

$$H = \left\{ x \mid a^T (x - x_0) = 0 \right\}$$

$x \in \mathbb{R}^n, x_0 \in H$

$Ax \leq b$

$a_1^T x \leq b_1$
 $a_2^T x \leq b_2$
 \vdots
 $a_m^T x \leq b_m$

(1). Degree of freedom H : $n-1$

(Simplest nontrivial) exp of \mathbb{R}^n space.

(2). H separate the space into 3 parts.

$$S_1 = \{x \mid a^T (x - x_0) > 0\}$$

$$S_2 = \{x \mid a^T (x - x_0) < 0\}$$

$H \cup S_1 \cup S_2$ whole space.

(3). H, S_1, S_2 all are convex

(4). $\forall y \in H, a^T (y - x_0) = 0.$

$\forall y \in S_1, a^T (y - x_0) > 0.$

$\forall y \in S_2, a^T (y - x_0) < 0.$

(5). $\tilde{H} = \left\{ x \mid \begin{bmatrix} a_1^T \\ a_2^T \end{bmatrix} x = 0, x \in \mathbb{R}^n \right\}$

$$S_4 = \left\{ x \mid \begin{bmatrix} a_1^T \\ a_2^T \end{bmatrix} x < 0, x \in \mathbb{R}^n \right\}$$

Cone: $S_5 = \left\{ [a_1, a_2] \begin{bmatrix} \theta_1 \\ \theta_2 \end{bmatrix} \mid \theta_1, \theta_2 \geq 0 \right\}$

Given a $y \in S_5, y^T x \leq 0, \forall x \in \tilde{H} \cup S_4$