CSE291 Convex Optimization: Review for Midterm Examination

1 Theorems and Proofs

We will test theorem statements and proofs. The following is a list of sample items.
Separating hyperplane theorem
First-order condition of convex function
Second-order condition of convex function
Convexity of Pointwise maximization of a set of convex functions
Convexity of minimization of a convex function $f(x,y)$ with respect to a subset of variables y in a convex set C
Concavity of the Lagrange dual function
KKT optimality conditions

Here are a few sample problems.
Problem State and prove the separating hyperplane theorem
Problem Show that conjugate function $f^*(y)$ can be used to identify a supporting hyperplane of the epigraph of the original function $f(x)$.
Problem Show that the dual function yields lower bounds on the optimal value p^* of the primal problem, i.e. for any Lagrange multipliers $\lambda \geq 0$ and any ν, we have the dual function,

$$g(\lambda,\nu) \leq p^*. \tag{1}$$

2 Case Studies

We go through a few cases to clarify the concept.
Problem Supporting Hyperplane: Given a set $\{x \mid x_1^2 + 2x_2^2 \leq 9\}$, find a supporting hyperplane at point $[x_1, x_2] = [1,2]$.
Problem Dual Cone: Given a cone $K = \{\theta_1 u_1 + \theta_2 u_2 \mid u_1 = [1,1]^T, u_2 = [0,1]^T, \theta_1 + \theta_2 = 1, \theta_1 \geq 0, \theta_2 \geq 0\}$, find the dual cone of K.
Problem Conjugate Function: Given a function $f(x) = x_1 x_2, x \in R_+^2$, find the dual function $f(y)^*, y \in R^2$.
Problem Primal Dual Formulation: Given a linear programming problem,
minimize $f_0(x) = c^T x$
subject to $Ax \leq b$, and $Cx = d$, where $x \in R^n$.
Derive the dual problem formulation.

3 Problems from Exercises and Other Sources

We may use problems from exercises and other sources. Here are some samples.
Problem exercise 3.13
Problem exercise 4.8