1 Almost perfect lattices

Prove that for any $n$, there exists an $n$-dimensional lattice $\Lambda \subset \mathbb{R}^n$ such that any point $t \in \mathbb{R}^n$ is within (Euclidean) distance $1.5 \cdot \lambda_1(\Lambda)$ from the lattice, i.e., the covering radius of the lattice is at most $1.5 \cdot \lambda_1(\Lambda)$.

[Hint: follow the same approach used in class to prove the bound $\mu(\Lambda) \leq 2 \cdot \lambda_1(\Lambda)$, and adapt it to the problem in the homework using vectors of the form $\frac{\sqrt{3}}{3}$ instead of $\frac{\sqrt{2}}{2}$. See lecture notes on Minkowski’s theorem for details.]