CSE 203B W22 Homework 2

Due Time : 11:50pm, Wednesday, Jan. 26, 2022. Submit to Gradescope
Gradescope: `https://gradescope.com/`

In this homework, we work on exercises from the textbook including midpoint convexity (2.3), Voronoi diagram (2.7, 2.9), quadratic function (2.10), general sets (2.12), cones and dual cones (2.28, 2.31, 2.32), and separation of cones (2.39). Extra assignments are given on convex sets.
Total points: 30. Exercises are graded by completion, assignments are graded by content.

I. Exercises from textbook chapter 2 (9 pts, 1pt for each problem)

2.3, 2.7, 2.9, 2.10, 2.12, 2.28, 2.31, 2.32, 2.39.

II. Assignments (21 pts)

II. 1. Qualification vs. enumeration of convex sets:
Given

$$A = \begin{bmatrix} 1 & 0 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & 0 & 1 \end{bmatrix},$$

$$b^T = \begin{bmatrix} 1 & 2 & 1 \end{bmatrix},$$

we describe the convex sets as follows.
II.1.1. Convert set $\{x|Ax \leq b, x \in R_+^6\}$ from a qualification oriented expression to an enumeration oriented expression in the format of $\{U\theta|1^T\theta = 1, \theta \in R_+^m\}$. (4 pts)
II.1.2. Convert set $\{x|Ax = 0, x \in R^6\}$ from a qualification oriented expression to an enumeration oriented expression in the format of $\{Px|x \in R^6\}$. (4 pts)
II.1.3. Derive the dual cone of the set $\{x|Ax \leq 0, x \in R^6\}$. (2 pts)

II. 2. Support vector machine (SVM): Given two sets of points $C = \{x_1, ..., x_c\}$ and $D = \{y_1, ..., y_d\}$, where $x_i, y_i \in R^n$, we find a hyperplane with vector $a \in R^n$ and bias $b \in R$ to minimize the following objective function.

$$\min ||a||_2^2, \ a \in R^n \tag{1}$$
$$\text{s.t. } a^T x_i \leq -1, \quad i = 1, ..., c \tag{2}$$
$$a^T y_i \geq 1, \quad i = 1, ..., d \tag{3}$$

(1) State the conditions with which the above formulation can have valid solutions. (2 pts)
(2) Rewrite equations 2 & 3 as two simultaneous linear inequalities in matrix format. (2 pts)
(3) Use enumeration oriented format to express the two convex hulls of sets C and D. (2 pts)
(4) Create a numerical example with $c = 5, \ d = 4, \ n = 2$. (2 pts)

(5) Use a nonlinear programming package (e.g. Matlab) to derive the solution of item (4). Demonstrate your result with a two-dimensional plot. (3 pts)