CSE 203B W23 Homework 2

Due Time : 11:50pm, Wednesday, Jan. 25, 2023. Submit to Gradescope
Gradescope: `https://gradescope.com/`

In this homework, we work on exercises from the textbook including midpoint convexity (2.3), Voronoi diagram (2.7, 2.9), quadratic function (2.10), general sets (2.12), cones and dual cones (2.28, 2.31, 2.32), and separation of cones (2.39). Extra assignments are given on convex sets.
Total points: 30. Exercises are graded by completion, assignments are graded by content.

I. Exercises from textbook chapter 2 (9 pts, 1pt for each problem)

2.3, 2.7, 2.9, 2.10, 2.12, 2.28, 2.31, 2.32, 2.39.

II. Assignments (30 pts)

II. 1. Qualification vs. enumeration of convex sets:
Given

$$A = \begin{bmatrix} 2 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 2 & 0 & 0 \\ 0 & 0 & 1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix},$$

$$b^T = \begin{bmatrix} 2 & 1 & 2 & 2 \end{bmatrix},$$

we describe the convex sets as follows.
II.1.1. Convert set $\{x|Ax \leq b, x \in R_+^6\}$ from a qualification oriented expression to an enumeration oriented expression in the format of $\{U\theta|1^T\theta = 1, \theta \in R_+^m\}$. (4 pts)
II.1.2. Convert set $\{x|Ax = 0, x \in R^6\}$ from a qualification oriented expression to an enumeration oriented expression in the format of $\{Px|x \in R^6\}$. (4 pts)
II.1.3. Derive the dual cone of the set $\{x|Ax \leq 0, x \in R^6\}$. (4 pts)

II. 2. Qualification vs. enumeration of convex sets:
Given

$$U = \begin{bmatrix} 2 & 0 & 1 \\ 0 & 1 & 0 \\ 0 & 0 & 2 \\ 0 & 1 & 0 \end{bmatrix},$$

we describe the convex sets as follows.
II.2.1. Convert set $\{U\theta|1^T\theta \leq 1, \theta \in R_+^3\}$ from an enumeration oriented expression to a qualification oriented expression to in the format of $\{x|Ax \leq b, x \in R^4\}$. (8 pts)
II.2.2. Derive the dual cone of the set $\{U\theta|\theta \in R_+^3\}$. (4 pts)

II. 3. Given p hyperplanes

$$a_i^T x = b_i, \text{ for } i = 1, 2, \ldots, p, \quad x \in \mathbb{R}^n.$$

List the maximum number of disjoint regions separated by the hyperplanes for the following cases. (6 pts)
(1) $n = 2, p = 2$.
(2) $n = 2, p = 3$.
(3) $n = 2, p = 5$.
(4) $n = 3, p = 3$.
(5) $n = 3, p = 6$.
(6) Generalize the problem to any given n and p and write down the equation. For example, $N(n, p) = 1 + p$ if $n = 1$.