CSE 105, Winter 2022 - Homework 6

Due: Monday (2/28/2022) 11:59 PM

Instructions

Upload a single file to Gradescope for each group. All group members’ names and PIDs should be on each page of the submission. Your assignments in this class will be evaluated not only on the correctness of your answers but on your ability to present your ideas clearly and logically. You should always explain how you arrived at your conclusions, using mathematically sound reasoning. Whether you use formal proof techniques or write a more informal argument for why something is true, your answers should always be well-supported. Your goal should be to convince the reader that your results and methods are sound.

Reading Sipser 3.1, 3.2, 4.1

Key Concepts TM
Problem 1 (10 points)

Let \( L = \{w \in \{a,b\}^* \mid |w|_a = 2^{|w|_b}\} \) where \( |w|_a \) denotes the number of occurrences of \( a \) in \( w \) and \( |w|_b \) denotes the number of occurrences of \( b \) in \( w \).

Provide a formal description of a Turing machine \( M \) which decides \( L \).

The transition function can be represented as a state diagram.

Problem 2 (10 points)

Show that a Turing machine with infinite tape in one direction is equivalent to a Turing machine with infinite tape in two directions. That is, show that if a language \( L \) has a TM deciding it in one model, then the same holds in the other model as well.

Problem 3 (10 points)

Prove the following statements

a) The class of decidable languages is closed under complementation  
b) The class of recognizable languages is closed under concatenation

Problem 4 (10 points)

The Hexagonal numbers, commonly denoted \( H_n \), form a sequence, where \( H_n = n^*(2n-1) \)

The beginning of the sequence is thus: 1, 6, 15, 28, 45, 66, 91, 120, 153, 190, 231…

\( L = \{1^n \mid n \text{ is a Hexagonal number and } n>0\} \). Construct a deterministic Turing Machine \( M \) that decides \( L \). Provide an implementation level description for \( M \).

Problem 5 (10 points)

Prove that a language is Turing-decidable if and only if some enumerator enumerates the strings of this language in lexicographic order.