CSE203B - Discussion Session

Po-Ya Hsu

02/05/21

Outline

- Convex Optimization
- CVX Basics
- Assignment Hints

Convex Optimization Problem in Standard Form

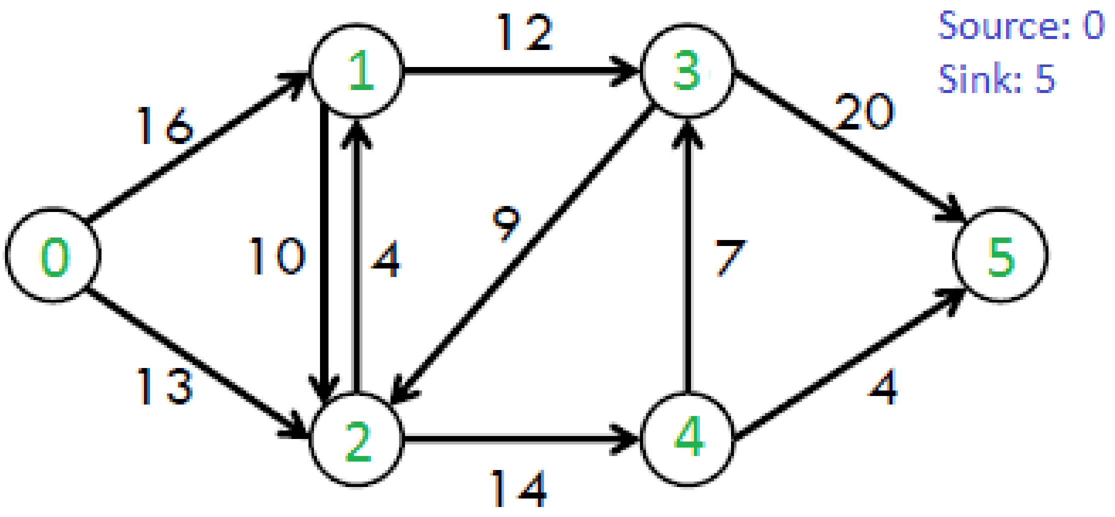
minimize $\quad f_0(x)$

Subject to $\quad f_i(x) \leq 0, i = 1, \dots, m$

$$a_i^T x = b_i, i = 1, \dots, p$$

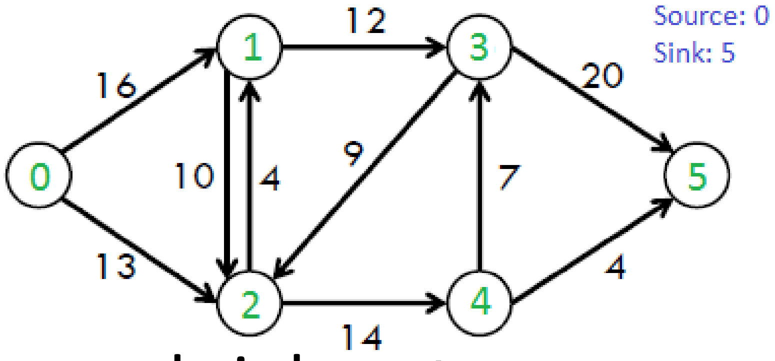
- The objective function $f_0(x)$ must be convex
- The inequality constraints $f_i(x)$ must be convex
- The equality functions $h_i(x) = a_i^T x - b_i$ must be affine.
- The feasible set of a convex optimization problem is also convex.

Example: Max-Flow Problem



- Source: node 0
- Sink: node 5
- Objective: maximize the flow from the source to the sink
- Constraints: flow capacity, conservation of flow

Example: Max-Flow Problem



- Denote the graph as $G(V, E)$, flow as f, source as s, and sink as t
- Objective function: maximize $\sum_{v:<s,v>\in E} f(s, v)$
- Constraints:
  1. capacity: $f(u, v) \leq c(u, v) \& f(u, v) \geq 0 \; for \; all \; v \in V - \{s, t\}$
  2. Conservation of flow

$$\sum_{<u,v>\in E} f(u, v) = \sum_{<v,w>\in E} f(v, w), \; for \; all \; v \in V - \{s, t\}$$

CVX Programming

- User Guide: http://web.cvxr.com/cvx/doc/
- CVXPY: https://www.cvxpy.org/

CVX Basics

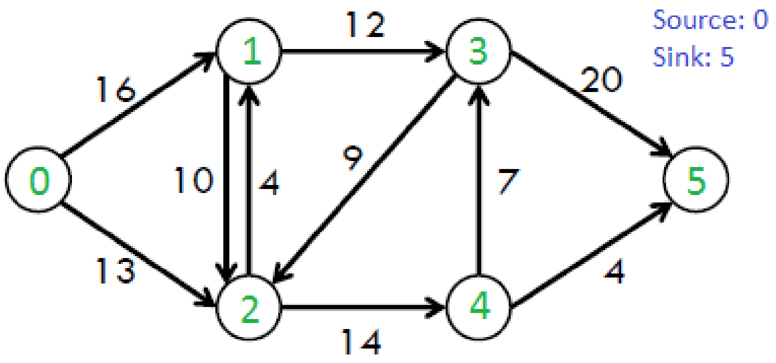
- To solve any convex optimization problem using CVX, your code should follow the structure shown below:

```
cvx_begin
        variable declaration
        objective function
        constraints
cvx_end
```

Revisit the Max-Flow Problem

- Find the solution using CVX
    1. Formulate the problem into a convex optimization problem (check slide 5)
    2. Instantiate the problem data
    3. Construct the convex optimization problem following CVX structure
    4. Run the script

Revisit the Max-Flow Problem

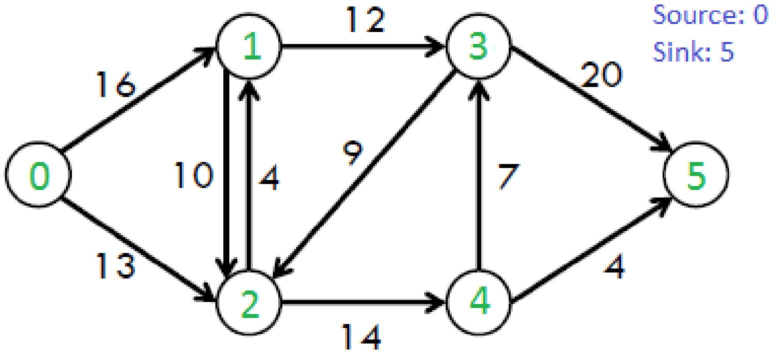


- Find the solution using CVX
  1. Formulate the problem into a convex optimization problem (check slide 5)
  2. **Instantiate the problem data**
  3. Construct the convex optimization problem following CVX structure
  4. Run the script

```
% capacity constraints
C = [16;13;10;4;12;9;14;7;20;4];

% conservation of flow
cf1 = [0,0,0,0,1,-1,0,1,-1,0];
cf2 = [0,0,0,0,0,0,-1,1,0,1];
cf3 = [0,1,1,-1,0,1,-1,0,0,0];
cf4 = [1,0,-1,1,-1,0,0,0,0,0];
```

Revisit the Max-Flow Problem



- Find the solution using CVX
  1. Formulate the problem into a convex optimization problem (check slide 5)
  2. Instantiate the problem data
  3. **Construct the convex optimization problem following CVX structure**
  4. Run the script

```
% cvxopt

cvx_clear;

n = 10;

cvx_begin
    variable x(n)
    maximize( x(1) + x(2)  )
    subject to
        -x <= 0;
        x <= C;
        cf1 * x == 0;
        cf2 * x == 0;
        cf3 * x == 0;
        cf4 * x == 0;
cvx_end
```

Revisit the Max-Flow Problem

- Find the solution using CVX
  1. Formulate the problem into a convex optimization problem (check slide 5)
  2. Instantiate the problem data
  3. Construct the convex optimization problem following CVX structure
  4. **Run the script**

```
>> run_max_flow

Calling SDPT3 4.0: 21 variables, 7 equality constraints
   For improved efficiency, SDPT3 is solving the dual problem.
------------------------------------------------------------

 num. of constraints =  7
 dim. of linear var  = 20
 dim. of free   var  =  1 *** convert ublk to lblk
*******************************************************************
   SDPT3: Infeasible path-following algorithms
*******************************************************************
 version  predcorr  gam  expon  scale_data
    NT      1      0.000   1        0
it pstep dstep pinfeas dinfeas  gap      prim-obj      dual-obj    cputime
-------------------------------------------------------------------
 0|0.000|0.000|6.7e-01|3.4e+00|7.5e+03| 1.090000e+03  0.000000e+00| 0:0:00| chol  1  1
 1|1.000|0.927|2.1e-06|2.6e-01|1.1e+03| 7.201429e+02 -1.569037e+01| 0:0:00| chol  1  1
 2|0.913|0.482|1.1e-06|1.4e-01|3.7e+02| 2.501349e+02 -1.852095e+01| 0:0:00| chol  1  1
 3|0.865|0.406|1.4e-07|8.0e-02|1.2e+02| 7.640093e+01 -1.739564e+01| 0:0:00| chol  1  1
 4|0.890|0.921|1.4e-07|6.3e-03|1.8e+01| 3.392250e+00 -1.441298e+01| 0:0:00| chol  1  1
 5|1.000|0.460|2.2e-08|3.4e-03|7.7e+00|-4.041973e+00 -1.146983e+01| 0:0:00| chol  1  1
 6|1.000|0.873|6.4e-08|4.3e-04|7.8e-01|-7.528634e+00 -8.287142e+00| 0:0:00| chol  1  1
 7|0.979|0.965|2.3e-09|1.5e-05|2.2e-02|-7.986511e+00 -8.007717e+00| 0:0:00| chol  1  1
 8|0.988|0.988|2.7e-10|1.8e-07|2.6e-04|-7.999843e+00 -8.000090e+00| 0:0:00| chol  1  2
 9|0.989|0.989|2.7e-11|1.9e-07|8.6e-06|-7.999998e+00 -8.000001e+00| 0:0:00| chol  1  1
10|0.996|0.989|1.1e-10|6.4e-09|3.0e-07|-8.000000e+00 -8.000000e+00| 0:0:00| chol  2  2
11|1.000|0.989|4.0e-12|2.2e-10|1.1e-08|-8.000000e+00 -8.000000e+00| 0:0:00|
```

```
  stop: max(relative gap, infeasibilities) < 1.49e-08
-------------------------------------------------------------------
 number of iterations   = 11
 primal objective value = -8.00000000e+00
 dual   objective value = -8.00000000e+00
 gap := trace(XZ)       = 1.05e-08
 relative gap           = 6.20e-10
 actual relative gap    = 1.92e-10
 rel. primal infeas (scaled problem)   = 4.00e-12
 rel. dual     "        "       "      = 2.25e-10
 rel. primal infeas (unscaled problem) = 0.00e+00
 rel. dual     "        "       "      = 0.00e+00
 norm(X), norm(y), norm(Z) = 2.2e+00, 1.2e+01, 3.4e+01
 norm(A), norm(b), norm(C) = 7.5e+00, 3.0e+00, 4.4e+01
 Total CPU time (secs)  = 0.32
 CPU time per iteration = 0.03
 termination code       =  0
 DIMACS: 6.0e-12  0.0e+00  5.1e-10  0.0e+00  1.9e-10  6.2e-10
-------------------------------------------------------------------

------------------------------------------------------------
Status: Solved
Optimal value (cvx_optval): +23
```

Assignment 1 - Hints

In a linear system,

$$Y = \Phi X$$
$$Y: d \times 1$$
$$\Phi: d \times S$$
$$X: S \times 1$$

When S>d, the system is underdetermined. However, if the sparsity of the sources X is guaranteed, then compressed sensing can be applied to reconstruct the original sources.

Assignment 1 - Hints

In this assignment,

$$Y = \Phi X + n$$
$$Y: data$$
$$\Phi: sinusoidal\ basis$$
$$X: variable, sparsity\ guaranteed$$
$$n: random\ noise$$

What you'll have to do:

1) Formulate the convex optimization problem
2) Use CVX or CVXPY to solve X
3) Experiment with the weight in your objective function

Clarification

- 1) the basis functions Φ: in general, when we consider a signal composed of K sinusoids, we express as $y(t) = a_0 + \sum_{k=1}^{K} a_k e^{-i2\pi f_k t}$. However, in the assignment, the basis function is simplified to$\sum \sin(2\pi f_k t)$
- 2) the objective function: minimize the errors in the data fitting and enforce the sparsity, so it should look like

$$minimize\ \alpha||?||_1 + ||? - ??||_2^2$$

- 3) you'll have the source as variables in your CVX programming
- 4) play with different weights α

Assignments 2

- Clarification
  - The minimum volume ellipsoid problem is not an SDP, because the objective is not linear
  - However, the log determinant is a convex function. So it can still be solved using an SDP solver

Hints

- Decompose R into $M^T M$ ($R = M^T M$)