Announcements

• Assignment 4 is due Mar 22, 11:59 PM
• Please complete course and TA evaluations
  – Must be completed by Mar 18, 8:00 AM (maybe by Mar 20, 11:59 PM)
• Reading
  – Sections 12.1, 12.2, 12.3, and 18.1
Additional topics

• Configurations of geometric primitives
• Robust cost functions
• Calibration, including lens distortion
• Auto-calibration
Configurations of geometric primitives
Configurations of geometric primitives

- **2D points**

<table>
<thead>
<tr>
<th>Rank</th>
<th>Configuration</th>
<th>Columns of null space</th>
</tr>
</thead>
<tbody>
<tr>
<td>3</td>
<td>general position</td>
<td>empty</td>
</tr>
<tr>
<td>2</td>
<td>collinear</td>
<td>line that points are on</td>
</tr>
<tr>
<td>1</td>
<td>coincident</td>
<td>2 lines that intersect at point</td>
</tr>
</tbody>
</table>
Configurations of geometric primitives

• 2D lines

\[
\begin{bmatrix}
\ell_1^T \\
\vdots \\
\ell_n^T
\end{bmatrix}
\]

<table>
<thead>
<tr>
<th>Rank</th>
<th>Configuration</th>
<th>Columns of null space</th>
</tr>
</thead>
<tbody>
<tr>
<td>3</td>
<td>general position</td>
<td>empty</td>
</tr>
<tr>
<td>2</td>
<td>pencil of lines</td>
<td>point that lines intersect at</td>
</tr>
<tr>
<td>1</td>
<td>coincident</td>
<td>2 points on line</td>
</tr>
</tbody>
</table>
## Configurations of geometric primitives

### 3D points

<table>
<thead>
<tr>
<th>Rank</th>
<th>Configuration</th>
<th>Columns of null space</th>
</tr>
</thead>
<tbody>
<tr>
<td>4</td>
<td>general position</td>
<td>empty</td>
</tr>
<tr>
<td>3</td>
<td>coplanar</td>
<td>plane that points are on</td>
</tr>
<tr>
<td>2</td>
<td>collinear</td>
<td>2 planes that intersect at line</td>
</tr>
<tr>
<td>1</td>
<td>coincident</td>
<td>3 planes that intersect at point</td>
</tr>
</tbody>
</table>
Configurations of geometric primitives

- **3D planes**

<table>
<thead>
<tr>
<th>Rank</th>
<th>Configuration</th>
<th>Columns of null space</th>
</tr>
</thead>
<tbody>
<tr>
<td>4</td>
<td>general position</td>
<td>empty</td>
</tr>
<tr>
<td>3</td>
<td>bundle of planes</td>
<td>point that planes intersect at</td>
</tr>
<tr>
<td>2</td>
<td>sheaf of planes</td>
<td>2 points on line that planes intersect at</td>
</tr>
<tr>
<td>1</td>
<td>coincident</td>
<td>3 points on plane</td>
</tr>
</tbody>
</table>

$$\begin{bmatrix} \pi_1^T \\ \vdots \\ \pi_n^T \end{bmatrix}$$
Robust cost functions
Cost functions

• Squared error cost function assumes the data is Gaussian distributed
  – Not robust to outliers

• Robust cost functions diminish weight of outliers (attenuation factor)
Statistically based cost functions

- Assumes the inlier data is Gaussian distributed with uniform (Blake-Zisserman) or Gaussian with larger standard deviation (corrupted Gaussian) of outliers.
Heuristic robust cost functions

- Noise immuneness properties

<table>
<thead>
<tr>
<th>Cost function</th>
<th>PDF</th>
<th>Attenuation factor</th>
</tr>
</thead>
<tbody>
<tr>
<td>Squared-error</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Cauchy</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$L1$          (sum of absolution error)</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
Heuristic robust cost functions

- Noise immuneness properties

<table>
<thead>
<tr>
<th>Cost function</th>
<th>PDF</th>
<th>Attenuation factor</th>
</tr>
</thead>
<tbody>
<tr>
<td>Squared-error</td>
<td><img src="image1.png" alt="Graph" /></td>
<td><img src="image2.png" alt="Graph" /></td>
</tr>
<tr>
<td>Huber (hybrid between squared-error and L1)</td>
<td><img src="image3.png" alt="Graph" /></td>
<td><img src="image4.png" alt="Graph" /></td>
</tr>
<tr>
<td>Pseudo-Huber (continuous derivatives)</td>
<td><img src="image5.png" alt="Graph" /></td>
<td><img src="image6.png" alt="Graph" /></td>
</tr>
</tbody>
</table>
Calibration
Camera projection matrix

Project points in camera coordinate frame

\[
\begin{bmatrix}
  x \\
  y \\
  w
\end{bmatrix}
= \begin{bmatrix}
  \alpha_x & s & x_0 \\
  0 & \alpha_y & y_0 \\
  0 & 0 & 1
\end{bmatrix}
\begin{bmatrix}
  1 & 0 & 0 & 0 \\
  0 & 1 & 0 & 0 \\
  0 & 0 & 1 & 0
\end{bmatrix}
\begin{bmatrix}
  X_{\text{cam}} \\
  Y_{\text{cam}} \\
  Z_{\text{cam}} \\
  T_{\text{cam}}
\end{bmatrix}
\]

\[
x = K[I \mid 0]X_{\text{cam}}, \text{ where camera calibration matrix } K =
\begin{bmatrix}
  \alpha_x & s & x_0 \\
  0 & \alpha_y & y_0 \\
  0 & 0 & 1
\end{bmatrix}
\]

Transform points from world coordinate frame to camera coordinate frame

\[
X_{\text{cam}} = \begin{bmatrix}
  R \\
  0^T \\
  1
\end{bmatrix}X
\]

Project points in world coordinate frame

\[
x = K[I \mid 0]
\begin{bmatrix}
  R \\
  0^T \\
  1
\end{bmatrix}X
\]

\[
x = K[R \mid t]X
\]

\[
x = PX, \text{ where camera projection matrix } P = K[R \mid t]
\]
Camera calibration matrix

\[
K = \begin{bmatrix}
\alpha_x & s & x_0 \\
0 & \alpha_y & y_0 \\
0 & 0 & 1
\end{bmatrix}
\]

where

\[
\begin{align*}
\alpha_x &= fm_x \\
\alpha_y &= fm_y \\
s &= \text{Skew} \\
(x_0, y_0)^	op &= (p_x m_x, p_y m_y)^	op
\end{align*}
\]

Focal length in \( x \) direction in terms of pixel dimensions
Focal length in \( y \) direction in terms of pixel dimensions
Skew
Coordinates of principal point in terms of pixel dimensions

where \( m_x \) and \( m_y \) are number of pixels per unit distance in \( x \) and \( y \) directions, respectively
Lens distortion

• Short focal length or wide field of view lenses commonly exhibit distortions along radial directions
  – Radial distortion appears as either pincushion or barrel distortion of the image
• Additionally, tangential distortion occurs when multiple or compound lenses are not aligned along their optical centers, a configuration referred to as lens decentering
Calibration

In order to model the radial and tangential lens distortion present in a non-ideal lens, non-distorted normalized 2D coordinates $\mathbf{\hat{x}} = (\hat{x}, \hat{y})^\top = (\hat{x}/\hat{w}, \hat{y}/\hat{w})^\top$ are mapped to distorted normalized 2D coordinates $\mathbf{\hat{x}}_d = (\hat{x}_d, \hat{y}_d)^\top$ by

$$
\begin{pmatrix}
\hat{x} \\
\hat{y}
\end{pmatrix} \mapsto 
\begin{pmatrix}
\hat{x}_d \\
\hat{y}_d
\end{pmatrix} = 
\left(1 + \kappa_1 r^2 + \kappa_2 r^4 + \kappa_5 r^6\right) 
\begin{pmatrix}
\hat{x} \\
\hat{y}
\end{pmatrix} + 
\begin{pmatrix}
2\kappa_3 \hat{x}\hat{y} + \kappa_4 (r^2 + 2\hat{x}_d^2) \\
\kappa_3 (r^2 + 2\hat{y}_d^2) + 2\kappa_4 \hat{x}\hat{y}
\end{pmatrix}
$$

where $\kappa_1$, $\kappa_2$, and $\kappa_5$ are radial distortion parameters, $\kappa_3$ and $\kappa_4$ are tangential distortion parameters, and $r = \sqrt{\hat{x}^2 + \hat{y}^2}$.

The final step of the imaging process is the mapping of distorted normalized coordinates $\mathbf{\hat{x}}_d = (\hat{x}_d, \hat{y}_d)^\top$ to distorted image coordinates $\mathbf{\tilde{x}}_d = (\tilde{x}_d, \tilde{y}_d)^\top$ by

$$
\begin{pmatrix}
\tilde{x}_d \\
\tilde{y}_d \\
1
\end{pmatrix} = 
\begin{bmatrix}
\alpha_x & s & x_0 \\
0 & \alpha_y & y_0 \\
0 & 0 & 1
\end{bmatrix} 
\begin{pmatrix}
\hat{x}_d \\
\hat{y}_d \\
1
\end{pmatrix}
$$

$$
\begin{pmatrix}
\tilde{x}_d \\
\tilde{y}_d
\end{pmatrix} = K \begin{pmatrix}
\hat{x}_d \\
1
\end{pmatrix}
$$

where $(x_0, y_0)^\top$ is the principal point, $s$ is the skew, and $\alpha_x$ and $\alpha_y$ are the focal lengths of the camera in the $x$ and $y$ directions.
Calibration

• Acquire many images of calibration target
  – Various orientations and distances from camera
Decomposition of camera projection matrix

If estimated from points in a metric coordinate frame
Solve for camera center \( \mathbf{C} \)
\[
\mathbf{PC} = \mathbf{0} \quad \text{(i.e., } \mathbf{C} \text{ is null space of } \mathbf{P})
\]

Solve for camera calibration matrix \( \mathbf{K} \) and camera rotation matrix \( \mathbf{R} \)
\[
\mathbf{P} = \mathbf{K}[\mathbf{R} | \mathbf{t}] = [\mathbf{KR} | \mathbf{Kt}] = [\mathbf{M} | \mathbf{Kt}], \text{ where } \mathbf{M} = \mathbf{KR}
\]

1. \( \mathbf{RQ} \) decomposition of \( \mathbf{M} \). \( \mathbf{M} = \mathbf{KR} \), where \( \mathbf{K} \) is upper triangular and \( \mathbf{R} \) is orthogonal.

2. Scale \( \mathbf{K} \) such that \( k_{33} = 1 \). \( \mathbf{K} = \frac{1}{k_{33}} \mathbf{K} \).

3. Ensure \( k_{11} \) and \( k_{22} \) are positive

\[
\mathbf{M} = \mathbf{KR} = \begin{bmatrix} \mathbf{k}_1 & \mathbf{k}_2 & \mathbf{k}_3 \end{bmatrix} \begin{bmatrix} \mathbf{r}_1^T \\ \mathbf{r}_2^T \\ \mathbf{r}_3^T \end{bmatrix}
\]

If \( k_{11} < 0 \), then negate \( \mathbf{k}_1 \) and \( \mathbf{r}_1^T \)
If \( k_{22} < 0 \), then negate \( \mathbf{k}_2 \) and \( \mathbf{r}_2^T \)

4. Ensure \( \mathbf{R} \) is a special orthogonal. If \( \det(\mathbf{R}) = -1 \), then negate \( \mathbf{R} \).
Calibration

• Nonlinear estimation of internal camera parameters, including lens distortion parameters, and camera poses
  – Use average of all resulting camera calibration matrices as initial estimate
Auto-calibration
Auto-calibration

• The recovery of affine and metric properties from images

• Approaches
  – Stratified: projective $\rightarrow$ affine $\rightarrow$ similarity
  – Direct: projective $\rightarrow$ similarity
2D stratified: projective $\rightarrow$ affine

- The vanishing line in the projective frame corresponds to the line at infinity in the Euclidean frame.
- Solve for planar projective transformation that maps line (back) to line at infinity.

The line at infinity is fixed under a planar affine transformation.
2D affine rectification using the vanishing line

- The vanishing line in the projective frame corresponds to the line at infinity in the Euclidean frame.
2D affine rectification using the vanishing line

- The vanishing line in the projective frame corresponds to the line at infinity in the Euclidean frame.

Vanishing points from equal length ratios

Vanishing line is join of vanishing points

Projective

Affine
2D stratified: affine $\rightarrow$ similarity

- Solve for absolute dual conic from two imaged orthogonal line pairs
2D direct: projective $\rightarrow$ similarity

- Solve for absolute dual conic from five imaged orthogonal line pairs
Plane at infinity in 3D is analogous to line at infinity in 2D

- **2D**
  - Solve for planar projective transformation that maps line (back) to line at infinity
  - The line at infinity is fixed under a planar affine transformation

- **3D**
  - Solve for 3D projective transformation that maps plane (back) to plane at infinity
  - The plane at infinity is fixed under a 3D affine transformation
Properties of the plane at infinity

• Two planes are parallel if, and only if, their line of intersection is on the plane at infinity
• A line is parallel to another line, or to a plane, if the point of intersection is on the plane at infinity
• A plane intersects the plane at infinity in a line on the plane that corresponds to the line at infinity
Parallel 3D lines and planes

Point of intersection is on the plane at infinity

Line of intersection is on the plane at infinity
Identify the plane at infinity

• Three or more points on the plane in the projective frame that corresponds to the plane at infinity in the Euclidean frame determine the plane

• Three or more sets of parallel lines in the projective frame determine three or more points on the plane that corresponds to the plane at infinity in the Euclidean frame

• Distance ratios on a line in 3D (similar to 2D)
Vanishing points and vanishing lines

• The image of a point on the plane at infinity is a vanishing point
• The image of a line on the plane at infinity is a vanishing line

Note that the vanishing point lies on the vanishing line
Vanishing points and vanishing lines
Absolute dual quadric in 3D is analogous to absolute dual conic in 2D

• 2D
  – Solve for absolute dual conic from images of orthogonal line pairs
  – The absolute dual conic is fixed under a planar similarity transformation

• 3D
  – Solve for absolute dual quadric
    • Solve for the image of the absolute conic (IAC) \( \mathbf{\omega} = (KK^T)^{-1} \)
  – The absolute dual quadric is fixed under a 3D similarity transformation
Imaging geometry applications
Motion capture
Matchmoving
CSE 252B

- Feature detection and matching (simple)
- Geometric primitives
- Single view
- Two view geometry
  - Rotation about the same camera center
  - Imaging a plane
  - Imaging a 3D scene
- Three view geometry
- (Four view geometry)
- $n$ view geometry
- Additional topics

- Outlier rejection
  - Uses minimal solution/estimate
- Nonlinear estimation
  - Uses linear estimation for initial estimate
  - Uses minimal parameterizations