CSE 105, Winter 2022 - Homework 3

Due: Monday (1/31/2022) 11:59 PM

Instructions

Upload a single file to Gradescope for each group. All group members’ names and PIDs should be on each page of the submission. Your assignments in this class will be evaluated not only on the correctness of your answers but on your ability to present your ideas clearly and logically. You should always explain how you arrived at your conclusions, using mathematically sound reasoning. Whether you use formal proof techniques or write a more informal argument for why something is true, your answers should always be well-supported. Your goal should be to convince the reader that your results and methods are sound.

Reading Sipser Sections 1.3 and 1.4

Key Concepts Regular expressions, equivalence of DFA and regular expressions, regular languages, the Pumping Lemma, pumping length, proofs of nonregularity
Problem 1 (10 points): Language of a RegEx

For each of the languages described by the regular expressions below, give an example of a string of length at least 5 that is in the language (if impossible, enumerate the language). Also, give a simplified English description of the language given by the expression.

a. \((1(0 \cup 1)^+0) \cup (0(1 \cup 0)^+1)\)

b. \((1 \cup 0)^*1(1 \cup 0)(1 \cup 0)\)

c. \((0) \cup ((1 \ 0 \ *) \ 00)\)

d. \(((1 \cup 0)(0 \cup 1)1)^+\)

e. \((1(1 \cup 0)) \cup ((0 \cup 010)^*(\ epsilon)^*0 \ 1^*0) \cup (0(\ epsilon \cup \ phi)^* (1 \cup 0))\)

Problem 2 (10 points): DFA to RegEx

a) Convert the following DFA into a regular expression. You may either do it by constructing the GNFA (Sipser section 1.3, Lemma 1.60, page 69), or using the inductive approach discussed in class. For full credit, show each step of your approach clearly.

![DFA Diagram]

b) We say that two regular expressions R and S are equivalent, if they describe the same language, i.e. \(L(R) = L(S)\). Give a smaller regular expression that is equivalent to the answer obtained in part a), i.e. your regular expression should have fewer symbols.
Problem 3 (10 points): Pumping Lemma

Use the pumping lemma to prove that the following languages are not regular:

a. $L = \{w \in \{a, b\}^* | n_a(w) = n_b(w)\}$, where $n_x(w)$ is the count of $x$ in $w$

b. $L = \{0^i1^j0^k | i, j, k \geq 1, i + j = 2k\}$

Problem 4 (10 points): Completion of proof using PL

In class we learned with examples that the string $s$ must be chosen carefully to complete the proof of non-regularity using the pumping lemma. In this question, you have to prove, for each instance below, whether the string $s$ completes the proof of non-regularity for language $L$ or not. Here, $p$ is the pumping length of $L$ assumed for the proof.

a. $L$ is the set of all palindromes over $\{0, 1\}$.
   $s = (10)^{p/2}1$ (assume $p$ is even)

b. $L = \{0^m1^n0^n | m, n \geq 0\}$
   $s = 0^p1100$

c. $L = \{0^m1^n | m, n \geq 0, m \leq n\}$
   $s = 0^{p/2}1^{p/2}$ (assume $p$ is even)

d. $L = \{0^m1^n0^n | m, n \geq 0, m \neq n\}$
   $s = 0^p1^{\text{LCM}(p,p-1,p-2,\ldots,2,1) + p}$
   where $\text{LCM}(x_1, x_2, \ldots, x_n)$ is the least common multiple of $x_1, x_2, \ldots, x_n$.

Problem 5 (10 points) Regularity

For each of the given languages, prove whether it is regular or not. If it is regular, give a regular expression for the language and justify your answer. If not, prove using the pumping lemma.

a. $L = \{wcw^R | w, c \in \{0, 1\}^+\}$
   b. $L = \{wcw^R | w, c \in \{0, 1\}^+, |c| = 3\}$