Morphological Image Processing
Announcements

• Assignment 5 is due today, 11:59 PM

• Assignment 6 will be released today
  – Due Mar 8, 11:59 PM

• Reading
  – Chapter 9: Morphological image processing
    • Sections 9.1, 9.2, 9.3, and 9.5 (through subsection connected components)
Morphological image processing

• Mathematical morphology
• Set theory
• Binary image
Reflection and translation

Reflection

$\hat{B} = \{ w \mid w = -b, \text{ for } b \in B \}$

Negate coordinates (rotate 180 degrees)

Translation

$(B)_z = \{ c \mid c = b + z, \text{ for } b \in B \}$
Structuring elements

• Small sets used to probe an image
  – Origin is commonly in the center of the structuring element (similar to spatial filtering kernels)
Sets of pixels: objects and structuring elements (SEs)

Objects represented as sets

Objects represented as a graphical image

Digital image

Structuring element represented as a set

Structuring element represented as a graphical image

Digital structuring element

Border of background pixels around objects

Tight border around SE
Reflection about the origin

Origin

Don’t care elements
Morphological operations

- Erosion
- Dilation
- Opening
- Closing
Erosion

• Example: square SE
  – The erosion of A by B

\[ A \ominus B = \{ z \mid (B)_z \subseteq A \} \]

\[ I \ominus B = \{ z \mid (B)_z \subseteq A \text{ and } A \in I \} \cup \{ A^c \mid A^c \subseteq I \} \]
Erosion

• Example: elongated SE
  – The erosion of A by B

\[ A \ominus B = \{ z \mid (B)_z \subseteq A \} \]

Complement of A (i.e., set of elements not in A)

\[ I \ominus B = \{ z \mid (B)_z \subseteq A \text{ and } A \in I \} \cup \{ A^c \mid A^c \subseteq I \} \]
Erosion

Equivalent to a min filter
Shrinks

11x11
45x45
15x15
Dilation

\[ A \oplus B = \{ z \mid (\hat{B})_z \cap A \neq \emptyset \} \]

Examples

Square SE

Elongated SE
Dilation

Equivalent to a max filter
Expands

Historically, certain computer programs were written using only two digits rather than four to define the applicable year. Accordingly, the company's software may recognize a date using "00" as 1900 rather than the year 2000.
Duality

• Erosion and dilation are duals of each other with respect to complementation and reflection

\[(A \ominus B)^c = A^c \oplus \hat{B} \text{ and } (A \oplus B)^c = A^c \ominus \hat{B}\]
Opening

• Opening a set $A$ by structuring element $B$
  – Erosion followed by dilation

\[ A \circ B = (A \ominus B) \oplus B \]

Smoothes the contour of an object
Breaks narrow isthmuses
Eliminates thin protrusions

Structuring element rolls along **inner** boundary
Closing

• Closing a set $A$ by structuring element $B$
  – Dilation followed by erosion

$$A \ast B = (A \oplus B) \ominus B$$

Fuses narrow breaks and long thin gulfs
Eliminates small holes
Fills gaps in contour of an object
Duality

• Opening and closing are duals of each other with respect to complementation and reflection
Basic morphological operations

- Opening
- Closing
- Dilation
- Erosion
Morphological image processing

- Dilation
- Erosion
- Opening
- Closing

Noisy input

Dilation

Erosion

Opening

Closing

Dilation

Erosion
Morphological algorithms

- Boundary extraction
- Hole filling
- Connected components
Boundary extraction

Erosion

Set difference
Boundary extraction
Hole filling

$X$ is same size as $I$
$X_0$ is all zeros, except ones at a given point in each hole
$X_k = (X_{k-1} \oplus B) \cap A^c \quad k = 1, 2, \ldots$
Looks for background points
Intersection with $A^c$ limits dilation
Stop when $X_k = X_{k-1}$
$X_{k_{\text{final}}} \cup A$ is $A$ with holes filled
Hole filling

Given points in holes

All holes filled
Connected components

$X$ is same size as $I$

$X_0$ is all zeros, except ones at a
given point in each connected component

$X_k = (X_{k-1} \oplus B) \cap A \quad k = 1, 2, \ldots$

Looks for foreground points

Intersection with $A^c$ limits dilation

Stop when $X_k = X_{k-1}$
Connected components

X-ray image

Threshold (negative)

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<th>Connected component</th>
<th>No. of pixels in connected comp</th>
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15 connected components
Next Lecture

• Image segmentation

• Reading
  – Chapter 10: Image segmentation I: edge detection, thresholding, and region detection
    • Sections 10.1, 10.2, and 10.3