CSE203B Convex Optimization: 1.1 Introduction

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Overview

• Optimization formulation without Constraints
  – Kuhn-Tucker Conditions

• Optimization formulation with Constraints
  – Primal Problem
  – Lagrangian Function
  – Lagrange Dual Problem

• Summary
Optimization without Constraints

Problem

$$\min f_0(x) \quad x \in R^n$$

Kuhn-Tucker Conditions

$$\nabla_x f_0(x^*) = 0$$
$$\nabla^2_x f_0(x^*) \geq 0$$

Solution $$x^*$$ is a locally optimal solution
If function $$f_0(\cdot)$$ is a convex function, then $$x^*$$ is a globally optimal solution
Optimization with Constraints

Problem
\[ \min f_0(x) \quad x \in \mathbb{R}^n \]

\[ \text{s. t. } f_i(x) \leq 0 \quad i = 1, \ldots, m \quad \text{domain } D \]

\[ h_i(x) = 0 \quad i = 1, \ldots, p \quad = \text{dom } f_0 \cap_i \text{dom } f_i \cap_i \text{dom } h_i \]

Lagrangian: \( L: \mathbb{R}^n \times \mathbb{R}^m \times \mathbb{R}^p \to \mathbb{R} \)

\[ L(x, \lambda, v) = f_0(x) + \sum_{i=1}^m \lambda_i f_i(x) + \sum_{i=1}^p v_i h_i(x) \]

\( \lambda_i, v_i: \text{Lagrange multiplier}, \lambda_i, \in \mathbb{R}_+, v_i \in \mathbb{R} \).

Lagrange dual function
\[ g(\lambda, v) = \inf_{x \in D} L(x, \lambda, v) \]

Dual Problem
\[ \max_{\lambda, v} g(\lambda, v) \quad \text{s. t. } \lambda \in \mathbb{R}_+^m, v \in \mathbb{R}^p \]
Summary

- Optimization without Constraints
  - Kuhn-Tucker Conditions
- Optimization with Constraints
  - Lagrange Multiplier/Duality
  - KKT Conditions
- Linear Algebra
  - High Dimensional Space
  - Matrix Properties
  - Matrix Operations
- Convexity
  - Convex Set
  - Convex Function
- Numerical Methods/Algorithms