Implicit: Qualification Explicit: Enumeration

Convex Set

$\{x \mid Ax \le b, x \in R^n\}$

$\{U\theta \mid \sum \theta_i = 1, \theta_i \ge 0 \ \forall i\}$

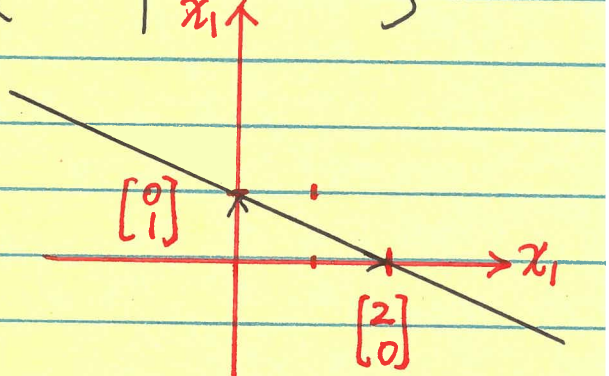
Affine Set

$\{x \mid Ax = b, x \in R^n\}$ (A: $m \times n$, x: $n \times 1$)

$\{U\theta \mid \sum \theta_i = 1\}$

Ex.

$\left\{ \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} \middle| x_1 + 2x_2 = 2 \right\}$



Ex. $\left\{ \begin{bmatrix} 2 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} \theta_1 \\ \theta_2 \end{bmatrix} \middle| 1^T\theta = 1, \theta \ge 0 \right\}$

Cone

$\{x \mid Ax \le 0, x \in R^n\}$

$\{U\theta \mid \theta_i \ge 0 \ \forall i\}$

Ex

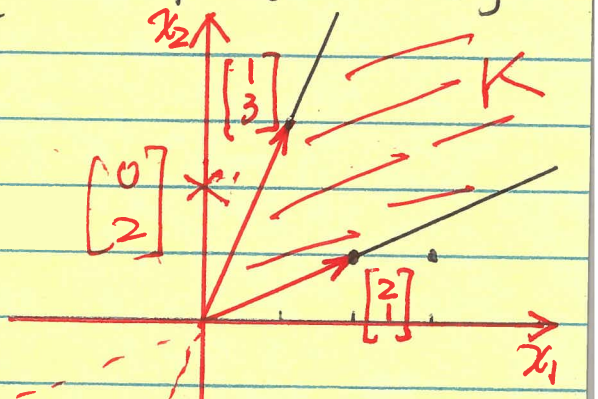
$\left\{ \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} \middle| \begin{bmatrix} -3 & 1 \\ 1 & -2 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} \le \begin{bmatrix} 0 \\ 0 \end{bmatrix} \right\}$

$-3x_1 + x_2 \le 0$

$\Rightarrow 3x_1 \ge x_2$

$x_1 - 2x_2 \le 0$

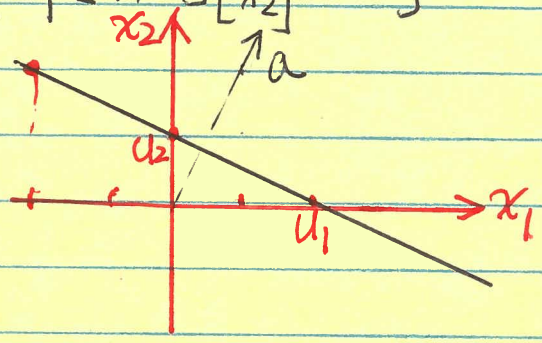
$x_1 \le 2x_2$



Ex. $\left\{ \begin{bmatrix} 2 & 1 \\ 1 & 3 \end{bmatrix} \begin{bmatrix} \theta_1 \\ \theta_2 \end{bmatrix} \middle| \theta_1, \theta_2 \ge 0 \right\}$

Hyperplane $\{x \mid a^T x = b,\ x \in R^n\}$ $\{U\theta \mid 1^T\theta = 1,\ U \in R^{n \times n}\}$

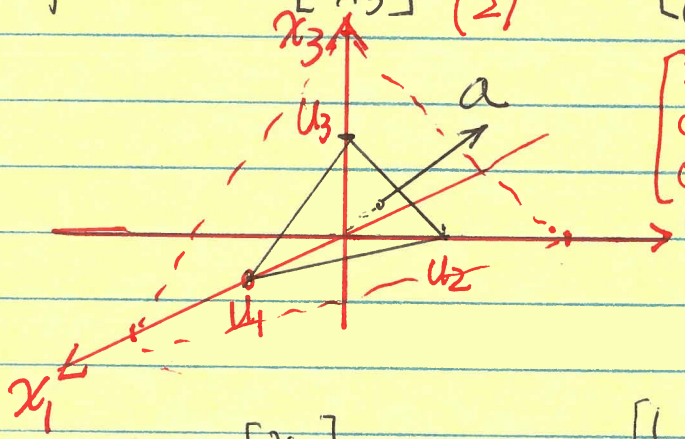
Ex. $\{x \mid [1,2]\begin{bmatrix}x_1\\x_2\end{bmatrix} = 2\}$ $\left\{\begin{bmatrix}2&0\\0&1\end{bmatrix}\begin{bmatrix}\theta_1\\\theta_2\end{bmatrix} \mid \theta_1+\theta_2=1\right\}$



$-1 \quad 2$

$-1\begin{bmatrix}2\\0\end{bmatrix}+2\begin{bmatrix}0\\1\end{bmatrix}=\begin{bmatrix}-2\\2\end{bmatrix}$

$\{x \mid [1,1,1]\begin{bmatrix}x_1\\x_2\\x_3\end{bmatrix} = 1\}$ (2) $\left\{\begin{bmatrix}1&0&0\\0&1&0\\0&0&1\end{bmatrix}\begin{bmatrix}\theta_1\\\theta_2\\\theta_3\end{bmatrix} \mid \theta_1+\theta_2+\theta_3=1\right\}$



$\begin{bmatrix}2&0&0\\0&2&0\\0&0&2\end{bmatrix}\begin{bmatrix}1&0&0\\0.5&2&0\\0.5&0&2\end{bmatrix}$

$\{x \mid [1,1,1,1]\begin{bmatrix}x_1\\x_2\\x_3\\x_4\end{bmatrix} = 1\}$ $\left\{\begin{bmatrix}1&0&0&0\\0&1&0&0\\0&0&1&0\\0&0&0&1\end{bmatrix}\begin{bmatrix}\theta_1\\\theta_2\\\theta_3\\\theta_4\end{bmatrix} \mid \Sigma\theta_i=1\right\}$

Freedom of n-1 degrees

intersect

union