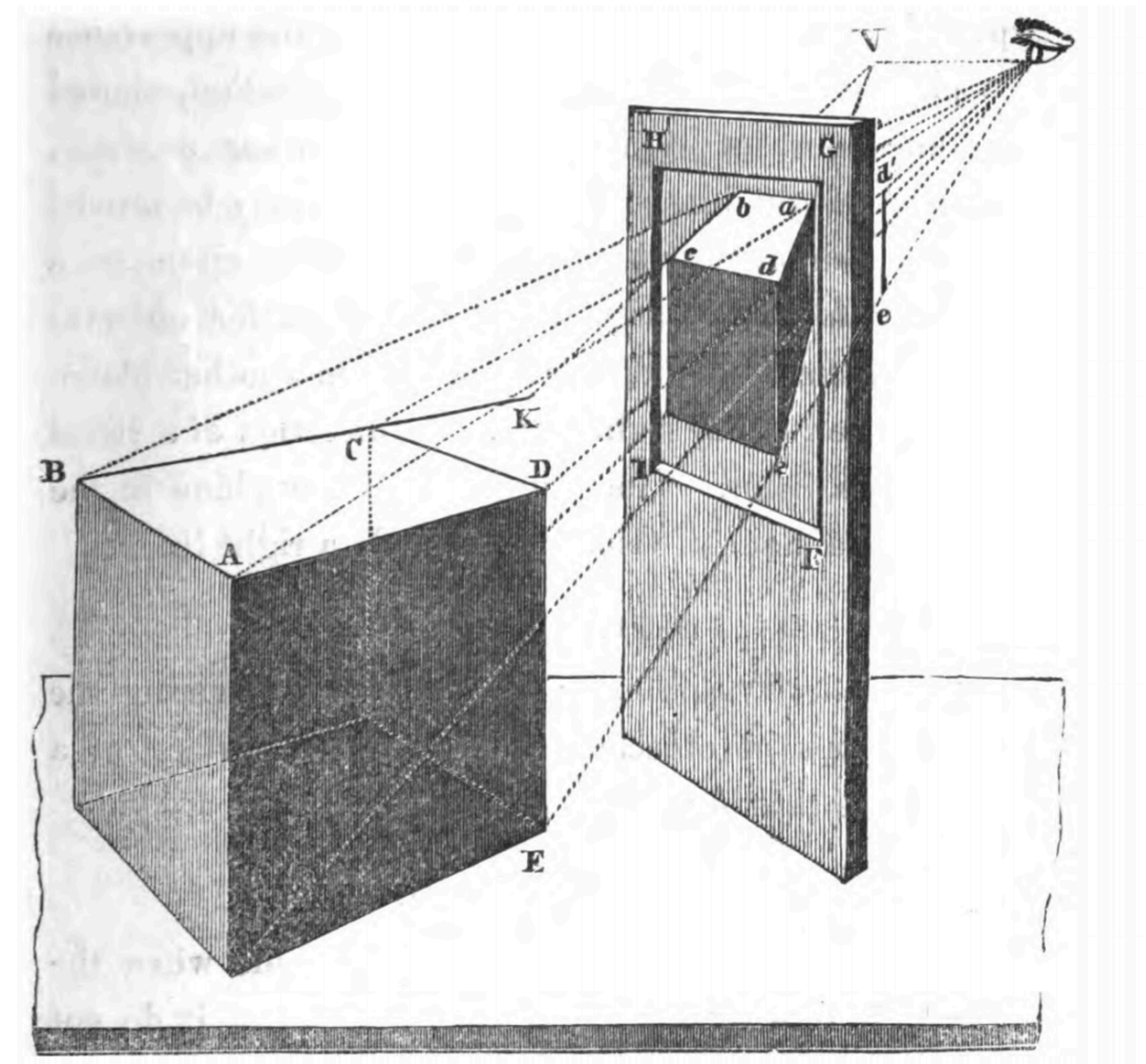
- ▶ A **projective plane** is the standard plane union a projective line of infinity.
- ▶ A **projective line** is a standard line union a point of infinity.
- ▶ Parallel lines in the plane meet at a point the line of infinity.
- ▶ Parallel lines of “infinite slope” meet at the “additional point” of the projective line of infinity.



Taylor's Principles of Linear Perspective

- Relation between the 3D object and 2D image

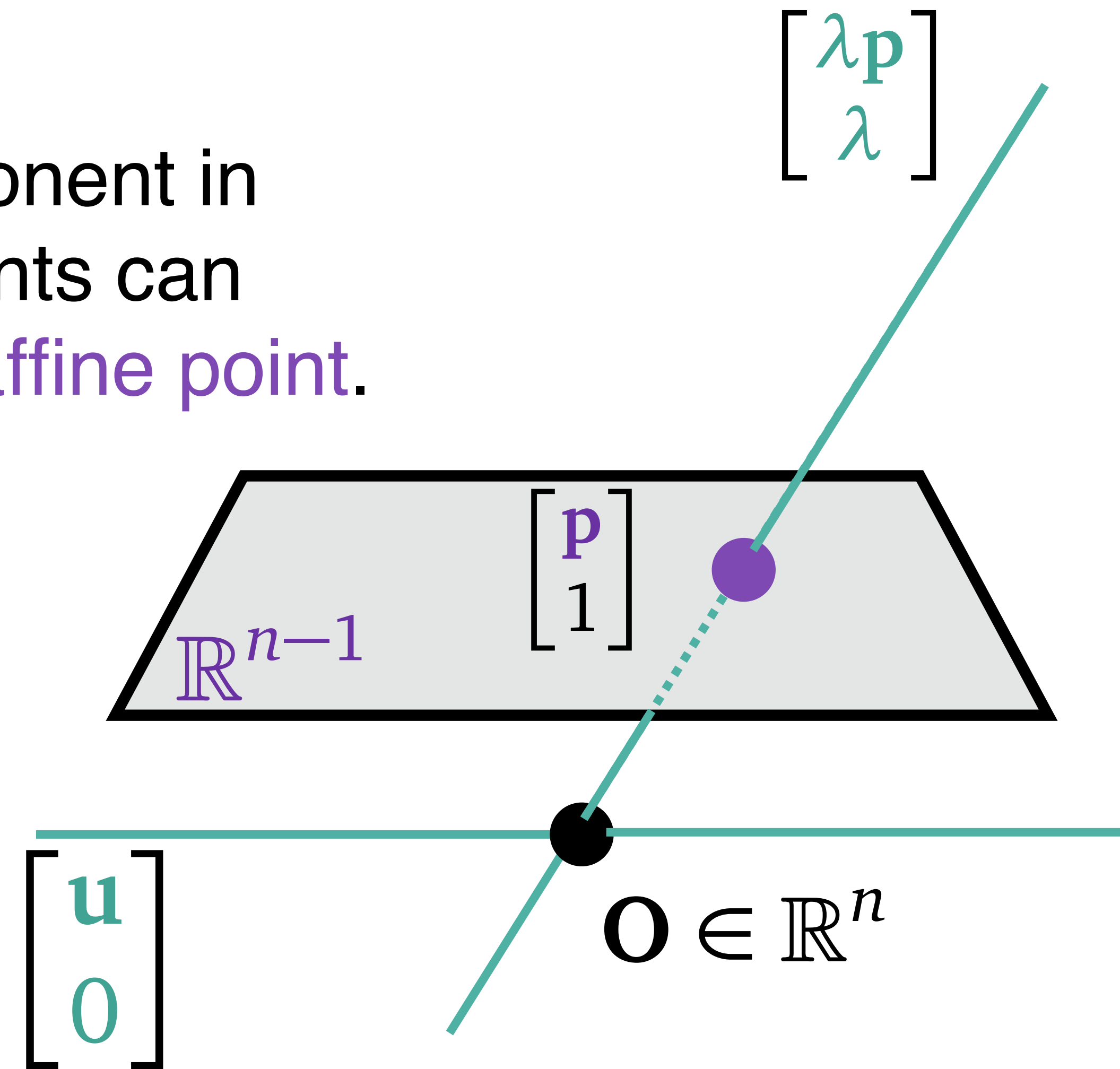
- ▶ Join line from a fixed point (eye) to the 3D object.
- ▶ Intersect the line on the image plane.
- ▶ The image plane is thought of as Desargues' projective plane where geometry of straight edges take place.
- ▶ Each **point** on the image plane is a line in 3D through the origin (eye).
- ▶ A line in 3D through the origin is a **3-proportion**.



Projective Space in Homogeneous Coordinates

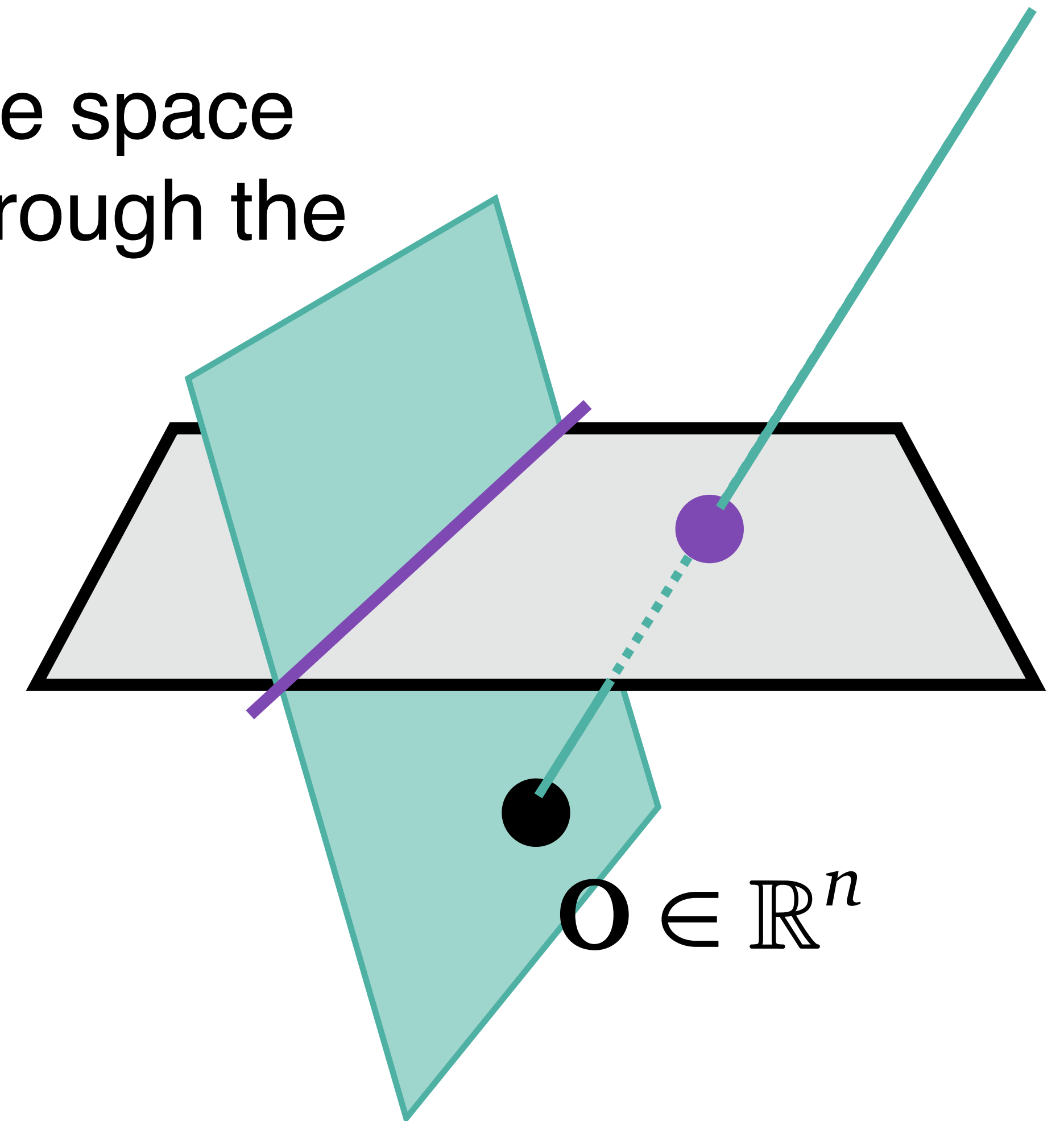
Homogeneous Coordinate

- A **point** in an $(n - 1)$ -dimensional projective space is an **n-proportion**.
- We call the point **finite** if the last component in the n-proportion is non-zero. Finite points can be dehomogenized and viewed as an **affine point**.
- A point is **infinite** if the last component is zero.



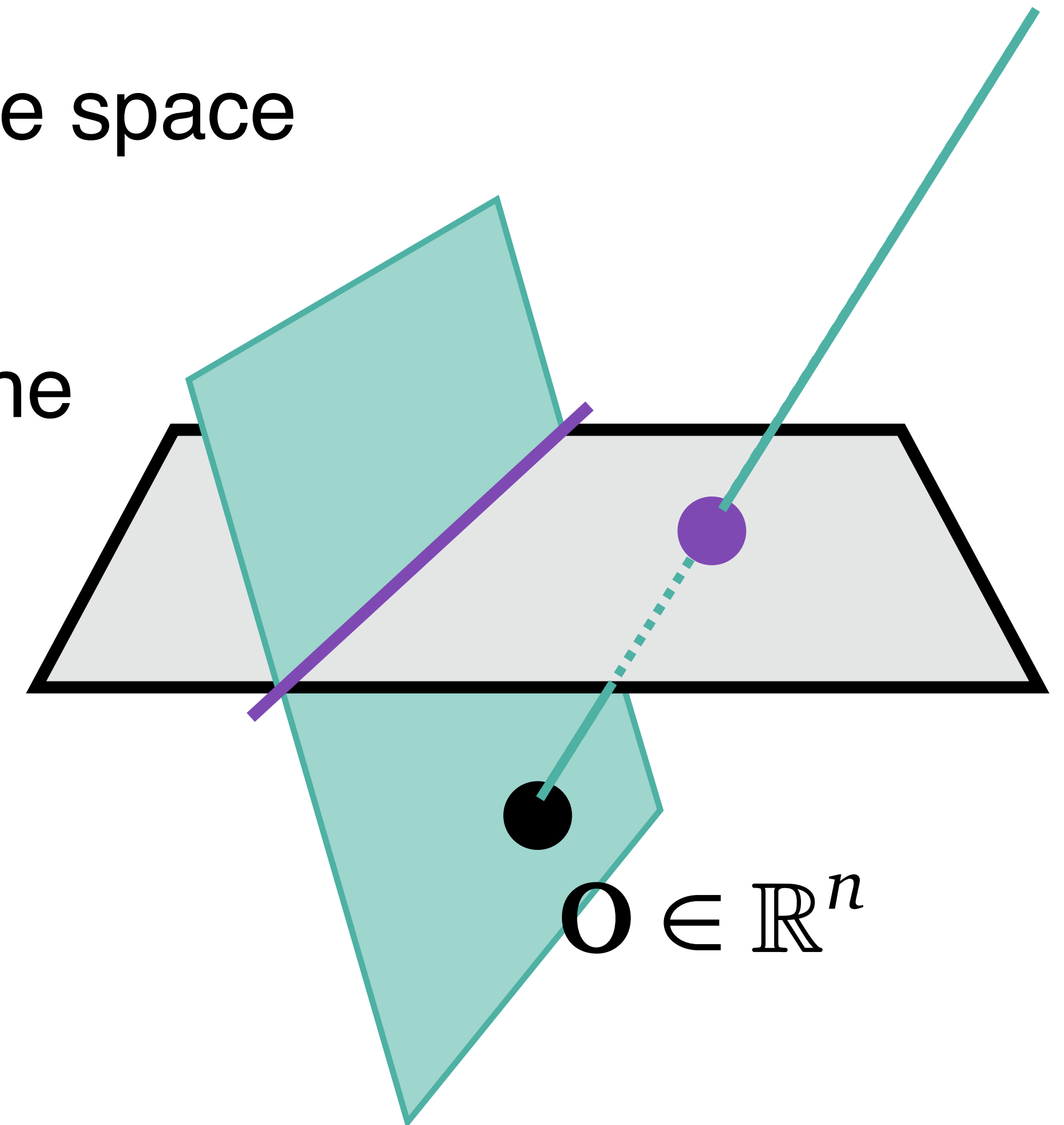
Homogeneous Coordinate

- A **point** in an $(n - 1)$ -dimensional projective space is an **n-proportion**.
- A **line** in this $(n - 1)$ -dimensional projective space is a 2D plane in the n -D space passing through the origin.



Homogeneous Coordinate

- A **point** in an $(n - 1)$ -dimensional projective space is a **1D subspace in nD** .
- A **line** in this $(n - 1)$ -dimensional projective space is a **2D subspace in nD** .
- The **point** lies on (incident to) the **line** if the corresponding 1D subspace is contained in the corresponding 2D subspace.

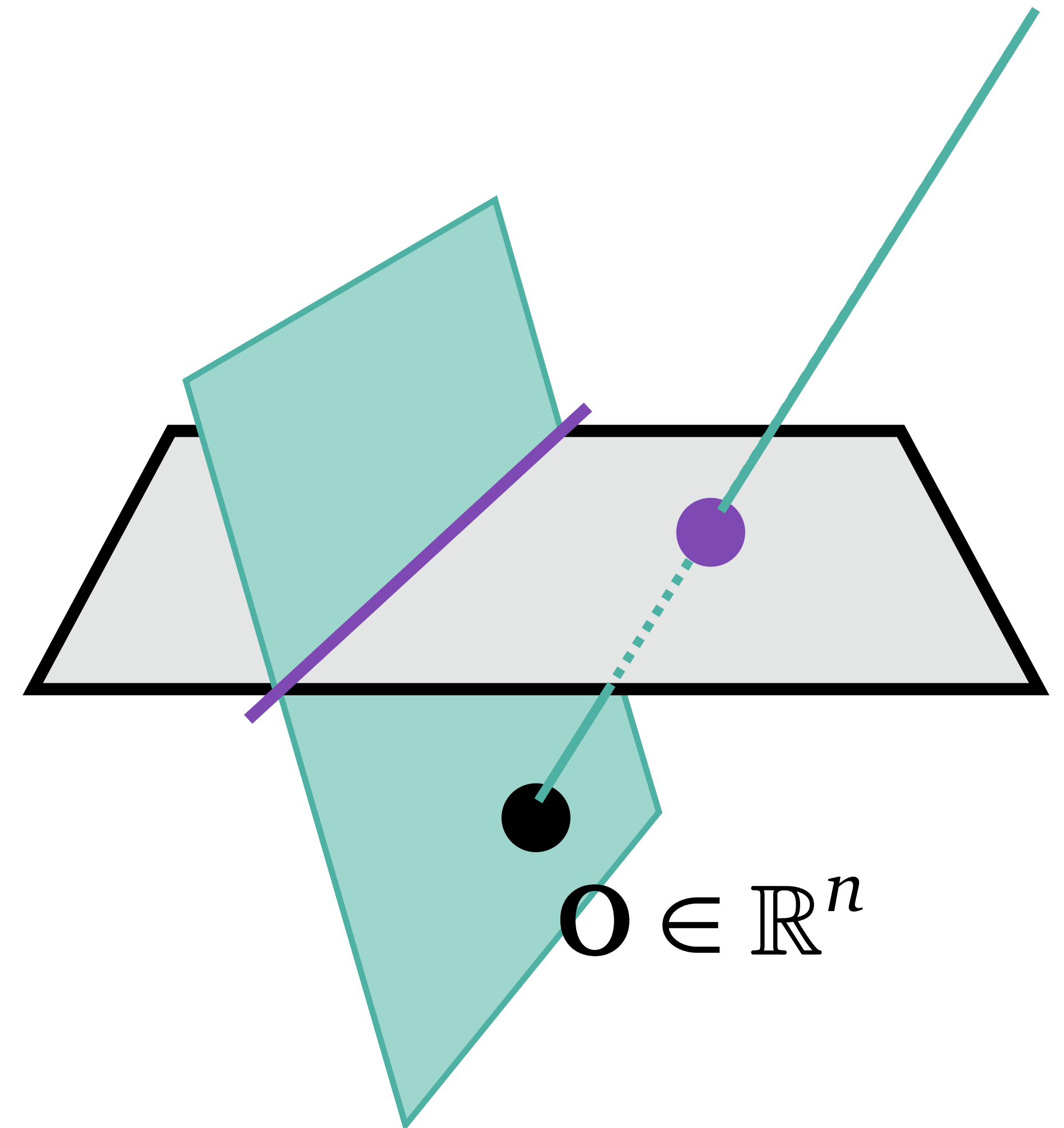


Projective Transformations

Projective Transformation

- A **projective transformation on the projective space** is a **linear transformation on the homogeneous coordinates**.
- Since linear transformations preserve the origin, a projective transformation maps points to points, lines to lines, and preserves all incidence relations.
- Affine transformations are special projective transformations that preserves the infinity.
 - ▶ Thus affine transformation preserves parallelism.

$$\begin{bmatrix} A & \mathbf{b} \\ 0 & 1 \end{bmatrix}$$



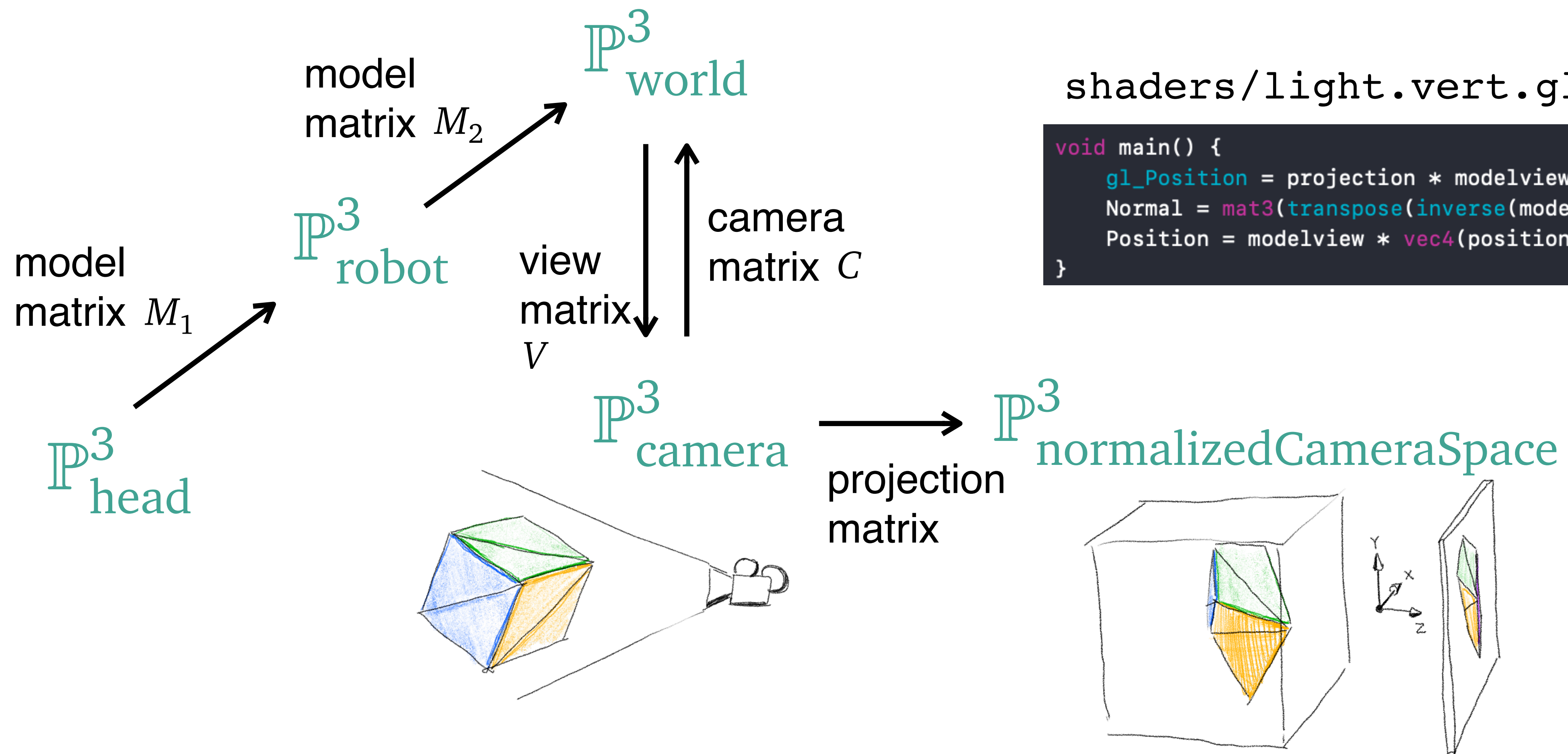
Projective Transformation

- Affine transformations are special projective transformations that preserves the infinity.
 - ▶ Thus affine transformation preserves parallelism.
- A general projective transformation can map a square to an arbitrary quadrilateral.
- In 3D projective space, a projective transformation can map a cube to a frustum.

$$\begin{bmatrix} A & \mathbf{b} \\ 0 & 1 \end{bmatrix}$$

Projective Transformation

- We will use a non-affine projective transformation for the perspective view.



shaders/light.vert.glsl

```
void main() {  
    gl_Position = projection * modelview * vec4(position, 1.0f);  
    Normal = mat3(transpose(inverse(modelview))) * normal;  
    Position = modelview * vec4(position, 1.0f);  
}
```

Next time

- Build the perspective projection matrix
- Basic OpenGL

