CSE203B Convex Optimization: 1.1 Introduction

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Overview

- Optimization formulation without Constraints
  - Kuhn-Tucker Conditions
- Optimization formulation with Constraints
  - Primal Problem
  - Lagrangian Function
  - Lagrange Dual Problem
- Summary

Optimization without Constraints

Problem

min $f_0(x)$ $\qquad x \in R^n$

Kuhn-Tucker Conditions

$\nabla_x f_0(x^*) = 0$
$\nabla_x^2 f_0(x^*) \geq 0$

Solution x^* is a locally optimal solution
If function $f_0(x)$ is a convex function, then x^* is a globally optimal solution

Optimization with Constraints

Problem

min $f_0(x)$ $x \in R^n$
s.t. $x \in X$ i.e. X is a feasible set of x

Let us denote x^* a locally optimal solution.
If function $f_0(x)$ is a convex function and X is a convex set then x^* is a globally optimal solution

Optimization with Constraints

Problem

$\min\ f_0(x) \qquad x \in R^n$

$s.t.\ f_i(x) \le 0 \quad i = 1, \dots, m$

$\qquad h_i(x) = 0 \quad i = 1, \dots, p$

$domain\ D = dom\ f_0 \cap_i dom\ f_i \cap_i dom\ h_i$

Feasible set $X =$?

Lagrangian: $L: R^n \times R^m \times R^p \to R$

$$L(x, \lambda, v) = f_0(x) + \sum_{i=1}^{m} \lambda_i f_i(x) + \sum_{i=1}^{p} v_i h_i(x)$$

λ_i, v_i: $Lagrange\ multiplier, \lambda_i, \in R_+, v_i \in R.$

Lagrange dual function

$$g(\lambda, v) = \inf_{x \in D} L(x, \lambda, v)$$

Dual Problem

$$max_{\lambda, v}\ g(\lambda, v) \quad s.t. \lambda \in R_+^m, v \in R^p$$

Summary

- Optimization without Constraints
  - Kuhn-Tucker Conditions
- Optimization with Constraints
  - Lagrange Multiplier/Duality
  - KKT Conditions
- Linear Algebra
  - High Dimensional Space
  - Matrix Properties
  - Matrix Operations
- Convexity
  - Convex Set
  - Convex Function
- Numerical Methods/Algorithms