Additional Topics

Computer Vision II

CSE 252B

Lecture 18

Announcements

- Final exam, Mar 16, 7:00 PM-9:59 PM
  - Only for CSE graduate students taking the course-hosted comprehensive exam
- Assignment 5 is due Mar 18, 11:59 PM
- Please complete course and TA evaluations
  - Must be completed by Mar 14, 8:00 AM
- Reading
  - Sections 2.7, 7.4, 8.5, 8.6, 8.7, 8.8, and A6.8, and chapter 19

Additional topics

- Configurations of geometric primitives
- Robust cost functions
- Auto-calibration
- Calibration, including lens distortion

Configurations of geometric primitives

Null space of a matrix

- (Right) null space

The (right) null space of a matrix $\mathtt{A} \in \mathbb{R}^{m \times n}$ is the set of all $\mathbf{x} \in \mathbb{R}^n$ for which

$$\mathtt{A}\mathbf{x} = \mathbf{0}$$

The matrix $\mathtt{Z} = \text{null}(\mathtt{A})$ such that

$$\mathtt{A}\mathtt{Z} = \mathtt{0}$$

where $\mathtt{Z} \in \mathbb{R}^{n \times (n-r)}$ with orthonormal columns and $r = \text{rank}(\mathtt{A}) \leq \min(m, n)$, is the orthonormal basis for the (right) null space of $\mathtt{A}$. The columns of $\mathtt{Z}$ span every $\mathbf{x}$ in the (right) null space of $\mathtt{A}$.

Configurations of geometric primitives

- 2D points

$$\begin{bmatrix} \mathbf{x}_1^\top \\ \vdots \\ \mathbf{x}_n^\top \end{bmatrix}$$

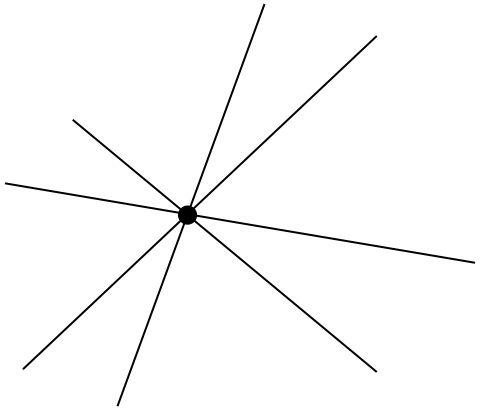
Rank	Configuration	Columns of null space
3	general position	empty
2	collinear	line that points are on
1	coincident	2 lines that intersect at point

Configurations of geometric primitives

- 2D lines

$$\begin{bmatrix} \boldsymbol{\ell}_1^\top \\ \vdots \\ \boldsymbol{\ell}_n^\top \end{bmatrix}$$

Rank	Configuration	Columns of null space
3	general position	empty
2	pencil of lines	point that lines intersect at
1	coincident	2 points on line



Pencil of lines

Configurations of geometric primitives

- 3D points

$$\begin{bmatrix} \mathbf{X}_1^\top \\ \vdots \\ \mathbf{X}_n^\top \end{bmatrix}$$

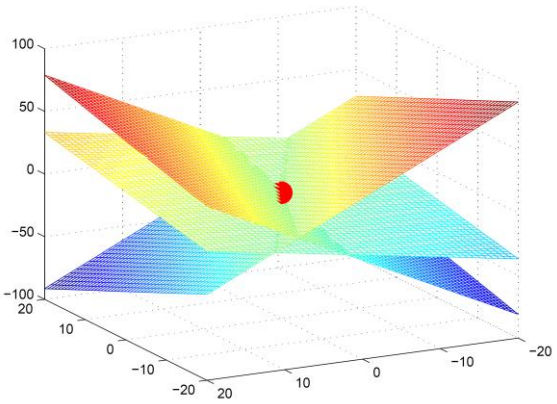
Rank	Configuration	Columns of null space
4	general position	empty
3	coplanar	plane that points are on
2	collinear	2 planes that intersect at line
1	coincident	3 planes that intersect at point

Configurations of geometric primitives

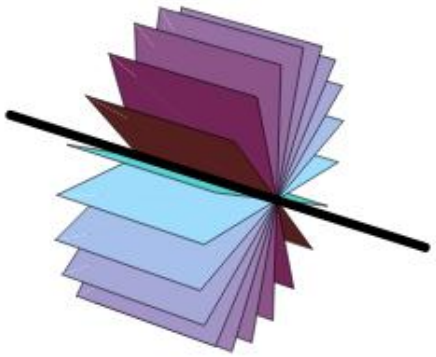
- 3D planes

$$\begin{bmatrix} \boldsymbol{\pi}_1^\top \\ \vdots \\ \boldsymbol{\pi}_n^\top \end{bmatrix}$$

Rank	Configuration	Columns of null space
4	general position	empty
3	bundle of planes	point that planes intersect at
2	sheaf of planes	2 points on line that planes intersect at
1	coincident	3 points on plane



Bundle of planes

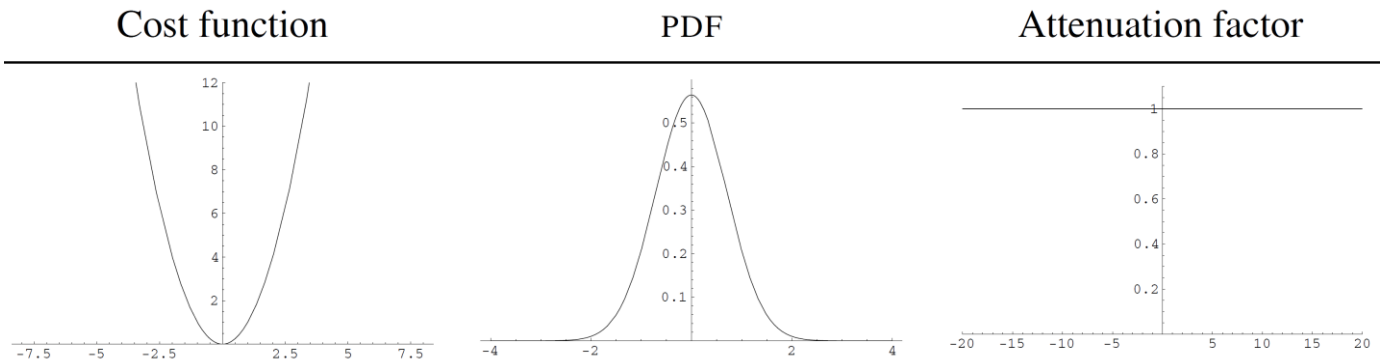


https://mathworld.wolfram.com/SheafofPlanes.html

Robust cost functions

Cost functions

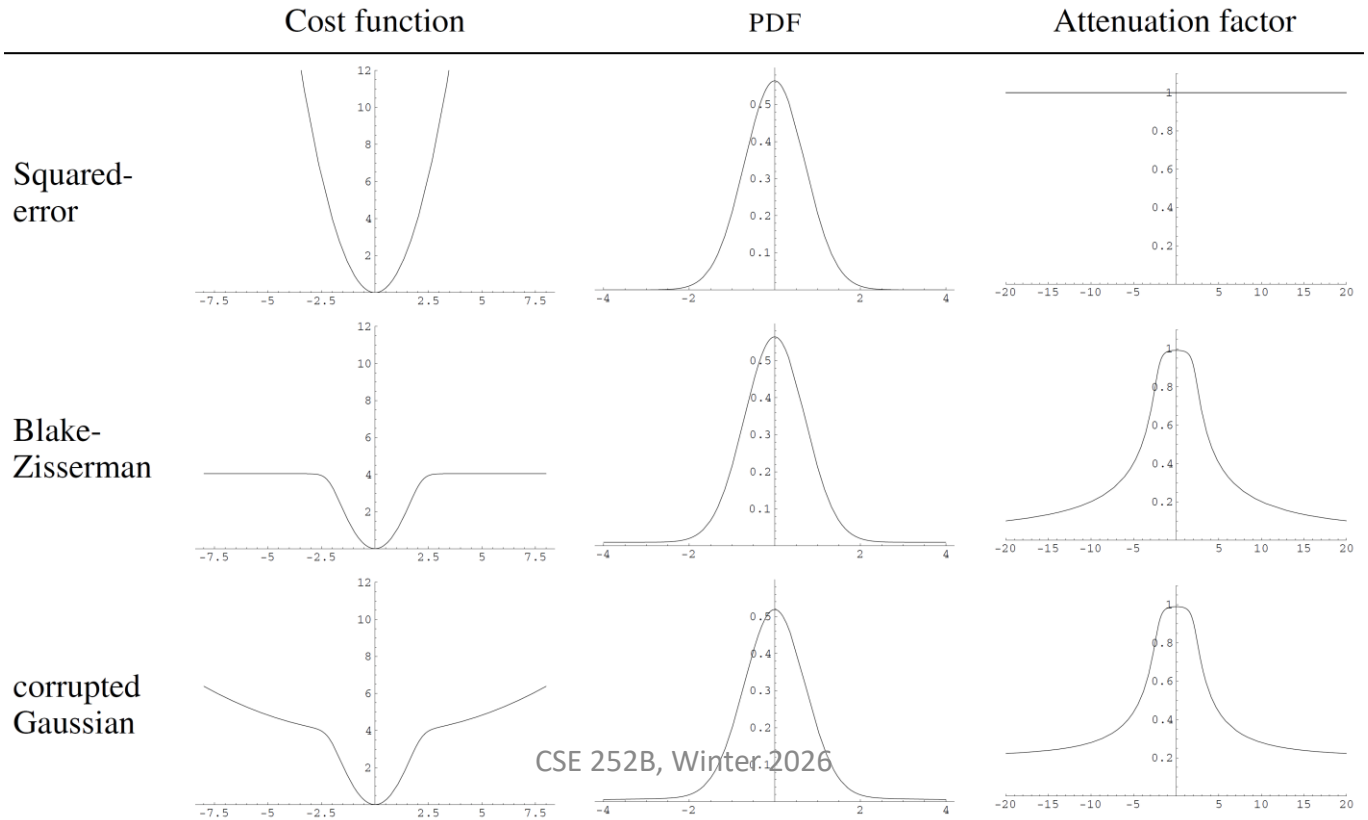
- Squared error cost function assumes the data is Gaussian distributed
  - Not robust to outliers



- Robust cost functions diminish weight of outliers (attenuation factor)

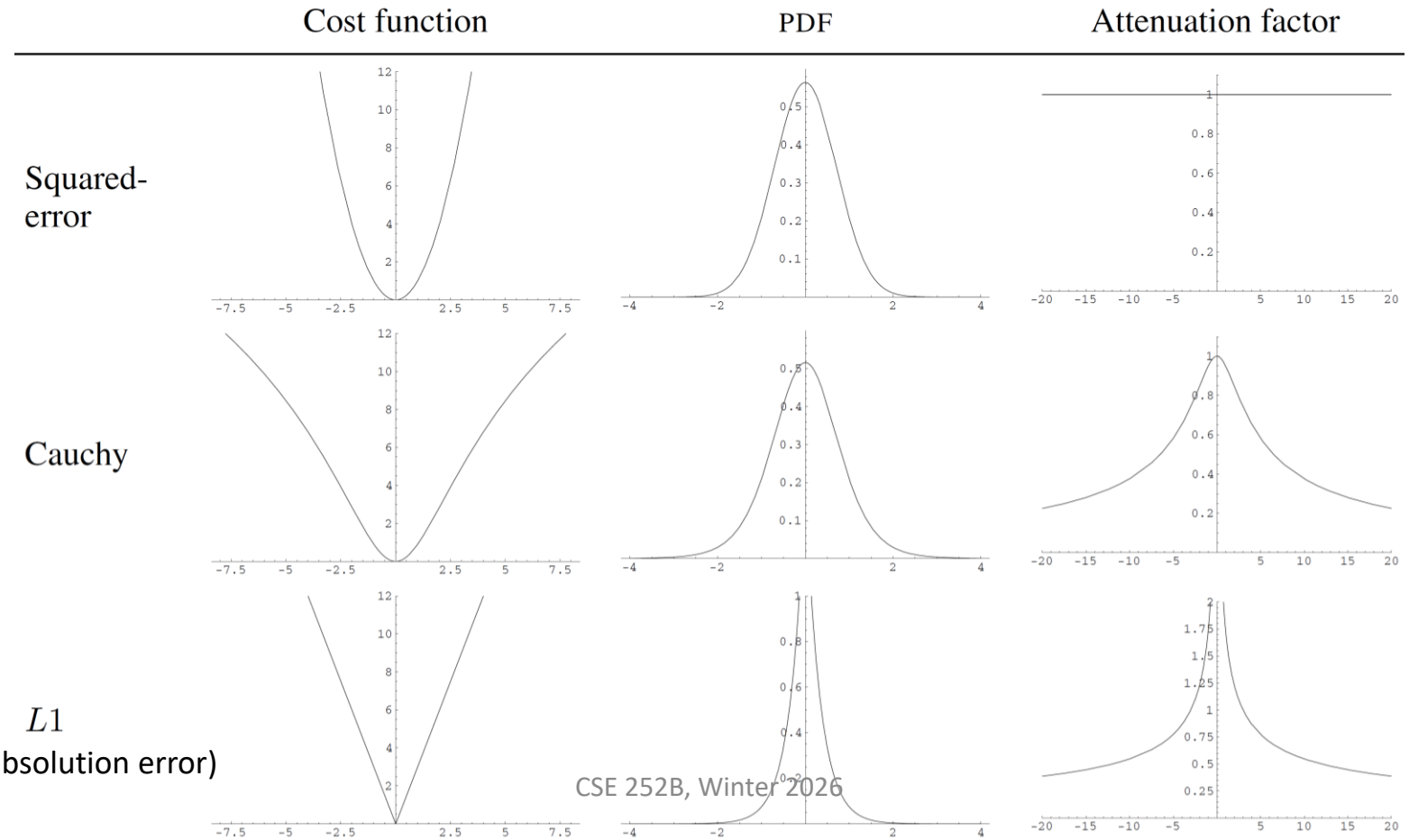
Statistically based cost functions

- Assumes the inlier data is Gaussian distributed with uniform (Blake-Zisserman) or Gaussian with larger standard deviation (corrupted Gaussian) of outliers



Heuristic robust cost functions

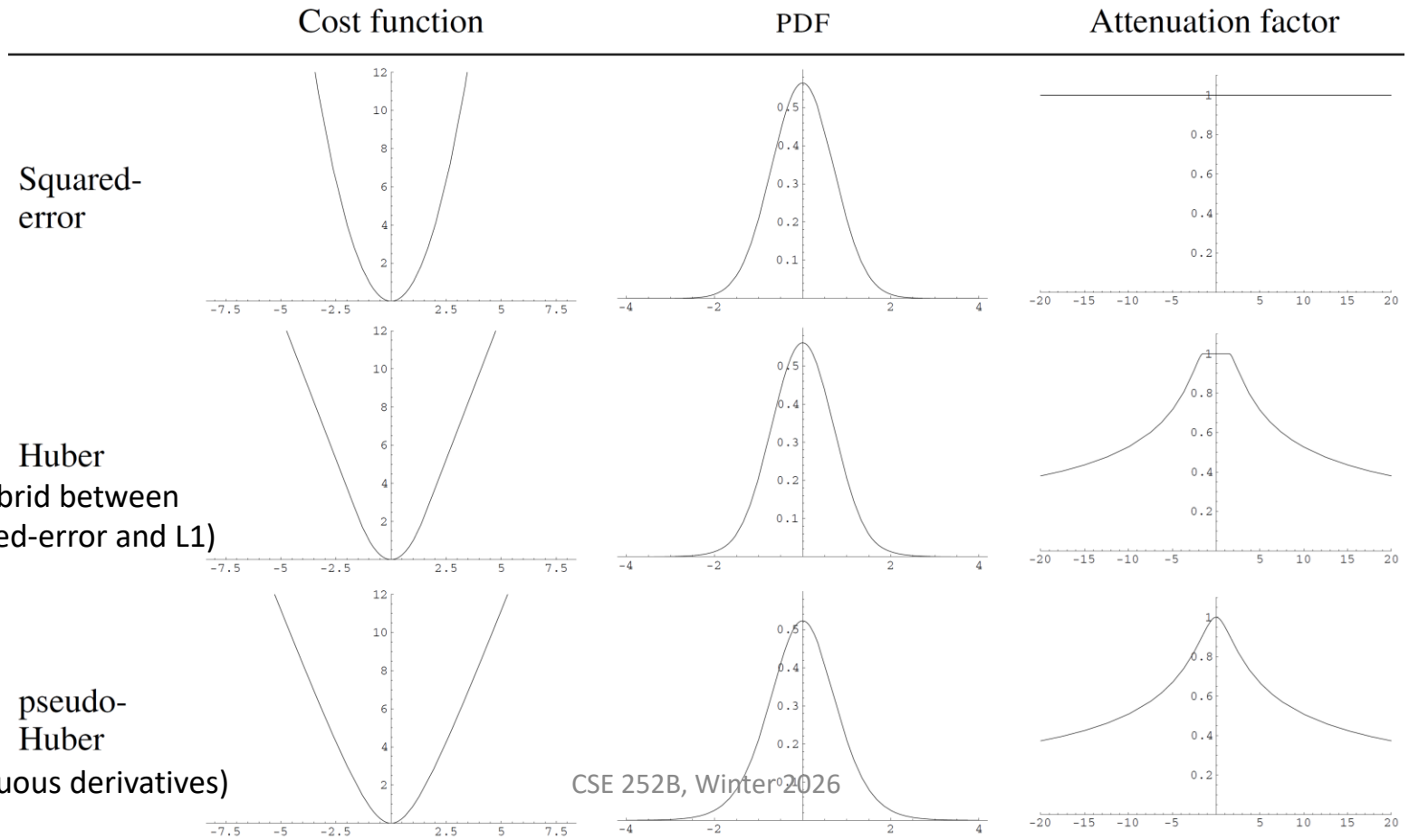
- Noise immuneness properties



(sum of absolution error)

Heuristic robust cost functions

- Noise immuneness properties



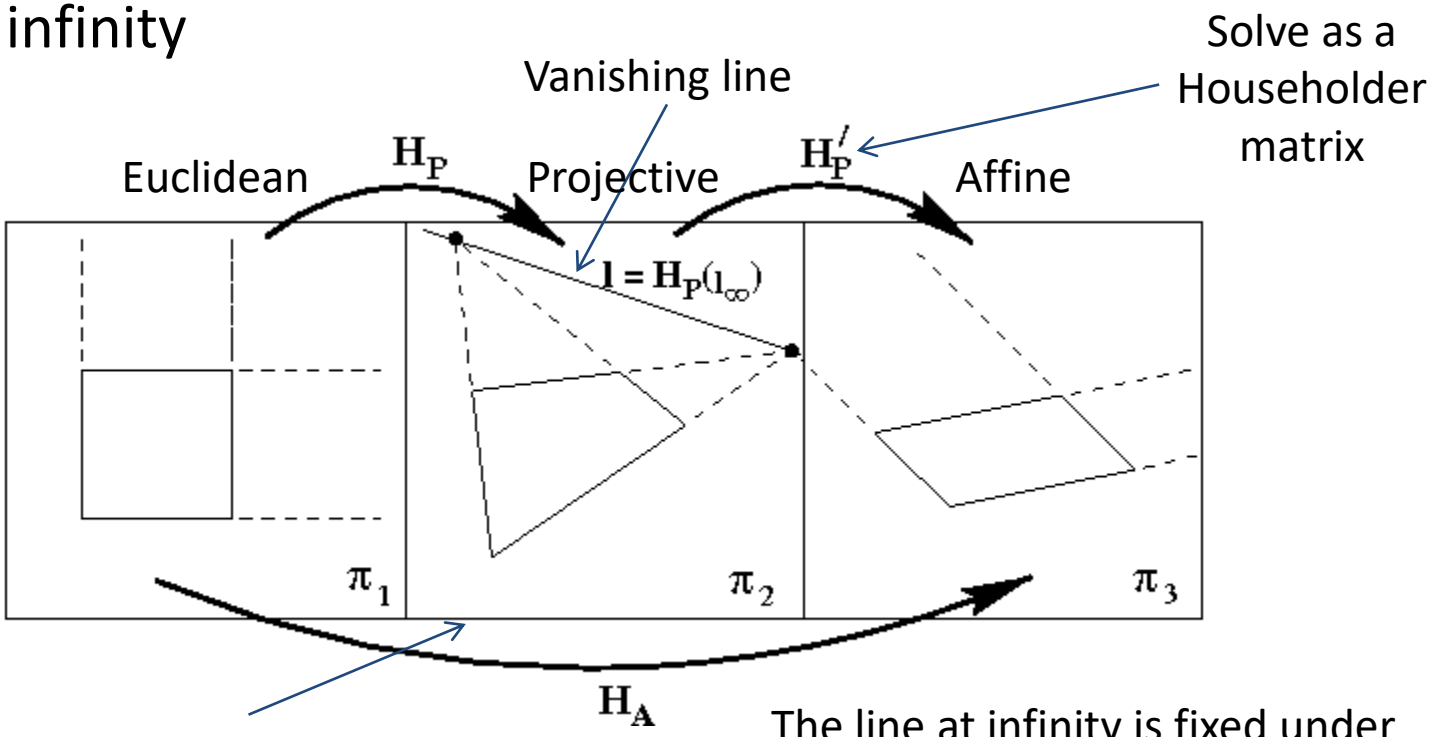
Auto-calibration

Auto-calibration

- The recovery of affine and metric properties from images
- Approaches
  - Stratified: projective → affine → similarity
  - Direct: projective → similarity

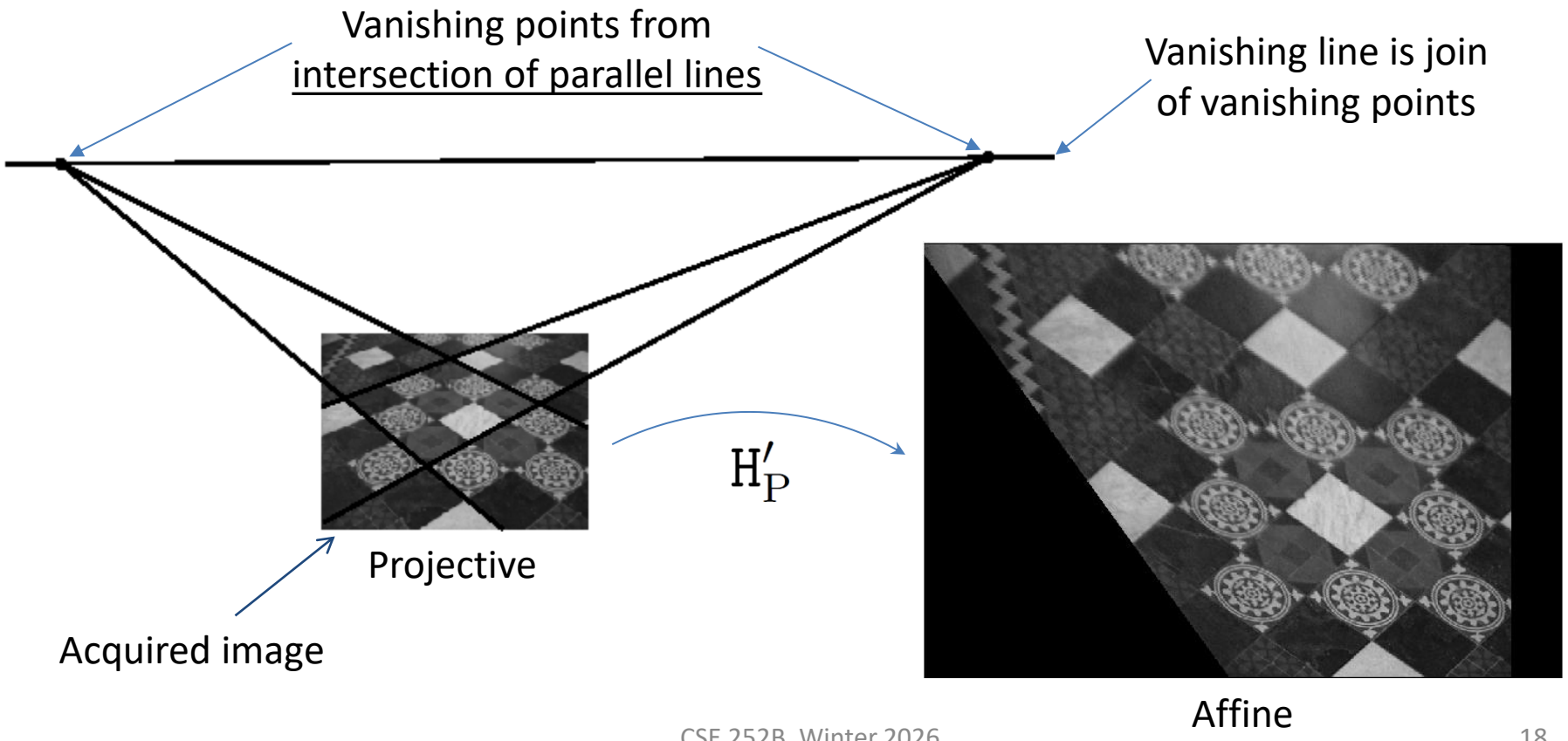
2D stratified: projective → affine

- The vanishing line in the projective frame corresponds to the line at infinity in the Euclidean frame
- Solve for 2D projective transformation that maps line (back) to line at infinity



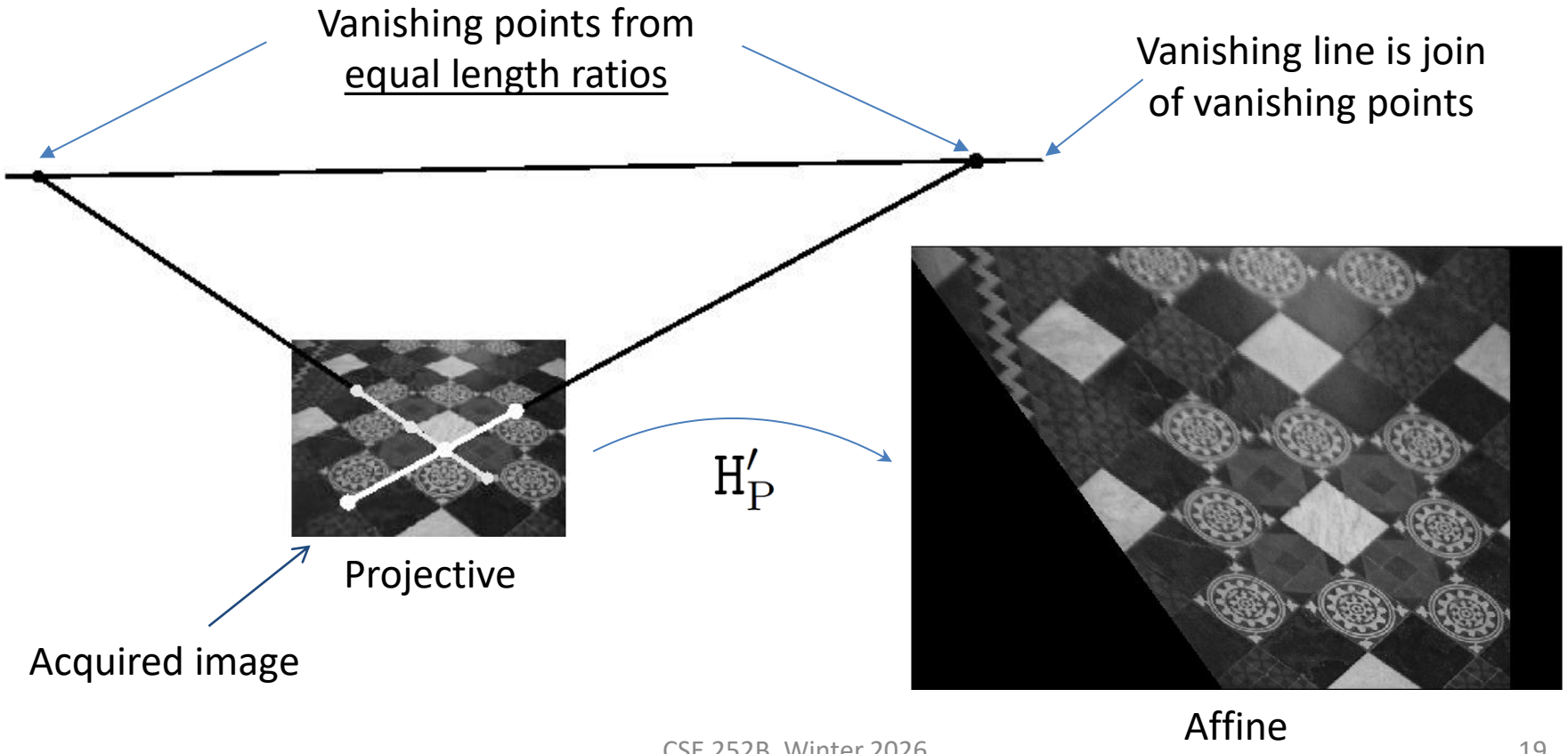
2D affine rectification using the vanishing line

- The vanishing line in the projective frame corresponds to the line at infinity in the Euclidean frame



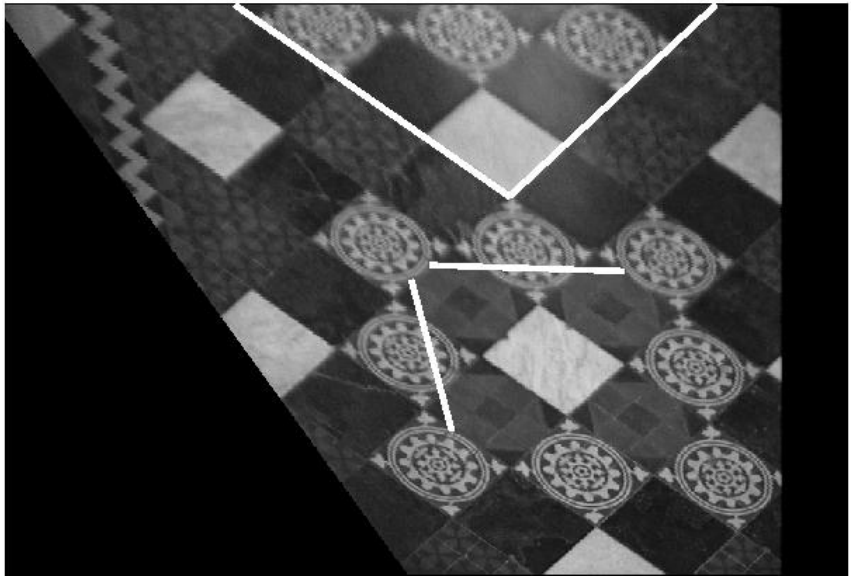
2D affine rectification using the vanishing line

- The vanishing line in the projective frame corresponds to the line at infinity in the Euclidean frame

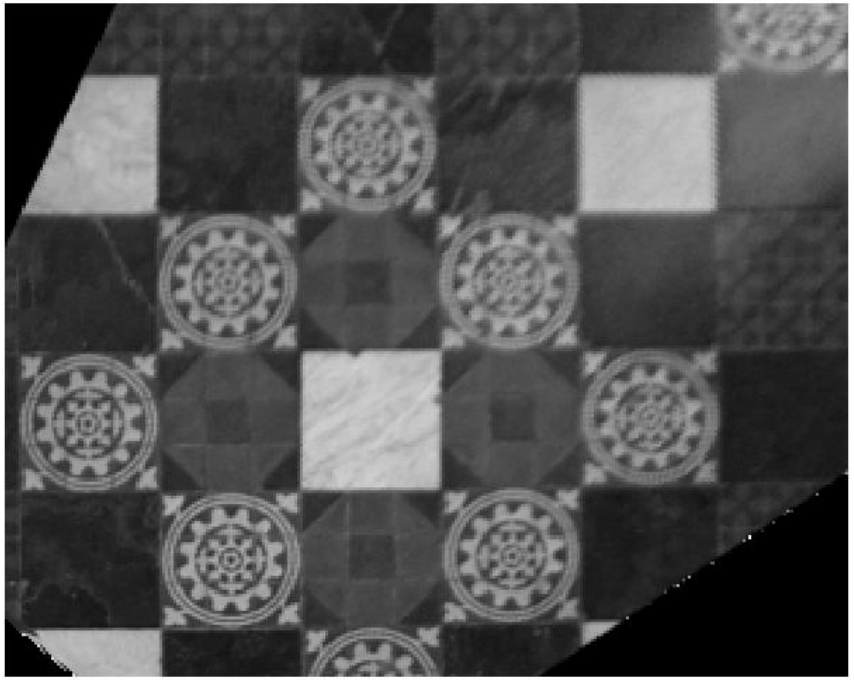


2D stratified: affine → similarity

- Solve for absolute dual conic from two imaged orthogonal line pairs



Affine



Similarity

2D direct: projective → similarity

- Solve for absolute dual conic from five imaged orthogonal line pairs



Projective



Similarity

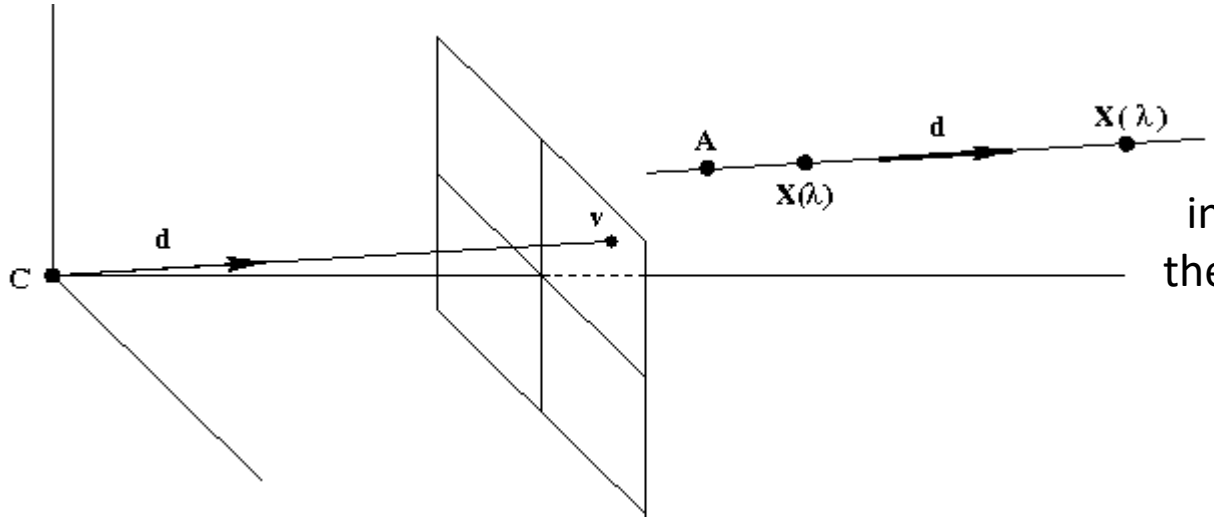
Plane at infinity in 3D is analogous to line at infinity in 2D

- 2D
  - Solve for 2D projective transformation that maps line (back) to line at infinity
  - The line at infinity is fixed under a 2D affine transformation
- 3D
  - Solve for 3D projective transformation that maps plane (back) to plane at infinity
  - The plane at infinity is fixed under a 3D affine transformation

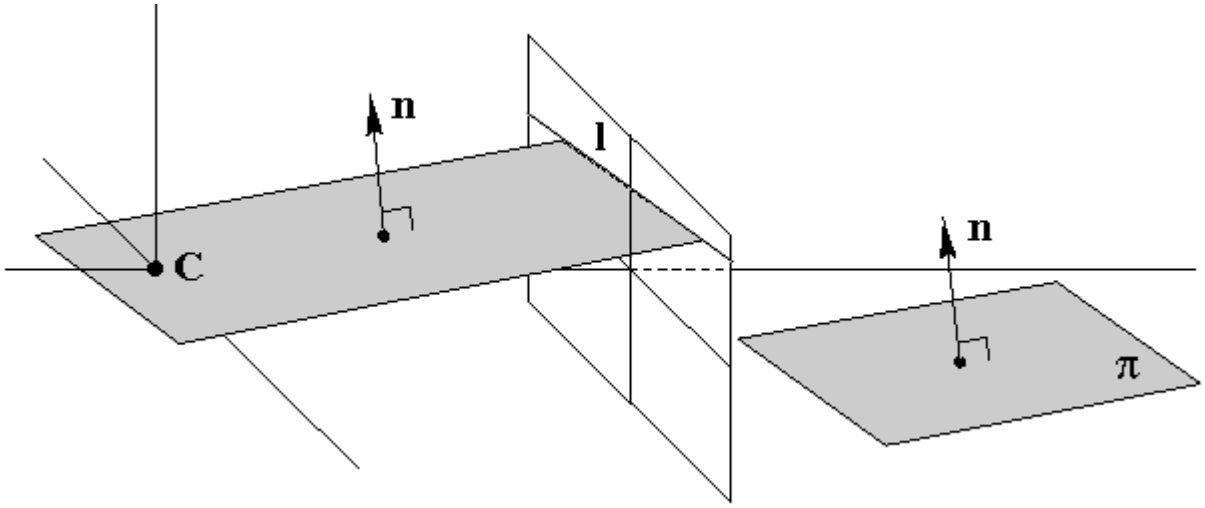
Properties of the plane at infinity

- Two planes are parallel if, and only if, their line of intersection is on the plane at infinity
- A line is parallel to another line, or to a plane, if the point of intersection is on the plane at infinity
- A plane intersects the plane at infinity in a line on the plane that corresponds to the line at infinity

Parallel 3D lines and planes



Point of intersection is on the plane at infinity



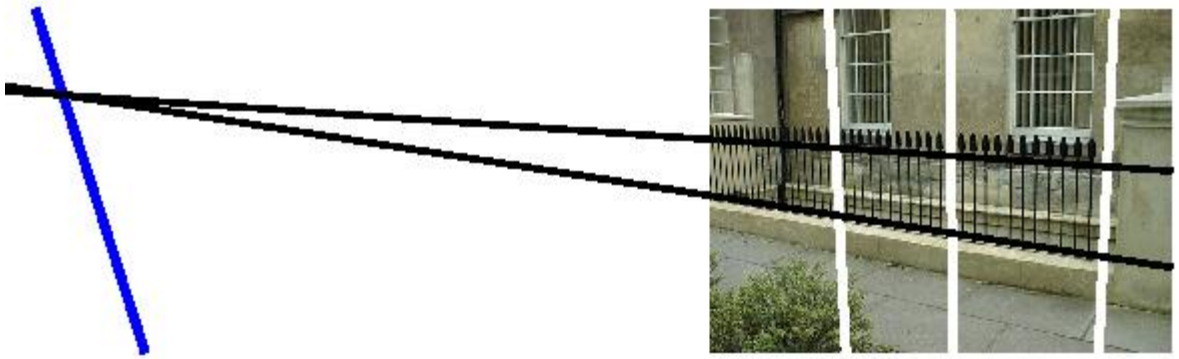
Line of intersection is on the plane at infinity

Identify the plane at infinity

- Three or more points on the plane in the projective frame that corresponds to the plane at infinity in the Euclidean frame determine the plane
- Three or more sets of parallel lines in the projective frame determine three or more points on the plane that corresponds to the plane at infinity in the Euclidean frame
- Distance ratios on a line in 3D (similar to 2D)

Vanishing points and vanishing lines

- The image of a point on the plane at infinity is a vanishing point
- The image of a line on the plane at infinity is a vanishing line



Note that the vanishing point lies on the vanishing line

Vanishing points and vanishing lines







Absolute dual quadric in 3D is analogous to absolute dual conic in 2D

- 2D
  - Solve for absolute dual conic from images of orthogonal line pairs
  - The absolute dual conic is fixed under a 2D similarity transformation
- 3D
  - Solve for absolute dual quadric
    - Solve for the image of the absolute conic (IAC) $\omega = (\mathrm{K}\mathrm{K}^\top)^{-1}$
  - The absolute dual quadric is fixed under a 3D similarity transformation

Camera calibration

Camera projection matrix

Project points in camera coordinate frame

$$\begin{bmatrix} x \\ y \\ w \end{bmatrix} = \begin{bmatrix} \alpha_x & s & x_0 \\ 0 & \alpha_y & y_0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} X_{\text{cam}} \\ Y_{\text{cam}} \\ Z_{\text{cam}} \\ T_{\text{cam}} \end{bmatrix}$$

$$\mathbf{x} = \mathtt{K}[\mathtt{I} \,|\, \mathbf{0}]\mathbf{X}_{\text{cam}}, \text{ where camera calibration matrix } \mathtt{K} = \begin{bmatrix} \alpha_x & s & x_0 \\ 0 & \alpha_y & y_0 \\ 0 & 0 & 1 \end{bmatrix}$$

Transform points from world coordinate frame to camera coordinate frame

$$\mathbf{X}_{\text{cam}} = \begin{bmatrix} \mathtt{R} & \mathbf{t} \\ \mathbf{0}^\top & 1 \end{bmatrix} \mathbf{X}$$

Project points in world coordinate frame

$$\mathbf{x} = \mathtt{K}[\mathtt{I} \,|\, \mathbf{0}] \begin{bmatrix} \mathtt{R} & \mathbf{t} \\ \mathbf{0}^\top & 1 \end{bmatrix} \mathbf{X}$$

$$\mathbf{x} = \mathtt{K}[\mathtt{R} \,|\, \mathbf{t}]\mathbf{X}$$

$$\mathbf{x} = \mathtt{P}\mathbf{X}, \text{ where camera projection matrix } \mathtt{P} = \mathtt{K}[\mathtt{R} \,|\, \mathbf{t}]$$

Camera calibration matrix

$$\mathtt{K} = \begin{bmatrix} \alpha_x & s & x_0 \\ 0 & \alpha_y & y_0 \\ 0 & 0 & 1 \end{bmatrix}$$

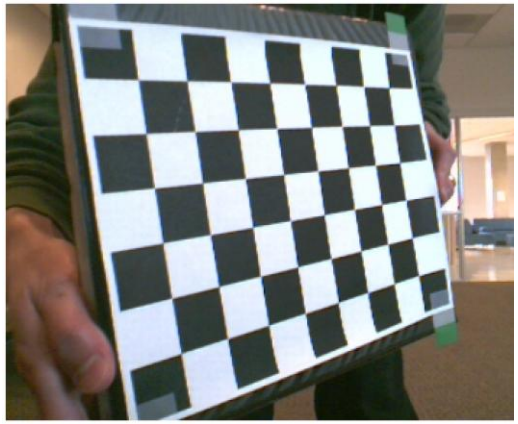
where

$\alpha_x = f m_x$	Focal length in x direction in terms of pixel dimensions
$\alpha_y = f m_y$	Focal length in y direction in terms of pixel dimensions
s	Skew
$(x_0, y_0)^\top = (p_x m_x, p_y m_y)^\top$	Coordinates of principal point in terms of pixel dimensions

where m_x and m_y are number of pixels per unit disance in x and y directions, respectively

Camera calibration

- Not preferred: acquire single image of calibration target
  - Coordinates of 3D points on calibration target are known to very high accuracy



- Estimate camera projection matrix

Decomposition of camera projection matrix

If estimated from points in a metric coordinate frame
Solve for camera center $\mathbf{C}$

$$\mathtt{P}\mathbf{C} = \mathbf{0} \text{ (i.e., } \mathbf{C} \text{ is null space of } \mathtt{P})$$

Solve for camera calibration matrix $\mathtt{K}$ and camera rotation matrix $\mathtt{R}$

$$\mathtt{P} = \mathtt{K}[\mathtt{R} \,|\, \mathbf{t}] = [\mathtt{K}\mathtt{R} \,|\, \mathtt{K}\mathbf{t}] = [\mathtt{M} \,|\, \mathtt{K}\mathbf{t}], \text{ where } \mathtt{M} = \mathtt{K}\mathtt{R}$$

1. RQ decomposition of $\mathtt{M}$. $\mathtt{M} = \mathtt{K}\mathtt{R}$, where $\mathtt{K}$ is upper triangular and $\mathtt{R}$ is orthogonal.
2. Scale $\mathtt{K}$ such that $k_{33} = 1$. $\mathtt{K} = \frac{1}{k_{33}}\mathtt{K}$.
3. Ensure k_{11} and k_{22} are positive

$$\mathtt{M} = \mathtt{K}\mathtt{R} = \begin{bmatrix} \mathbf{k}_1 & \mathbf{k}_2 & \mathbf{k}_3 \end{bmatrix} \begin{bmatrix} \mathbf{r}^{1\top} \\ \mathbf{r}^{2\top} \\ \mathbf{r}^{3\top} \end{bmatrix}$$

If $k_{11} < 0$, then negate $\mathbf{k}_1$ and $\mathbf{r}^{1\top}$
If $k_{22} < 0$, then negate $\mathbf{k}_2$ and $\mathbf{r}^{2\top}$

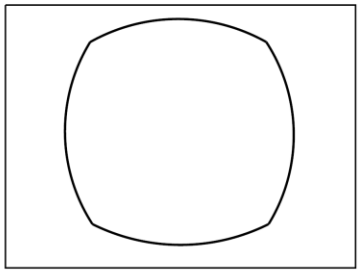
$$\mathtt{K} = \begin{bmatrix} \alpha_x & s & x_0 \\ 0 & \alpha_y & y_0 \\ 0 & 0 & 1 \end{bmatrix}$$

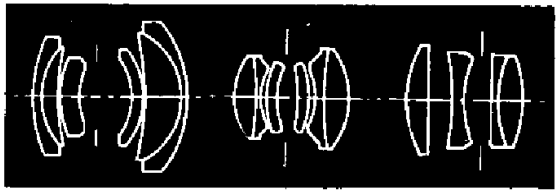
4. Ensure $\mathtt{R}$ is a special orthogonal. If $\det(\mathtt{R}) = -1$, then negate $\mathtt{R}$.

Lens distortion

- Short focal length or wide field of view lenses commonly exhibit distortions along radial directions
  - Radial distortion appears as either pincushion or barrel distortion of the image
- Additionally, tangential distortion occurs when multiple or compound lenses are not aligned along their optical centers, a configuration referred to as lens decentering







Camera calibration

In order to model the radial and tangential lens distortion present in a non-ideal lens, non-distorted normalized 2D coordinates $\hat{\tilde{\mathbf{x}}} = (\hat{\tilde{x}}, \hat{\tilde{y}})^\top = (\hat{x}/\hat{w}, \hat{y}/\hat{w})^\top$ are mapped to distorted normalized 2D coordinates $\hat{\tilde{\mathbf{x}}}_d = (\hat{\tilde{x}}_d, \hat{\tilde{y}}_d)^\top$ by

$$\begin{pmatrix} \hat{\tilde{x}} \\ \hat{\tilde{y}} \end{pmatrix} \mapsto \begin{pmatrix} \hat{\tilde{x}}_d \\ \hat{\tilde{y}}_d \end{pmatrix} = (1 + \kappa_1 r^2 + \kappa_2 r^4 + \kappa_5 r^6) \begin{pmatrix} \hat{\tilde{x}} \\ \hat{\tilde{y}} \end{pmatrix} + \begin{pmatrix} 2\kappa_3 \hat{\tilde{x}}\hat{\tilde{y}} + \kappa_4(r^2 + 2\hat{\tilde{x}}^2) \\ \kappa_3(r^2 + 2\hat{\tilde{y}}^2) + 2\kappa_4 \hat{\tilde{x}}\hat{\tilde{y}} \end{pmatrix}$$

where κ_1, κ_2, and κ_5 are radial distortion parameters, κ_3 and κ_4 are tangential distortion parameters, and $r = \sqrt{\hat{\tilde{x}}^2 + \hat{\tilde{y}}^2}$.

The final step of the imaging process is the mapping of distorted normalized coordinates $\hat{\tilde{\mathbf{x}}}_d = (\hat{\tilde{x}}_d, \hat{\tilde{y}}_d)^\top$ to distorted image coordinates $\tilde{\mathbf{x}}_d = (\tilde{x}_d, \tilde{y}_d)^\top$ by

$$\begin{pmatrix} \tilde{x}_d \\ \tilde{y}_d \\ 1 \end{pmatrix} = \begin{bmatrix} \alpha_x & s & x_0 \\ 0 & \alpha_y & y_0 \\ 0 & 0 & 1 \end{bmatrix} \begin{pmatrix} \hat{\tilde{x}}_d \\ \hat{\tilde{y}}_d \\ 1 \end{pmatrix}$$

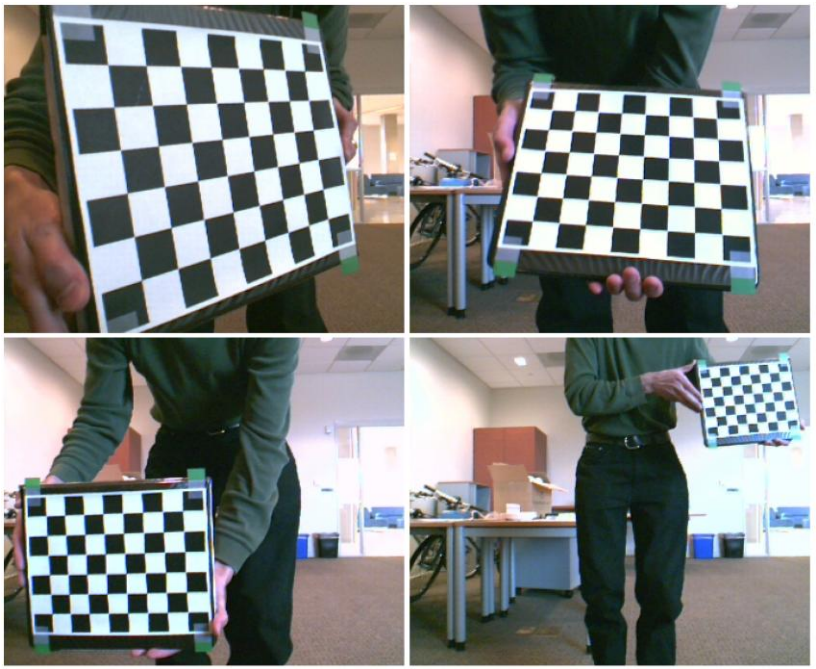
$$\begin{pmatrix} \tilde{\mathbf{x}}_d \\ 1 \end{pmatrix} = \mathtt{K} \begin{pmatrix} \hat{\tilde{\mathbf{x}}}_d \\ 1 \end{pmatrix}$$

$$\mathbf{x}_d = \mathtt{K}\hat{\mathbf{x}}_d$$

where $(x_0, y_0)^\top$ is the principal point, s is the skew, and α_x and α_y are the focal lengths of the camera in the x and y directions.

Camera calibration

- Preferred: acquire many images of planar calibration target
  - Various orientations and distances from camera

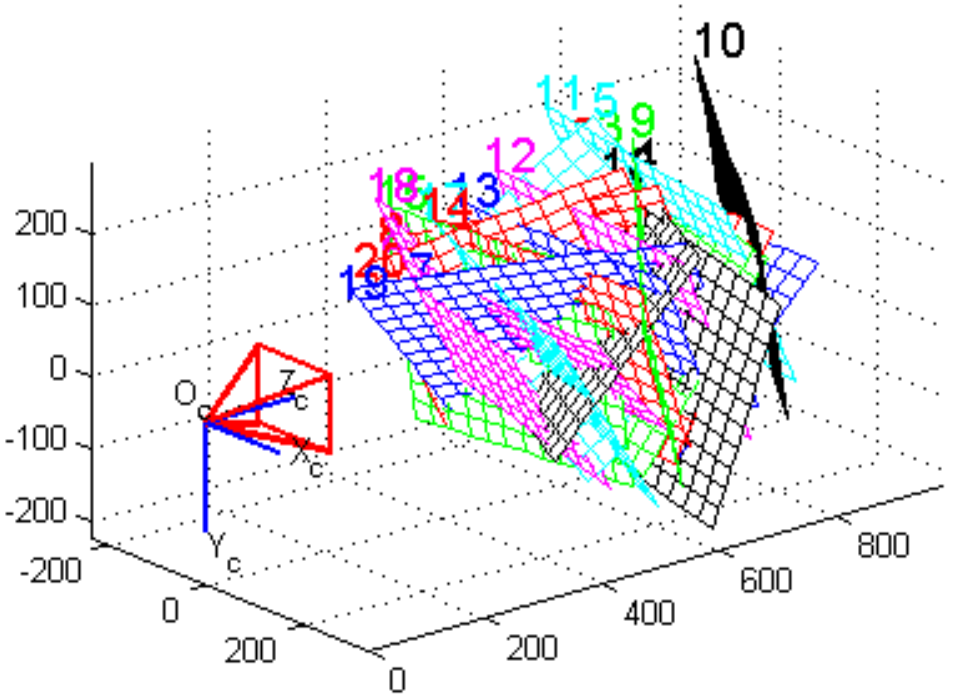


Camera calibration

1. For each image, estimate the 2D projective transformation from the **2D** points on the calibration target to the corresponding image pixel coordinates
2. Linear estimate of the image of the absolute conic (IAC) from the resulting set of 2D projective transformations
3. Calculate initial estimate of the camera calibration matrix from the IAC $\omega = (\mathtt{K}\mathtt{K}^\top)^{-1}$
4. For each image, initial estimate of the camera pose from the **3D** points on the calibration target to the corresponding image normalized coordinates (using initial estimate of the camera calibration matrix)

Camera calibration

5. Nonlinear estimation of internal camera parameters, including lens distortion parameters, associated with all images and camera pose associated with each image



Imaging geometry applications

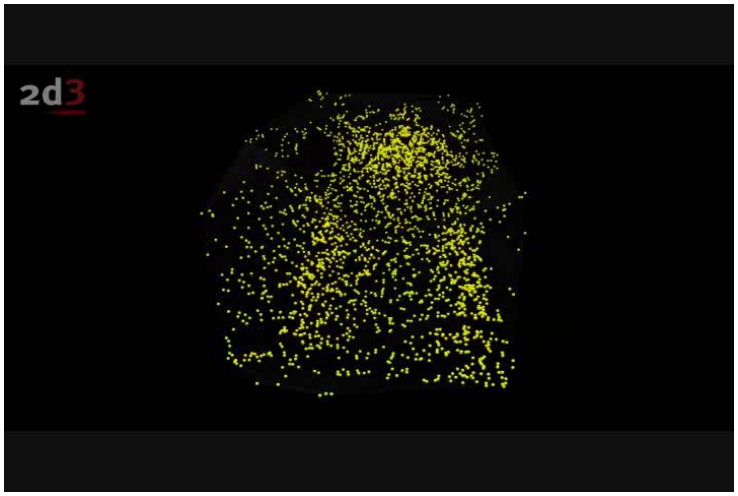
Motion capture



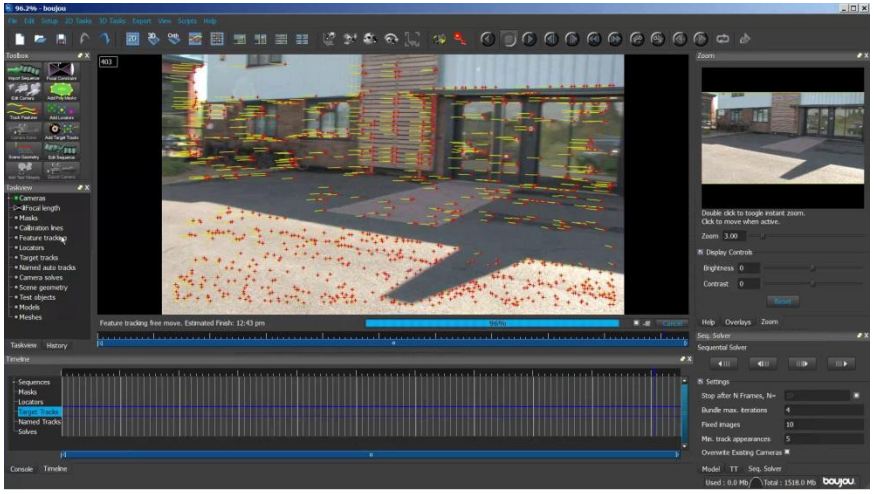


Matchmoving











CSE 252B

- Feature detection and matching (simple)
- Geometric primitives
- Single view
- Two view geometry
  - Rotation about the same camera center
  - Imaging a plane
  - Imaging a 3D scene
- Three view geometry
- (Four view geometry)
- n view geometry
- Additional topics

- Outlier rejection
  - Uses minimal solution/estimate
- Nonlinear estimation
  - Uses linear estimation for initial estimate
  - Uses minimal parameterizations