

1 True or False

Circle your choice of true or false. Use one sentence to explain your choice. (10 points)

1. The union of two convex sets is convex.

True False

False, a counterexample can be provided

2. Function $f(x) = \log \sum_{i=1}^n e^{a_i x_i}$, where $x \in \mathbb{R}^n$ and $a_i \in \mathbb{R}$ for $i = 1, \dots, n$, is convex.

True False

True, second order derivative of $f(x)$ is PSD

3. The dual problem of a nonconvex primal problem is a convex optimization problem.

True False

True/False, both are correct. It depends on how one interprets this problem.

4. Geometric Programming: A local minimal solution of an optimization problem minimizing a posynomial function is a global minimal solution even without the transformation into a convex problem (\log).

True False

False, a nonconvex example can be found.

5. The figure below shows some level sets of a function f . The labels indicate the value of $f(x)$ at the curve. Could f be convex?

True False False, we can see it is not convex with the approach used in Exercise 3.2.

Grading Rubrics

One point is taken off for each wrong choice.

2 Theorems and Proofs

Problem 2.1 State and prove the first-order condition of convex functions. (20 points)

Please refer to the class note for the answer.

Grading Rubrics

Five points are taken off for missing necessary or sufficient condition.

Two points are taken off if the statement is incorrect.

One point is taken off for minor mistake.

Problem 2.2 Given an arbitrary function $f(w, z)$ for $w \in W, z \in Z$, prove that $\max_{w \in W} \min_{z \in Z} f(w, z) \leq \min_{z \in Z} \max_{w \in W} f(w, z)$. (10 points)

Please refer to the class note for the answer.

3 Case Studies

Problem 3.1 Dual Cone: Given a cone $K = \{\theta_1 u_1 + \theta_2 u_2 \mid u_1 = [1, -1]^T, u_2 = [0, 1]^T, \theta_1 \geq 0, \theta_2 \geq 0\}$, find the dual cone of K . (10 points)

$$K^* = \{\theta_1 v_1 + \theta_2 v_2 \mid v_1 = [1, 1]^T, v_2 = [1, 0]^T, \theta_1 \geq 0, \theta_2 \geq 0\}$$

Grading Rubrics

Two points are taken off for wrong answer.

One point is taken off for minor mistake.

Problem 3.2 Conjugate Function: Given a function $f(x) = x_1^2 x_2$, $x \in \mathbb{R}_+^2$, find the dual function $f^*(y)$, $y \in \mathbb{R}^2$. (20 points)

Since $\nabla_x^2 f^*(y) = \begin{bmatrix} -2x_2 & -2x_1 \\ -2x_1 & 0 \end{bmatrix}$ is not PSD,

$$\begin{cases} y \preceq 0, f^*(y) = \lim_{t \rightarrow 0} t = 0 \\ \text{otherwise, } \infty \end{cases} \quad (1)$$

Grading Rubrics

As long as the section is not left blank, not a single point is taken off for wrong answer.

Problem 3.3 Primal Dual Formulation: Given a linear programming problem,

$$\text{minimize } f_0(x) = c^T x$$

$$\text{subject to } Ax \leq b, \text{ and } Px = q, \text{ where } x \in \mathbb{R}_+^n.$$

Derive the dual problem formulation. (10 points)

Use Lagrangian method and we get

$$\text{maximize } f_0(x) = -\lambda^T b - v^T q$$

$$\text{subject to } c + A^T \lambda + P^T v = 0, \text{ and } \lambda \geq 0$$

Grading Rubrics

For each missing condition, three points are taken off.

One point is taken off for minor mistake.

4 Problem from Exercises

Problem 4.1 Demonstrate (give proof) if the following function is either convex or not.

$f(x_1, x_2) = e^{x_1} + x_1^2/x_2$ where $x_1 \in \mathcal{R}$, $x_2 \in \mathcal{R}_{++}$. (10 points)

The function is convex. Please refer to Exercise 3.16 for details.

Grading Rubrics

Two points are taken off for incorrect solution.

5 Feedback of the Class

Problem 5.1 Provide one sentence suggestion to improve the class. (10 points)

Grading Rubrics

Any feedback worths full score.