CSE 107: Applied Cryptography

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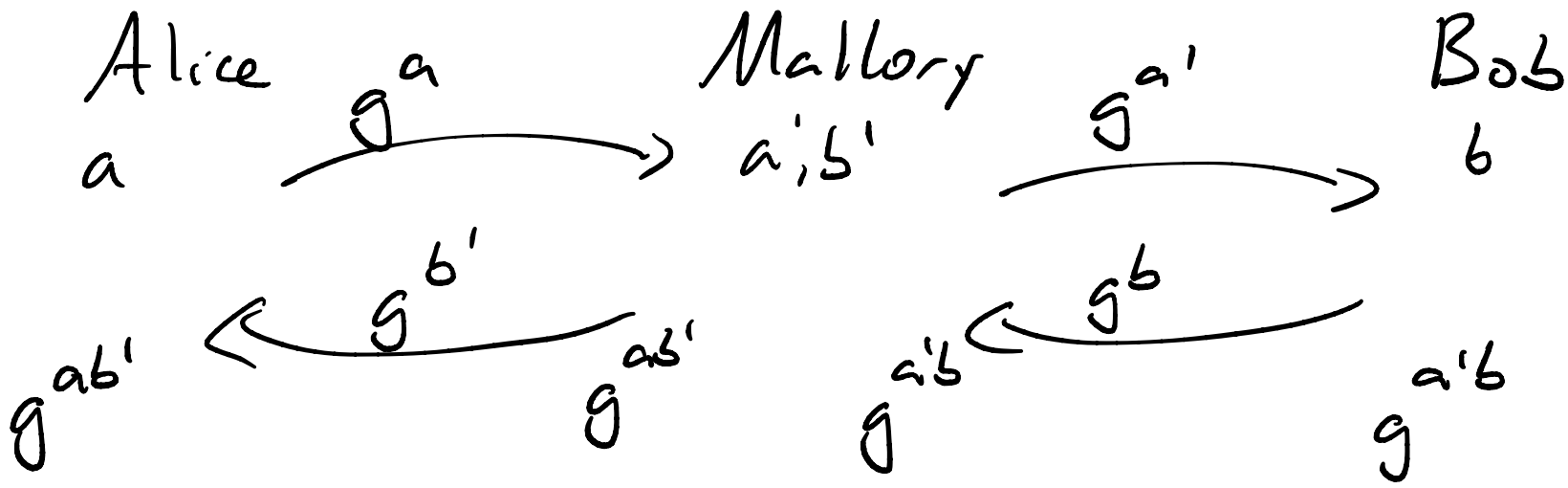
Last time:

- Public-key and hybrid encryption
- Attacks on RSA

This time:

- Digital signatures

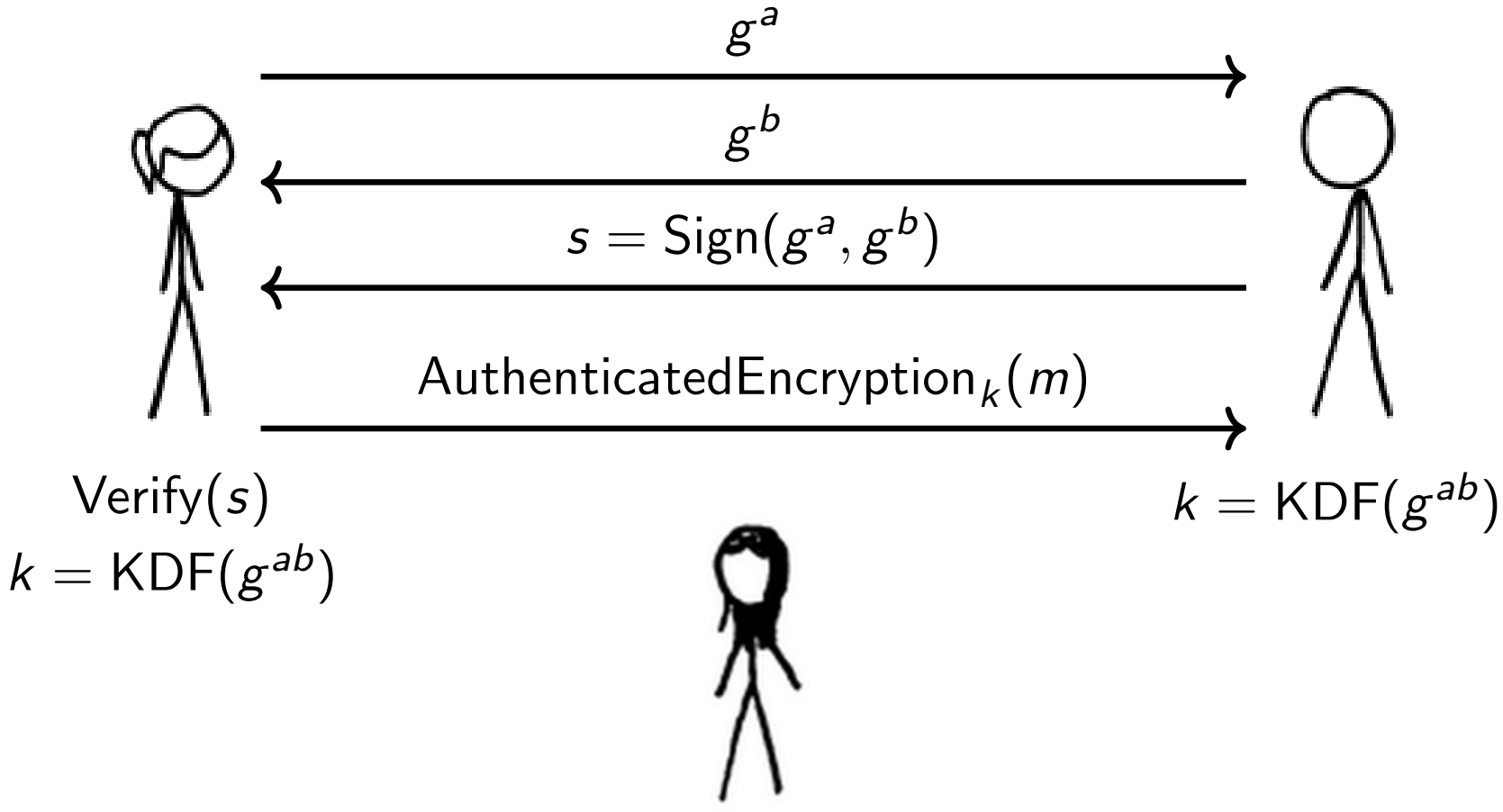
Man-in-the-middle attacks against Diffie-Hellman



An active man-in-the-middle attacker can modify Diffie-Hellman key exchange messages in transit, and Alice and Bob will never know.

Public Key Idea #3: Digital Signatures

Solving the authentication problem.



* Still a toy protocol; not secure.

Digital Signatures

- Key generation: Generate key pair (pk, sk)
- Sign: $\mathsf{sign}_{sk}(m) = \sigma$
- Verify:

$$\mathsf{verify}_{pk}(m, \sigma) = \begin{cases} \texttt{accept} & \text{if valid} \\ \texttt{reject} & \text{otherwise} \end{cases}$$

Correctness: $\Pr[\mathsf{verify}_{pk}(m, \mathsf{sign}_{sk}(m)) = \texttt{accept}] = 1.$

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Differences between handwritten signatures and cryptographic signatures:

- Only person with secret key can generate signature.
- Anyone with public key can verify signature.
- Cryptographic signature is a function of the signed message.

UF-CMA Security of a digital signature scheme

Intent: Adversary should not be able to get a verifier to accept σ as Alice's signature of M unless Alice has previously signed M, even if adversary can obtain Alice's signatures on messages of the adversary's choice.

A change from UF-CMA for MACs: Adversary gets the verification key.

As with MA schemes, the definition does not require security against replay. That is handled on top, via counters or time stamps.

UF-CMA game

Let $\mathcal{DS} = (\mathcal{K}, \mathcal{S}, \mathcal{V})$ be a signature scheme and A an adversary.

Game $\mathrm{UF\text{-}CMA}_{\mathcal{DS}}$

procedure Initialize
$(vk, sk) \xleftarrow{\$} \mathcal{K}$; $S \leftarrow \emptyset$
return vk

procedure Finalize(M, σ)
$d \leftarrow \mathcal{V}_{vk}(M, \sigma)$
return $(d = 1 \wedge M \notin S)$

procedure Sign(M)
$\sigma \xleftarrow{\$} \mathcal{S}_{sk}(M)$
$S \leftarrow S \cup \{M\}$
return σ

The uf-cma advantage of A is

$$\mathrm{Adv}^{\text{uf-cma}}_{\mathcal{DS}}(A) = \Pr\left[\text{UF-CMA}^{A}_{\mathcal{DS}} \Rightarrow \text{true}\right]$$

Differences between signatures and MACs

- Signatures are publicly verifiable
- MACs are non-binding: either Alice or Bob could have generated
- Signatures are usually non-repudiable
- Signatures might allow an adversary to create a new valid pair (m, σ') from (m, σ), which isn't allowed for a MAC
- Signer may not know what m is

Digital signature applications

- Authenticated key exchange
- Software signing
- Digital certificates and public key infrastructure
- Email spam prevention (DKIM)
- Cryptocurrencies

Textbook RSA Signatures

[Rivest Shamir Adleman 1977]

Public Key

$N = pq$ modulus

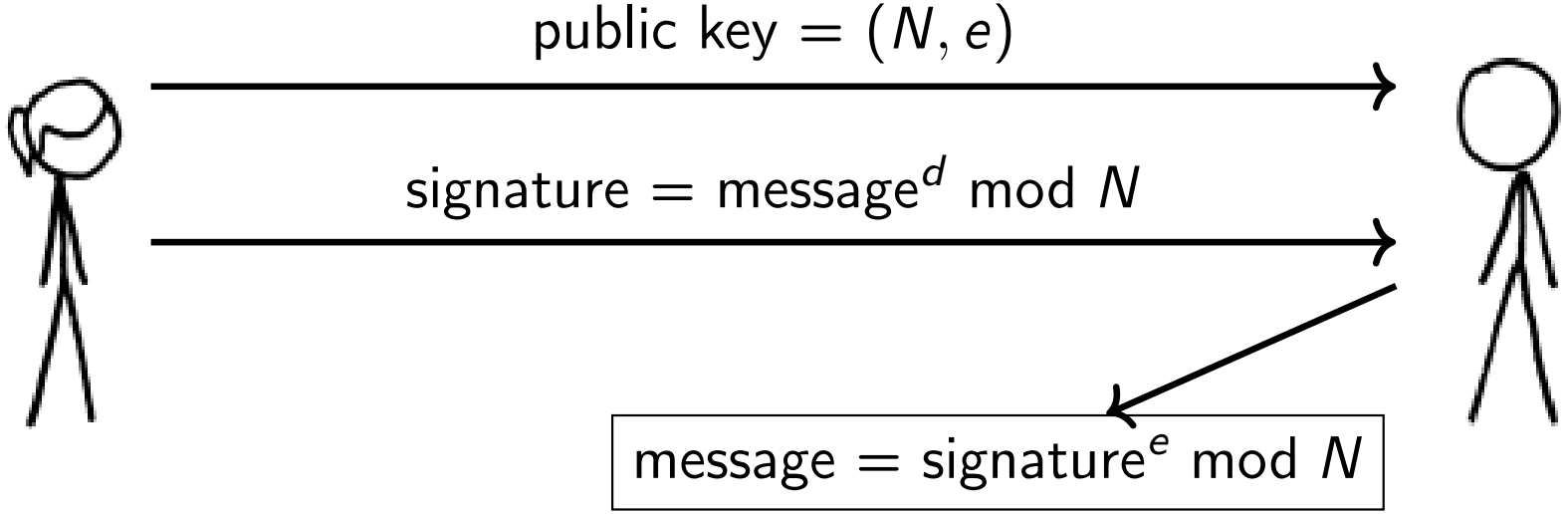
e verifying exponent

Private Key

p, q primes

d signing exponent

$(d = e^{-1} \bmod (p-1)(q-1))$



Textbook RSA signatures are insecure

Forgery from public key:

1. Choose arbitrary value σ.
2. Output $(m = \sigma^e \bmod N, \sigma)$.

Textbook RSA signatures are insecure

Arbitrary forgery for any m:

1. Pick r. Compute $m' = r^e m \bmod N$
2. Query signature for m': $\sigma' = (r^e m)^d \bmod N = r m^d$
3. Output $(m, \sigma' r^{-1} \bmod N)$.

Signature blinding: Allows signer to sign without seeing message.

"Hash-and-sign" paradigm

- Allows signing of arbitrary length messages.
- Protects against signature forgery

Use a secure signature scheme $(\text{sign}, \text{verify})$ and a collision-resistant hash function H.

- Key generation: Generate key pair (pk, sk)
- Sign': $\sigma = \text{sign}_{sk}(H(m))$
- Verify':

$$\text{verify'}_{pk}(m, \sigma) = \begin{cases} \texttt{accept} & \text{if } \text{verify}_{pk}(H(m), \sigma) \\ \texttt{reject} & \text{otherwise} \end{cases}$$

Theorem

If the signature scheme is existentially unforgeable under adaptive chosen-message attack and the hash function is collision resistant, then this construction is existentially unforgeable under an adaptive chosen-message attack.

Construction on previous page requires the hash function to be a "full-domain hash". For RSA, this means we want, e.g., a hash function $H : \{0,1\}^* \to \mathbb{Z}/N\mathbb{Z}$ for e.g. 2048-bit N.

Needs new hash constructions.

So nobody does this. Instead, everyone uses padding schemes.

PKCS #1 v.1.5 padding for RSA signatures

SHA256 has output length 256, but we would like to use it with 2048-bit RSA keys.

PKCS#1 v. 1.5 padding is commonly used.

```
m = 00 01 [FF FF ...FF] 00 [DI] [H(m)]
```

DI specifies the hash function that was used, and $H(m)$ is the hash of the message to be signed.

Bleichenbacher RSA signature forgery attack

PKCS#1 v. 1.5 padding:

`m = 00 01 [FF FF ...FF] 00 [DI] [H(m)]`

Bleichenbacher noticed that implementations that:

1. Use small public exponent e
2. Do not check length of `FF FF FF...` padding

Would be vulnerable to signature forgery.

Attack: Generate value r such that
$r^e =$ `00 01 FF 00 DI H(m) || garbage` over $\mathbb{Z}$.

Padding length is wrong, but verifier doesn't notice.

FIPS PUB 186-3

FEDERAL INFORMATION PROCESSING STANDARDS PUBLICATION

Digital Signature Standard (DSS)

CATEGORY: COMPUTER SECURITY **SUBCATEGORY: CRYPTOGRAPHY**

DSA (Digital Signature Algorithm)

Public Key

- p prime
- q prime, divides $(p-1)$
- g generator of subgroup of order q mod p
- $y = g^x \bmod p$

Private Key

- x private key

Verify

$u_1 = H(m)s^{-1} \bmod q$

$u_2 = rs^{-1} \bmod q$

$r \stackrel{?}{=} g^{u_1} y^{u_2} \bmod p \bmod q$

Sign

Generate random k.

$r = g^k \bmod p \bmod q$

$s = k^{-1}(H(m) + xr) \bmod q$

DSA security

Theorem (Pointcheval, Vaudenay 96)

Breaking DSA is equivalent to computing discrete logs in the random oracle model.

Prime-field DSA is not used much anymore. RSA historically more common for SSL/TLS.

Elliptic curve DSA is becoming much more common: used for SSH, growing in popularity for TLS.

DSA Nonce Vulnerability

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- p prime
- q prime, divides $(p-1)$
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Private Key

- x private key

Sign

Generate random k.

$r = g^k \bmod p \bmod q$

$s = k^{-1}(H(m) + xr) \bmod q$

The nonce k must remain secret, or else the secret key x is revealed.

$$x = (sk - H(m))r^{-1} \bmod q$$

DSA Repeated Nonce Vulnerability

Public Key

- p prime
- q prime, divides $(p-1)$
- g generator of subgroup of order q mod p
- $y = g^x \bmod p$

Private Key

- x private key

Sign

Generate random k.

$r = g^k \bmod p \bmod q$

$s = k^{-1}(H(m) + xr) \bmod q$

If k is ever reused to sign distinct message hashes $H(m_1)$, $H(m_2)$, it is easy to compute:

$$k = (H(m_1) - H(m_2))(s_1 - s_2)^{-1} \bmod q$$

Then the secret key can be computed as before:

$$x = (s_1 k - H(m_1))r_1^{-1} \bmod q$$

DSA Repeated Nonce Vulnerabilities in the Wild

- 1% of SSH DSA host keys compromised by poor RNGs in 2012.
- 2013 Android RNG issue caused Bitcoin thefts due to ECDSA repeated nonce vulnerability.
- Sony PS3 code signing key computed in 2011 due to fixed nonce used to sign ECDSA

Countermeasure: Use "deterministic" nonce generation:

$$k = H(m||x)$$

or "hedged" nonce generation:

$$k = H(m||x||\text{random})$$

(As standardized, H is HMAC_0)