**CSE203B Convex Optimization Midterm Examination, 2/18/2020** Name ____________

# 1 True or False

Circle your choice of true or false. Use a short sentence to explain your choice. (20 points)

1. The union of two convex sets is convex.
   True False

2. Given two convex sets $S_1$ and $S_2$ in the same domain, then set $S_3 = \{x|x \in S_1, x \notin S_2\}$ is convex.
   True False

3. Given two convex sets $S_1, S_2 \subset R^n$, then set $S_3 = \{x_1 - x_2|\ x_1 \in S_1,\ x_2 \in S_2\}$ is also convex.
   True False

4. Function $f(x) = -xlogx,\ x \in R_{++}$ is a convex function
   True False

5. The conjugate function $f^*(y)$ is convex even if function $f(x)$ is not convex.
   True False

6. Function $f(x) = log\sum_{i=1:n} e^{a_i \times x_i}$, where $a_i$ and $x_i \in R$ for $i = 1, ..., n$, is convex.
   True False

7. The equation $x_1^3 x_2^{-1} + x_3^2 x_4^5 = 0$ for $x \in R_+^4$ can be converted into a linear equality constraint using the standard geometric programming formulation.
   True False

8. In geometric programming, a posynomial function $f(x)$ may not be convex, but can be converted into convex form.
   True False

9. In second order cone programming, the set $\{x|||Ax + b||_2 \le c^T x + d\}$, where $A \in R^{mn}$, $x, b, c \in R^n$ and $d \in R$, is convex.
   True False

10. The inequality $sup_{z \in Z} inf_{w \in W} f(w,z) \le inf_{w \in W} sup_{z \in Z} f(w,z)$ is true even when function $f(w,z)$ is not convex.
    True False

## 2 Theorems and Proofs

**Problem 2.1** Prove the following optimality criterion for a convex optimization problem. Suppose that the problem is convex and the objective function $f_0(x)$ is differentialble, prove that $\bar{x}$ is optimal if and only if $\bar{x}$ is feasible and $\nabla f_0(\bar{x})^T(y-\bar{x}) \geq 0$ for all feasible $y$. (10 points)

**Problem 2.2** Prove that the Lagrange dual function is concave even if the primal problem is not convex. (10 points)

## 3 Case Studies

**Problem 3.1** Dual Cone: Given a cone $K = \{\theta_1 u_1 + \theta_2 u_2 \mid u_1 = [2,-1,3]^T,\ u_2 = [-2,1,0]^T,\ \theta_1 \geq 0, \theta_2 \geq 0\}$, find the dual cone of $K$. (15 points)

**Problem 3.2** Conjugate Function: Given a function $f(x) = x_1 + 2x_2 + 3x_3^2,\ x \in R^3$, derive the conjugate function $f^*(y), y \in R^3$. (15 points)

**Problem 3.3** Primal Dual Formulation: Given a linear programming problem,

minimize $f_0(x) = c^T x$

subject to $Ax \leq b$, and $Px = q$, where $x \in R^n_+$ (i.e. $x \succeq 0$).

Derive the dual problem. (10 points)

# 4 Problems from Exercises

**Problem 4.1** Let $C \subset R^n$ be the solution set of a quadratic inequality, $C = \{x \in R^n | x^T Ax + b^T x + c \leq 0\}$, with $A \in S^n, b \in R^n$, and $c \in R$.
Prove that $C$ is convex if $A \succeq 0$. (10 points)

**Problem 4.2** Prove that the following function is convex. (10 points)
$f(x) = 1/(x_1 x_2)$, where $x \in R^2_{++}$.