

## CSE 203B W25 Homework 3

Due Time : 11:50pm, Thursday Feb. 6, 2025 Submit to Gradescope  
Gradescope: <https://gradescope.com/>

In this homework, we work on exercises from text book including level sets of convex, concave, quasi-convex, quasi-concave functions (3.1, 3.2), second-order conditions for convexity on affine sets (3.9), Kullback-Leibler divergence (3.13), saddle points of convex-concave functions (3.14) determination of convex, concave, quasi-convex, quasi-concave functions (3.16), conjugate functions (3.36), and gradient and Hessian of conjugate functions (3.40). Extra assignments are given on Kullback-Leibler divergence, and softmax functions.

Please make two separate submissions on Gradescope:

### Exercises:

- Graded by completion, you may work in a group of up to four students.
- Submit a single PDF file and add all group members to the submission.
- Describe each member's contributions at the beginning of your report.

### Assignments:

- Graded by content and must be completed individually.
- Submit a single PDF file.

#### I. Exercises from textbook chapter 3 (8 pts)

3.1, 3.2, 3.9, 3.13, 3.14, 3.16, 3.36, 3.40.

#### II. Assignments (42 pts)

II. 1. Convex Functions. (19 pts)

I.1.1. Determine if the following functions are convex or concave, and provide proofs. (12 pts)

(a)  $f(x) = \sqrt{x}$ ,  $\text{dom } f = \mathbb{R}_{++}$ . (3 pts)

(b)  $f(x) = \sum_{i=1}^n e^{x_i}$ ,  $\text{dom } f = \mathbb{R}^n$ . (3 pts)

(c)  $f(x_1, x_2) = x_1^\alpha x_2^{1-\alpha}$ ,  $0 \leq \alpha \leq 1$ ,  $\text{dom } f = \mathbb{R}_{++}^2$ . (3 pts)

(d)  $f(x) = \max\{a_1^T x + b_1, \dots, a_k^T x + b_k\}$ ,  $\text{dom } f = \mathbb{R}^n$ . (3 pts)

II.1.2. Consider the function:

$$f(x) = \left( \sum_{i=1}^n x_i^p \right)^{1/p}, \text{ dom } f = \mathbb{R}_{++}^n, \quad p < 1, p \neq 0.$$

Answer the following questions related to this function:

- (a) Derive the Hessian matrix of  $f(x)$ . (3 pts)
- (b) Is the function  $f(x)$  convex or concave? Show your proof. (4 pts)

II. 2. Conjugate Functions. (23 pts)

Find the conjugate function of the following functions.

II.2.1.  $f_1(x) = 2x + 1$ , where  $x \in \mathbb{R}$ . (2 pts)

II.2.2. Let  $f_2(x) = \frac{1}{3}x^T Qx$ , where  $Q \in S_{++}^n$  (a symmetric positive definite matrix) and  $x \in \mathbb{R}^n$ . (3 pts)

II.2.3. Let  $f_3(x) = -\log(ax^2 + bx + c)$ , where  $x \in \mathbb{R}$ ,  $a > 0$ ,  $b \in \mathbb{R}$ ,  $c > 0$ . (6 pts)

II.2.4. Let  $f_4(x) = \sum_{i=1}^n e^{a_i x_i + b_i}$ , where  $x \in \mathbb{R}^n$ ,  $a_i > 0$ , and  $b_i \in \mathbb{R}$  for all  $i$ . (6 pts)

II.2.5. Let  $f_5(x) = \log \sum_{i=1}^n \exp\left(\frac{x_i}{\gamma}\right)$ , where  $x \in \mathbb{R}^n$ . (6 pts)