CSE 203B W21 Homework 3

Due Time : 11:50pm, Wednesday Feb. 2, 2022 Submit to Gradescope
Gradescope: `https://gradescope.com/`

In this homework, we work on exercies from text book including level sets of convex, concave, quasi-convex, quasi-concave functions (3.1, 3.2), second-order conditions for convexity on affine sets (3.9), Kullback-Leibler divergence (3.13), saddle points of convex-concave functions (3.14) determination of convex, concave, quasi-convex, quasi-concave functions (3.16), conjugate functions (3.36), and gradient and Hessian of conjugate functions (3.40). Extra assignments are given on conjugate function and compressed sensing using different norms.

Total points: 30. Exercises are graded by completion, assignments are graded by content.

I. Exercises from textbook chapter 3 (8 pts, 1pt for each problem)

3.1, 3.2, 3.9, 3.13, 3.14, 3.16, 3.36, 3.40.

II. Assignments (22 pts)

II. 1. Conjugate Functions. (4 pts)

Find the conjugate functions of the following functions.

(1) $f(x) = x^T Ax + b^T x$, where $x \in R^n, A \in R^{nn}, b \in R^n$.
Discuss your solution for the following three cases: i. matrix A is symmetric and positive definite, ii. A is positive definite but not symmetric, iii. A is not positive definite.

(2) $f(x) = \begin{cases} \frac{1}{2}x^T x, & for\ ||x||_2 \leq 1, \\ (x^T x)^{1/2} - \frac{1}{2}, & for\ ||x||_2 > 1, \end{cases}$ where $x \in R^n$.

II. 2. Compressed Sensing. (18 pts)

For compressed sensing, we formulate the sensed signal y_t with an equation $y = Ax + b$, where $y \in R^m, A \in R^{mn}, x \in R^n, b \in R^m$. We have vector y contains the input (hw3_signal.txt), which is generated by a sparse linear combination of sinusoidal waves, $y_t \approx \sum_k a_{2k} sin(2\pi f_k t) + a_{2k+1} cos(2\pi f_k t)$, matrix A is a dictionary of *sine* and *cosine* patterns and your job is to figure out the coefficients a_k using vector x with noise b. It is known that the data is a 5 second data sampled at 1k Hz. Also, the frequencies (in the unit of Hertz) are in the set of $\{i | i \in \mathbb{N}, 1 \leq i \leq 100\}$.

Please formulate the problem into an optimization problem that minimizes the sum of the squared L_2 norm of the noise b and the L_α norm of the frequency components x. Once you derive the formulation, use the convex optimization programming tool to compute the numerical values of the amplitudes and figure out the frequencies of the signal.

II.2.1. Work on the following with $\alpha = 2$, e.g. $min_{x,b} ||x||_{\alpha=2} + \lambda ||b||_2^2$, such that $y = Ax + b$. (6 pts)
(i) Write the formulation. Note that we need a weight λ to balance between the squared L_2 norm of the noise and the L_α norm of the frequency components in the objective function.
(ii) Show your results with three or more samples of weight λ.
(iii) Show and explain your best choice of the weight.

II.2.2. Repeat the items of 1 with $\alpha = 1$. (6 pts)
II.2.3. Try to repeat the items of 1 with $\alpha = 0.5$. Explain your solution. (6 pts)
Hint: You should get three or more frequency components in your solution.
ps: CVX is a Matlab-based modeling system for convex optimization created by Professor Stephen Boyd. For more details, please refer to the tutorial for CVX:
`https://web.stanford.edu/class/ee364a/lectures/cvx_tutorial.pdf`