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Title

On Optimization of Iso-spectralization.

Motivation

The problem of optimization involving the spectrum of a matrix has been historically important for both theoretical and practical reasons. Given some matrix-value function of the free variables, the problem is to minimize an objective that involves the spectrum of the function or subject to constraints related to the spectrum. For instance, rank minimization and low rank approximation problem of matrices can be formulated via relaxation as convex problems that involve the spectrum. In our research project, we work on the problem of shape recovery of geometric objects; specifically, the problem can be formulated as an optimization problem over the Laplacian spectrum of the graphs that represent the shapes. We plan to make improvement over the optimization process proposed by [1].

Background

Whether one can recover the shape of an object from its Laplacian spectrum is a classical problem in spectral geometry. Although the spectrum carries many geometric and topological information, it is still impossible to recover the full metric from it. Counter-examples of non-isometric isospectral manifolds have been constructed in 1960s. The theoretical existence of these counter-examples cannot preclude the possibility of reconstruction from spectrum in practice, which is exactly what we will explore.

[1] introduces a numerical procedure called iso-spectralization, which consists of deforming a mesh in order to align a given Laplacian spectrum of the target shape. This technique can be widely used in some cutting-edge topics of computer vision and graphics, such as style and pose transfer across objects. To be specific, the solution is to apply iso-spectralization on the source shape to align the eigenvalues with the target shape, and the resulting shape has the similar pose with the source shape, and similar geometric details with the target shape.

Recently, there have been attempts at reconstructing 3D shapes from full Laplacian matrix or other intrinsic operators [3, 4]. Such methods differ from ours since they leverage the complete information encoded in the input operator matrix, while we only have the operator's eigenvalues as input. What's more, in computer graphics, several shape modeling methods involve the known mesh connectivity and additional extrinsic information, such as user-provided landmarks and hand-crafted features, while our method doesn't rely on them.

There are additional methods that related to our approach that explore the possibility of reconstructing shapes from their spectrum in the case of coarsely triangulated surfaces, such as [5, 6]. But [5] and [6] study shapes with a low number of degrees of freedom, while our methods allow every vertex in the mesh can move.

Statement of problem

A mesh could be represented by a graph $X = \langle V, E, F \rangle$. The Laplacian spectrum of X is defined as the eigenvalues of the Laplacian of V . The discrete Laplacian operator is denoted as $\Delta_X(V)$.

Given a mesh $X = \langle V, E, F \rangle$, and the Laplacian eigenvalues μ of the target mesh Y , iso-spectralization is defined as the following optimization problem

$$\min_{V \in R^{n \times d}} \|\lambda(\Delta_X(V)) - \mu\|_w + \rho_X(V)$$

where $\lambda(\cdot)$ is the operator to calculate eigenvalues ordered by the magnitude and $\rho_X(V)$ is a regularizer over V ; $|V| = n$, d is the dimension of vertex, say, 2 for a plane and 3 for a point cloud. Notice that $\Delta_X(V)$ is a non-linear and non-convex operator of V .

Data

Random Graph/Mesh

FAUST inter-subject dataset [7]

ShapeNet [8]

Plan and Innovative Methods

1. Implement iso-spectralization according the paper [1]. Test our algorithm on synthetic 2D flat shapes and surfaces.
2. Improve optimization methods. The authors of [1] resort to modern deep learning tools, like Tensorflow, to solve the highly non-linear optimization problem directly.

There are several directions to explore:

- a. Decompose the original problem into two optimization problems. And alternative optimization might be applied.
 - i. Norm optimization: $\min_{L_X \in R^{n \times n}} \|\lambda(L_X) - \mu\|_w$, which is a convex optimization problem.
 - ii. Non-convex optimization: $\min_{V \in R^{n \times d}} \|\lambda(\Delta_X(V)) - L\|_w + \rho_X(V)$
 - b. The choices of regularizer $\rho_X(V)$ to ease the optimization.
 - c. The convex approximation of $\Delta_X(V)$
3. Apply the modified version to point clouds apart from meshes.

Task Assignment: TBD

Reference

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